May 2013

National Exams

04-Soft-A1 Algorithms & Data Structures

3 hours duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made;

2. This is a "Closed Book" exam. Candidates may use one of two calculators, the Casio or the Sharp approved models;

3. You can pick any 4 of the first five questions, but you have to solve questions 6,7, and 8.

4. The value of each question is stated.

Marking Scheme:

4 out of 5

1. 10 points (4,3,3)
2. 10 points (4,4,2)
3. 10 points (2; 2,2,2,2)
4. 10 points (2,5,3)
5. 10 points (3,4,3)

Required

6. 20 points (5,3,3,5,2,2)
7. 20 points (five out of 6 requested, 5 points each)
8. 20 points (5 points each)

1) Linked Lists (10 points)

You are asked to implement a linked list using pointers using the following structure in C:

```
struct list_el {
   int val;
   struct list_el * next;
};

typedef struct list_el item;
```

Implement the functions

```
int Insert(item** head, int insertMe)
int Delete(item** head, int deleteMe)
```

(note, the pointer to the list is passed as a var parameter) for

1. Singly-linked linked list
2. Sorted (smallest first) linked list
3. Circular linked list

2) Tree Implementation (10 points)

The array

```
int data[max_size]
```

was used to represent a binary tree containing integers.

1. If n is the index of an element, what is the index of the parent? Give a C or pseudo code implementation of the function
   ```
   int parent(int* data, int* out)
   ```
   which returns 0 if there is no parent and 1 if there is one, and the value of it in out.
2. If n is the index of an element, what is the index of the right child? Give a C or pseudo code implementation of the function
   ```
   int right_child(int* data, int* out)
   ```
   which returns 0 if there is no child and 1 if there is one, and the value of it in out.
3. If n is the number of total nodes in the tree, how many internal (non-leave) nodes doe the tree have?

3) Heap (10 points)

1) Give the definition of a max heap

2) We used an array to implement a max heap.
   I. Describe the process of adding a node to a max heap in 2-3 sentences
   II. Describe the process of deleting the element form a max heap in 2-3 sentences.
   III. Show pictures illustrating each step adding 4, 6, 3, 7, 1 to a max heap starting with an empty heap.
   IV. What is the complexity of adding a node to a max heap, what is the complexity of deleting from a max-heap? Give a 1-2 sentence reason for you answer.

4) Spanning Tree (10 points)

1. Draw the simple weighted graph G = (V, E, w), where the edges/weights are given by
   E =
   {(a,b,1), (a,c,3), (b,c,3), (c,d,6), (b,e,4), (c,e,5), (d,f,4), (d,g,4), (e,g,5), (f,g,2), (f,h,1), (g,h,2)}.
2. Give a short description of an algorithm of your choice to find a minimum spanning tree for the graph
3. Use the algorithm to find the minimum spanning tree for the above graph for the graph. Show all your steps.

5) Sorting (10 points)

1. Describe the merge-sort algorithm by sorting the data [5,8,1,4,7,2,3,9,5]. Show your operations step by step.
2. Give a recursive implementation of Merge sort in C or pseudo code
3. Derive the asymptotic complexity of merge sort.

6) Design an Algorithm (20 points)

Jerry works in a company based in Yellowknife providing services all over North America and he has to travel to cities. He is asked to find the cheapest flight or flights that would take him to Kingston. He knows that a direct flight from Yellowknife to Kingston, if one exists, would be absurdly expensive, so he is willing to tolerate stopovers (we ignore the waiting times in each airport). However, there are multiple airlines in North America with so many different flights between different cities now, which makes it very difficult for Jerry to find the least expensive way to Kingston! Since this situation occurs repeatedly Jerry decides to write a program that will solve this problem once and for all.

You will be given a list of cities in North America. The cities will be given in order of "nearest" to "farthest' from Yellowknife.
You will also be given a list of flights between pairs of cities, and the associated cost for each flight, taxes included. There will never be a flight from a farther city to a nearer city - Jerry has already discarded those flights, deeming them to be a waste of time and money. Bear in mind, however, that there may be more than one flight between any two cities, as Jerry is considering flights from all airlines.

1. Represent the problem into a tree or graph problem by clearly explaining what edges and vertices represent, and how they would be labeled.
2. Describe the property of a solution (meaning a sequence a flight route/sequence of cities) using your representation above.
3. Give a data structure (C or pseudo code) used to represent the data
4. Given and explain an algorithm which will solve the problem
5. Discuss the complexity of your algorithm
6. Discuss how your solution changes if Jerry wants to limit the number of stopovers he is willing to tolerate to some given number n?

7) Algorithm Concepts (20 points)

For five of the following concepts, explain the concept and give examples of problems and algorithms to solve them.

1. Divide and conquer
2. Breadth-first search
3. Use of randomness in algorithms
4. "Greedy" algorithms
5. Dynamic programming
6. Tail recursive

8) Complexity (20 points)

Explain the following five concepts, and give a sample problem/illustrative example for each to illustrate it.

1. What is a polynomial problem?
2. What is a NP-problem?
3. What is a NP-hard problem
4. What does asymptotic complexity vs. worst-case complexity mean?
5. What is the relation between Space and Time complexity?