National Exams December 2019

17-Phys-A3, Electromagnetics

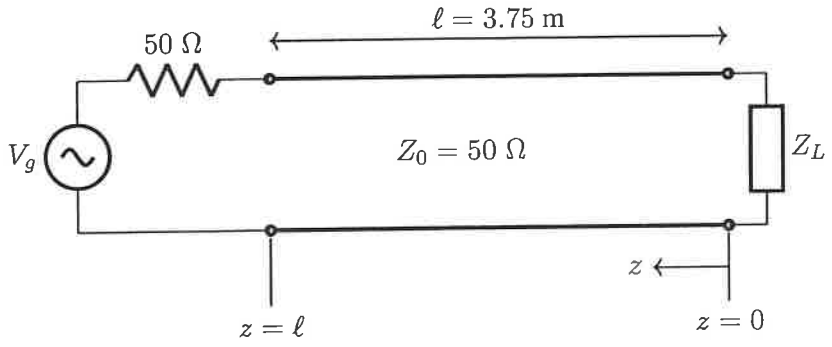
3 hours duration

NOTES:

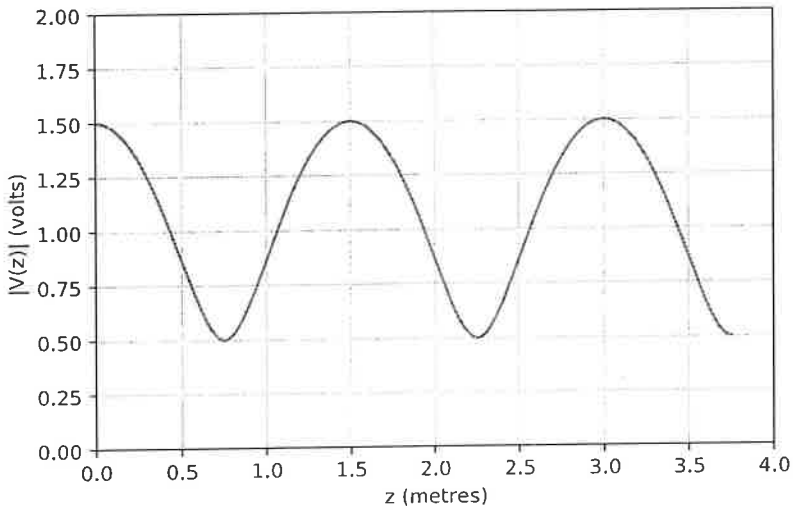
1. If doubt exists as to the interpretation of any question, the candidate is urged to submit clear statements of any assumptions made.
2. An approved Casio or Sharp calculator is permitted. This is a **closed book** exam.
3. FIVE (5) questions constitute a complete exam paper. The first five completed questions as they appear in the answer book will be marked.
4. Each question is of equal value.
5. Fully justify your answers. Inadequate justification will lead to only partial marks.

Question 1

An unknown complex load terminates a 50 Ω transmission line as shown in Figure 1(a). The frequency of the source is 50 MHz. When measuring the amplitude of the voltage as a function of distance from the load z, the graph shown in Figure 1(b) is observed.



(a) Terminated transmission line



(b) Measured voltage amplitude as a function of distance from the load z

Figure 1 Transmission line and measurement

(a) What is the speed of light along the line? **[4 marks]**

(b) What standing wave ratio is observed along the line? **[4 marks]**

(c) Determine the load impedance Z_L. **[6 marks]**

(d) What is the input impedance observed looking into the line by the source at $z = \ell$? **[6 marks]**

Question 2

A dielectric-dielectric interface shown in Figure 2. The medium in the region $z < 0$ is lucite ($\epsilon_r = 2.8$), while the medium in the region $z > 0$ is free space. Both media are lossless. A 2 GHz plane wave is incident as shown, having a polarization in the y-direction. The transmitted (refracted) and reflected electric fields are labelled $\mathbf{E}^t$ and $\mathbf{E}^r$, respectively.

