National Exams – May 2018

16-Mec-A6 Advanced Fluid Mechanics

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question the candidate is urged to submit with the answer paper a clear statement of the assumptions made.
2. Candidates may use any approved Sharp/Casio calculator.
   The exam is OPEN BOOK.
3. Any FIVE (5) out of the 6 questions constitute a complete exam paper for a total of 100 MARKS.
   The first five questions as they appear in the answer book will be marked.
4. Each question is of equal value (20 marks) and question items are marked as indicated.
5. Clarity and organization of the answer are important.

(20) **Question 1**

Air ($\gamma = 1.4$, $R = 287$ J/(kg K)) flows from a very large reservoir through a convergent-divergent nozzle. The air in the reservoir is kept at a constant temperature of $T_0 = 300$ K. The exit area is $A_E = 10$ cm^2 and the throat area is $A_T = 6.45$ cm^2. When the back pressure is $P_b = 100$ kPa (absolute), the pressure distribution as measured along the centre-line shows a discontinuity at the exit as shown schematically in Figure 1. The flow may be assumed adiabatic for all conditions and isentropic in the absence of shocks.

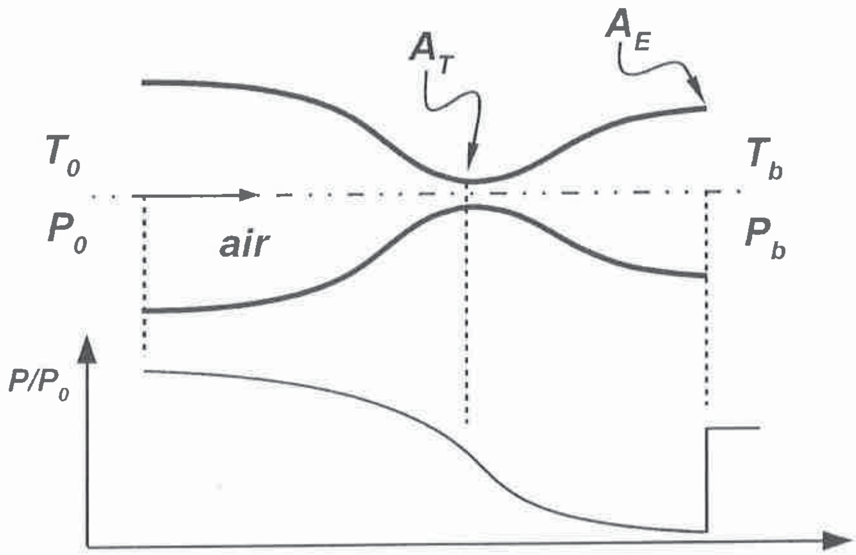

Figure 1: Top: Convergent-divergent nozzle attached to a large air reservoir. Bottom: Pressure distribution along the nozzle centre-line for a back pressure of P_b.

(5) (a) What is the total pressure P_0 in the reservoir.

(5) (b) What is the speed of the flow and the temperature directly downstream of the exit?

(5) (c) What is the mass flow rate?

(5) (d) What is the lowest back pressure for which the flow will be subsonic throughout the channel? What will be the mass flow rate at this back pressure?

(20) **Question 2**

Consider the ideal flow given by the velocity potential function

$$\phi = \frac{-A}{2\pi} \ln r$$

where A is a positive constant.

(5) (a) Determine the stream function ψ.

(5) (b) Sketch the equipotential lines and the stream lines of this flow.

(5) (c) Calculate the radial velocity V_r and identify the flow pattern.

(5) (d) Give the physical meaning of the constant $(2\pi\Gamma)$.

(20) **Question 3**

Air flows through an insulated constant diameter horizontal duct as shown in Figure 2. The inlet conditions are P_1=680 kPa, T_1=60°C and V_1=110 m/s. The diameter of the pipe is D=13 cm and the flow is "choked" at the exit. Determine the net force of the pipe on the fluid.

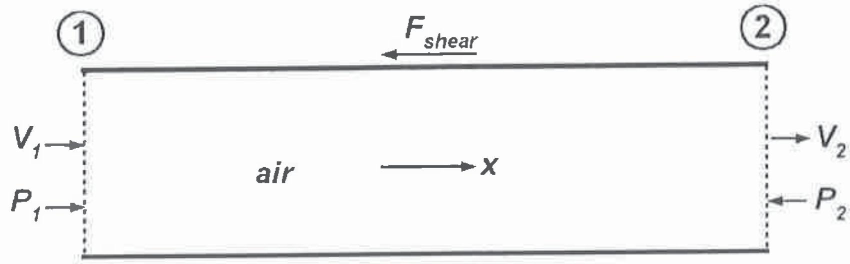

Figure 2: Pipe section with choked flow at the exit.

(20) **Question 4**

Water flows vertically and at a constant rate in a pipe of length L, as shown in Figure 3. The pipe consists of a circular outer pipe of radius R and a central rod of radius $\delta = R/4$. The walls of the outer pipe are coated with Teflon so that the water does not wet the surface (negligible shear). The inner rod is not coated, so that the water does wet the surface. The walls are non-porous. The inlet and outlet of the pipe are exposed to atmosphere, such that the inlet and outlet static pressures are the same. Assuming that $L/R >> 10$ and that the flow is laminar and axisymmetric, determine the following.

