98-Phys-A7, Optics December 2016

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National Exams December 2016

98-Phys-A7, Optics

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate should include in the answer clear statements of the interpretation and any assumptions made.

2. This is a CLOSED BOOK EXAM. **Some formulae are listed on pages 6, 7, and 8.**

3. Candidates may use one of two calculators, the Casio or Sharp approved models.

4. Answers to questions 1 and 2 plus any two of questions 3 to 6 constitute a complete exam paper.

5. Answer question 1 in the space provided on the exam paper.

6. If more than two of questions 3 to 6 are attempted, only the first two, as they appear in the answer book, of questions 3 to 6 will be marked.

7. Each question is of equal value. Questions 1 and 2 are mandatory.

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1. A phrase, or a diagram and a phrase, is all that is required in most cases. [15 marks total, one mark for each letter] **QUESTION 1 IS MANDATORY**. Answer question 1 on these pages, in the space provided.

a) This examination is about light and optics. Define *light*.

b) Define *optics*.

c) Define *geometrical optics*.

d) Define *physical optics*.

e) What problems with *geometrical optics* led to the development of *physical optics*?

f) Define *plane of incidence*.

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g) State the *law of reflection*. Include *plane of incidence* in the statement of the law.

h) State the *law of refraction*. Include *plane of incidence* in the statement of the law.

i) Complete the following table:

		wavelength range (nm)	frequency range (Hz)
	UV light		
	red light		
	blue light		
	IR light		

j) Define *spherical aberration*.

k) Define *chromatic aberration*.

l) Define *astigmatism* as it relates to an aberration.

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m) Define *index of refractive*.

n) Define *group index of refraction.*

o) Define *f-number*.

Answer the remaining questions in an answer book.

2. [15 marks total] **QUESTION 2 IS MANDATORY**

a) A collimated beam of light is normally incident on a single slit. The beam of light is composed of light with wavelength components at 500 nm and an unknown wavelength. If, for the diffraction pattern in the far field, the fifth minimum of the unknown wavelength overlaps the fourth minimum of the 500 nm light, calculate the wavelength of the unknown component. [3 marks]

b) A 50 groove grating with a groove separation of 0.01 mm is illuminated normally with light of 500 nm and placed in front of a lens of focal length 100 cm. What is the separation, in the focal plane of the lens, between the second-order principal maximum and the neighbouring minima on either side for the diffraction pattern? [3 marks]

c) Assume a mono-chromatic plane wave of wavelength λ is normally incident on an aperture of diameter D. Determine the distance from the aperture to an observation plane for which the diameter of the beam transmitted through the aperture is essentially twice the diameter of the aperture. Select and justify your criterion for *essentially twice the diameter*. [4 marks]

d) i) State Rayleigh's criterion for resolution of an imaging system and sketch a diagram to demonstrate Rayleigh's criterion. [2 mark]

ii) Assume the pupils of an observer adapt under changes in ambient light from 2 mm to 7 mm diameter. Given the adaptation of the pupils of the observer, over what range of distances can the observer just resolve two lights that are separated laterally by 1 m and that emit at a wavelength of 500 nm? [3 marks]

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3. [15 marks total] **ANSWER ANY TWO OF QUESTIONS 3, 4, 5, OR 6.**

a) A microscope is focussed on a small scratch on the top surface of a glass plate. Assume that the top and bottom surfaces of the plate are parallel and separated by 1.50 mm. If the plate is raised by 2 mm, a focussed image of the scratch is observed (again). Sketch rays for the optical measurements and determine the refractive index of the glass plate. [4 marks]

b) Given a system ray-transfer matrix $[M]$ and input and output arrays $[y_{in}, \alpha_{in}]^T$ and $[y_{out}, \alpha_{out}]^T$ where the superscript T indicates transpose, draw a diagram to indicate the physical meaning of the elements of the input and output arrays. [2 marks]

c) A thin lens of focal length 10 cm is separated by 5 cm from a thin lens of focal length −10 cm. Using the matrix method, **explain** how to and then **calculate** the equivalent focal length of the lens combination, the linear magnification, the angular magnification, and the locations of the focal planes. [9 marks]

4. [15 marks total] **ANSWER ANY TWO OF QUESTIONS 3, 4, 5, OR 6.**

a) Write equations for the electric field and magnetic fields for TE and TM polarized plane waves. Let the plane of incidence be the *y-z* plane and allow the waves to be propagating at an angle of β with respect to the z axis. Assume that the waves propagate in an isotropic, homogeneous medium with refractive index of n. Remember to draw the coordinate system. [6 marks]

b) Check your answers for the **E** and **H** fields and for the vector **k** for part a) by taking limiting cases and by calculating **E**×**H**. **Explain** your reasoning. [4 marks]

c) Allow two coherent, monochromatic plane waves of light that are propagating in the *y-z* plane to interfere. Assume that one plane wave is propagating at an angle of β with respect to the z axis and the other plane wave is propagating at an angle of $-\beta$ with respect to the z axis. **Derive** (i.e., show your work and explain your reasoning) a simplified expression for the time average of the Poynting vector in the plane $z = 0$ that can be used to determine by inspection (hence the request for a simplified expression) the period of the interference fringes in the $z = 0$ plane. [3 marks]

d) Assume that the fringes in part c) are observed in a material with a refractive index $> n$, and that this material has a planar interface starting at the plane $z = 10$. **Derive** (i.e., show your work and explain your reasoning) the period of the interference fringes when observed in the material with refractive index $> n$. [2 marks]