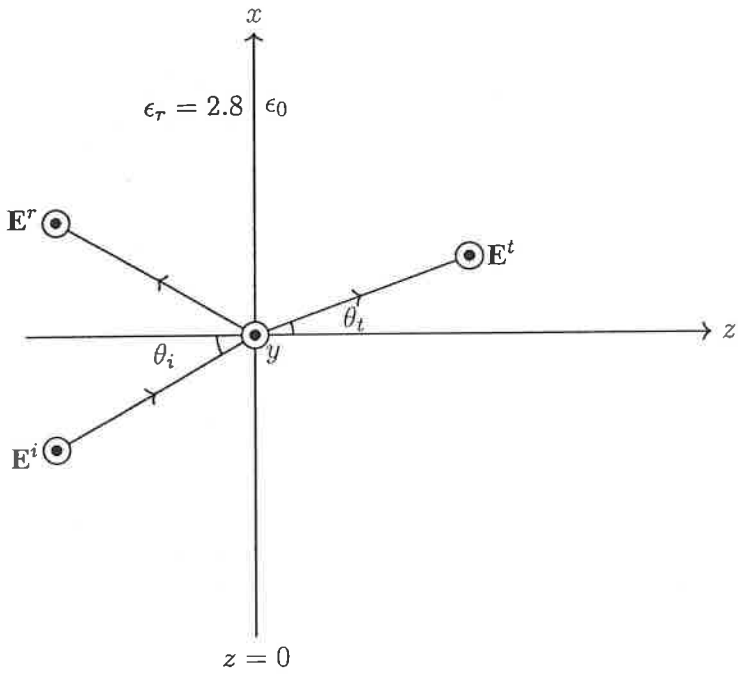


Figure 2 Dielectric media interface and plane waves

(a) If the angle of incidence is $\theta_i = 30°$, determine the angle of refraction, θ_t. **[4 marks]**

(b) Write a suitable time-domain expression for the incident electric field vector in part (a), if the incident electric field strength is $|\mathbf{E}^i| = 10$ V/m. Numerically evaluate all symbolic constants in the expression. **[4 marks]**

(c) If the angle of incidence is $\theta_i = 30°$, determine the reflection coefficient (E^r/E^i) and transmission coefficient (E^t/E^i) associated with the electric field. **[8 marks]**

(d) Determine the critical angle associated with this problem and explain what is special about this angle. **[4 marks]**

Question 3

A lossless transmission line is connected to a generator and load as shown in Figure 3. The line length is 2 m and the speed of light along the line is 2×10^8 m/s. Its characteristic impedance is 100 Ω.

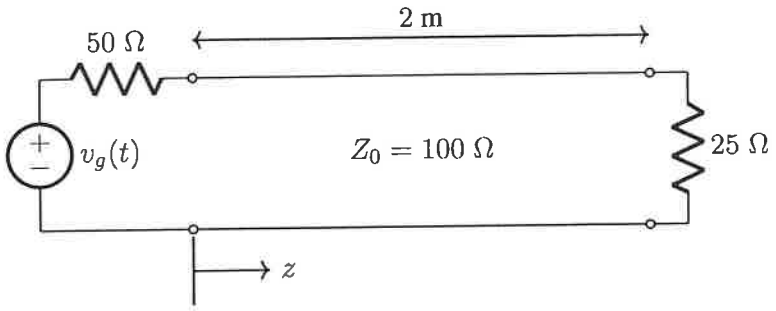


Figure 3 Transmission line circuit

(a) What is the one-way transit time through the transmission line? **[4 marks]**

(b) The generator voltage is described by $v_g(t) = 10u(t)$, where $u(t)$ is a unit step function. Graph the voltage at the following time instants, as a function of position z. In each graph, indicate the position of the leading or falling edge of the waveform, as well as its direction of travel (towards the source or towards the load). Clearly label all voltage levels in your graphs.

a) $t = 4$ ns; **[8 marks]**

b) $t = 14$ ns. **[8 marks]**

Question 4

A WR-90 rectangular waveguide has cross-sectional dimensions 22.86 mm × 10.16 mm. It is filled with free space.