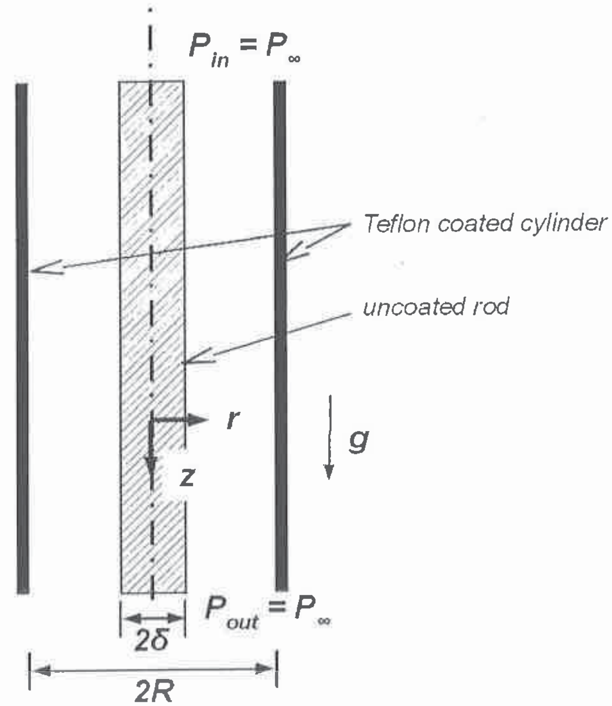

Figure 3: Schematic of vertical pipe with central rod.

(6) (a) A formula for the radial distribution of the radial velocity component v_r.

(7) (b) A formula for the radial distribution of the vertical velocity component v_z.

(7) (c) The force per unit length (magnitude and direction) acting on the inner rod.

(20) **Question 5**

Consider the flow over a very small object in a viscous fluid. Analysis of the equations of motion shows that the inertial terms are much smaller than the viscous and pressure terms and they can be neglected. As a result, the density drops out of the equations of motion. Such flows are called creeping flows. The remaining parameters in the problem are the velocity U, the dynamic viscosity μ, and the length scale of the body. While this approach works well for three-dimensional bodies, it is unclear if it can work also for two-dimensional bodies, such as an infinitely long cylinder of small diameter D.

(6) (a) Using the Buckingham Pi theorem, generate an expression for the two-dimensional drag force F (per unit length!) as a function of the other 3 parameters in the problem.

(4) (b) Is this expression physically plausible as the functional dependence of the drag? Explain.

(6) (c) Now, repeat the analysis adding the density back as a parameter for the problem.

(4) (d) Explain if the new nondimensional relationship is better and why.

(20) **Question 6**

A rectangular cross-section suction wind tunnel is designed with a curved upper wall and a straight bottom wall. We are interested in characterising the boundary layer development along the bottom wall. The shape of the upper wall is designed such that the external (irrotational, or main) flow varies along the tunnel according to:

$$U_0 = A\,x^{1/3}$$

where A is a constant.

The flow can be assumed to be two-dimensional and laminar, and the properties of air to be constant (ρ is density, μ is dynamic viscosity, and $\nu = \mu/\rho$ is kinematic viscosity). The boundary layer thickness is represented by δ and it is assumed that at $x = 0$, $\delta = 0$. The velocity distribution in the boundary layer is assumed to be approximated by:

$$\frac{u}{U_0} = 2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2 \qquad \text{such that: } \frac{\delta^*}{\delta} = \frac{1}{3} \quad \text{and} \quad \frac{\theta}{\delta} = \frac{2}{15}$$

where δ^* and θ are the momentum and displacement thickness, respectively.

The boundary layer thickness can be assumed to satisfy: $\delta = B\,x^n$.

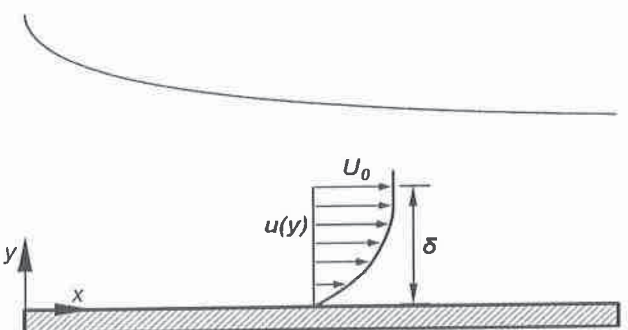

Figure 4: Boundary layer developing over bottom wall of suction wind tunnel.

(5) (a) Find the value of n to satisfy the integral boundary layer equations.

(5) (b) Express δ/x in terms of $\mathrm{Re}_x = \dfrac{U_0\,x}{\nu}$.

(5) (c) What is the local wall shear stress coefficient, $C_{fx} = \dfrac{\tau_w}{\rho U_0^2/2}$, in terms Re_x.

(5) (d) What is the average coefficient $\overline{C_f} = \overline{\tau_w}/(\rho U_0^2/2)$, where $U_0 = A\,L^{1/3}$?