(a) What is the mode with the lowest cut-off frequency ("dominant") mode of this waveguide? Calculate its cut-off frequency, in GHz. **[4 marks]**

(b) Determine the phase velocity of the waveguide at 10 GHz. **[4 marks]**

(c) Determine the guide wavelength at 10 GHz. **[4 marks]**

(d) Determine the wave impedance of the waveguide at 10 GHz. **[4 marks]**

(e) Calculate the cutoff frequencies of the first 3 higher order modes of the waveguide and identify whether they are TE, TM, or both. **[4 marks]**

Question 5

A time-varying magnetic flux density is described by

$$\mathbf{B}(t) = 0.2e^{5t}\mathbf{a}_z \text{ [T]}.$$

A circular wire loop of radius $\rho = 40$ cm is placed in xy-plane.

(a) Determine an expression for the electromotive force induced in the wire loop, as a function of time. **[6 marks]**

(b) Determine the curl of the electric field, $\nabla \times \mathbf{E}$, as a function of time. **[4 marks]**

(c) If the wire has a resistance of 10 Ω, determine a function describing the current induced in the wire as a function of time. **[4 marks]**

(d) Sketch a diagram of the loop of wire and clearly show the direction the current flows with respect to the B-field. **[3 marks]**

(e) If the magnetic flux density is instead described by

$$\mathbf{B}(t) = 0.2e^{5t}\mathbf{a}_x \text{ [T]}$$

and everything else remains unchanged, repeat part (a). **[3 marks]**

Question 6

A parallel-plate capacitor is formed by sandwiching a lossless dielectric with dielectric constant $\epsilon_r = 10.6$ between two square perfectly conducting plates, of dimensions 4 mm $\times$ 4 mm. The plates are separated by 0.25 mm.

(a) Determine the capacitance of the capacitor formed by the plates and the dielectric. **[4 marks]**

(b) Determine the electric field strength between the plates, if a DC potential of 5 V is applied between the plates. Also determine the electric flux density established between the plates. **[6 marks]**

(c) Determine the surface charge density established on each plate of the capacitor, for the situation described in part (b). **[4 marks]**

(d) Determine how much i) RMS displacement current and ii) RMS conduction current flows in the capacitor if a 5 Vrms sinusoidal source is applied to the plates of the capacitor. **[6 marks]**

Question 7

A conducting spherical shell with a radius of 1 cm is charged such that a surface charge density is 20 μC/m^2 on the surface of the sphere. The sphere is placed at the origin and surrounded by free space.

(a) Determine the total charge on the sphere's surface. **[4 marks]**

(b) Derive and calculate, using Gauss' Law, the vector electric field **E** produced by the sphere in the free space region $r > 1$ cm, in a coordinate system of your choice. **[6 marks]**

(c) How much work is done moving a 1 nC test charge from a point (5 cm, 0, 0) to (3 cm, 0, 0)? **[6 marks]**

(d) What is the electric potential between the two points in part (c)? **[4 marks]**

Marking Scheme

1) a) 4 marks; b) 4 marks; c) 6 marks; d) 6 marks.

2) a) 4 marks; b) 4 marks; c) 8 marks; d) 4 marks.

3) a) 4 marks; b) i) 8 marks; ii) 8 marks.

4) a) 4 marks; b) 4 marks; c) 4 marks; d) 4 marks; e) 4 marks.

5) a) 6 marks; b) 4 marks; c) 4 marks; d) 3 marks; e) 3 marks.

6) a) 4 marks; b) 6 marks; c) 4 marks; d) 6 marks.

7) a) 4 marks; b) 6 marks; c) 6 marks; d) 4 marks.