5. [15 marks total] **ANSWER ANY TWO OF QUESTIONS 3, 4, 5, OR 6.**

a) Assume a power of 1 mW at some cross-sectional plane A of a long fibre. The guided power inside the fibre at 100 m past the cross-sectional plane A is 0.2 mW. What is the absorption coefficient of the fibre in dB/km? [4 marks]

b) A fibre has an attenuation loss of 1.2 dB/km at 900 nm owing to Rayleigh scattering. What minimum attenuation loss in dB/km can be expected at 1550 nm? [2 marks]

c) Determine the lengths and transit times of the longest and shortest possible paths in a step-index fibre. Assume that the fibre has a core index of 1.46, cladding index of 1.45, a core diameter of 50 μm, and that the fibre is perfectly straight (i.e., the optical axis of the fibre is a straight line). **Draw** diagrams and **explain** your reasoning. [6 marks]

d) Determine the numerical aperture and the maximum acceptance angle of the fibre in c). **Draw** diagrams and **explain** your reasoning. [3 marks]

6. [15 marks total] **ANSWER ANY TWO OF QUESTIONS 3, 4, 5, OR 6.**
A thin film of refractive index 1.38 is deposited on a substrate of refractive index 1.50 and functions as an antireflection coating (i.e, exhibits a minimum in the reflection) at 500 nm under normal incidence.

a) Determine the thickness of the thin film. Draw a ray diagram of the thin film and substrate combination, and label the appropriate quantities that are used in your determination of the thickness of the thin film. **Explain** clearly your reasoning. [5 marks]

b) Determine the wavelength that is minimally reflected when the filter is angle tuned through θ, i.e., determine the wavelength that is minimally reflected when the angle of incidence of mono-chromatic plane-wave illumination is θ. Draw a ray diagram of the thin film and substrate combination, and label the appropriate quantities. **Explain** clearly your reasoning. [10 marks]

Question 6 b) is the last question. Some formulae follow.

$$E(x,y,z) = \frac{ik}{2\pi}\frac{e^{ikz}}{z} e^{i\frac{k}{2z}(x^2+y^2)} \iint E(x_a, y_a, 0) e^{i\frac{k}{2z}(x_a^2+y_a^2)} e^{-i\frac{k}{z}(x\,x_a + y\,y_a)}\, dx_a\, dy_a$$

The field in the neighbourhood of the focus of a circular lens of radius a is given in the usual paraxial approximation as

$$E(u,v) = \int_0^1 J_0(2\pi v \rho_a)\, e^{-i\pi u \rho_a^2}\, \rho_a\, d\rho_a$$

with $\rho_a = \sqrt{(x_a^2 + y_a^2)}$, $\quad u = \frac{1}{\lambda}\frac{a^2}{q\,(q+\Delta)}\Delta, \quad v = \frac{1}{\lambda}\frac{a\sqrt{(x^2+y^2)}}{(q+\Delta)}$.

$J_0(0) = 1$; $J_0(2.4048) = 0$; $J_0(5.5201) = 0$; $J_0(8.6537) = 0$; $J_0(11.7915) = 0$;

$\gamma = \frac{1}{2} k D \sin(\theta)$. The zeros for $J_1(\gamma)$ occur for γ = 0, 3.832, 5.136, 7.016, 8.417, 10.173, 11.620, 13.324, . . .

$$\frac{d}{dx} x^n J_n(x) = x^n J_{n-1}(x)$$

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$$(1 + \epsilon)^{\xi} = 1 + \frac{\xi}{1!}\epsilon + \frac{\xi \times (\xi - 1)}{2!}\epsilon^2 + \ldots$$

zone plate radii $R_m = (m\, r_o\, \lambda)^{0.5}$

The intensity in the far-field as a function of the angle θ_m from the normal of a diffraction grating of N lines, line spacing of a, and line width of b, for illumination with a plane wave with $k = 2\pi/\lambda$ and an angle of incidence of θ_i is

$$I(\theta) = I_o \left[\frac{\sin(\beta)}{\beta}\right]^2 \left[\frac{\sin(N\alpha)}{\sin(\alpha)}\right]^2$$

$$\beta = \frac{k\,b}{2}\left(\sin(\theta_i) + \sin(\theta_m)\right)$$

$$\alpha = \frac{k\,a}{2}\left(\sin(\theta_i) + \sin(\theta_m)\right)$$

For a blazed grating, $2\theta_b = \theta_i - \theta_m$

The resolution R and the dispersion D for a grating with N lines and order m are

$$R = \frac{\lambda}{\Delta\lambda} = mN \qquad D = \frac{m}{a\,\cos(\theta)}\,.$$

double angle formulae:

$$\begin{aligned}
\sin(A+B) &= \sin(A)\sin(B) + \cos(A)\cos(B)\\
\sin(A-B) &= \sin(A)\sin(B) - \cos(A)\cos(B)\\
\cos(A+B) &= \cos(A)\cos(B) - \sin(A)\sin(B)\\
\cos(A-B) &= \cos(A)\cos(B) + \sin(A)\sin(B)
\end{aligned}$$

$$\begin{aligned}
\sin(A) + \sin(B) &= 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\\
\sin(A) - \sin(B) &= 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\\
\cos(A) + \cos(B) &= 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\\
\cos(A) - \cos(B) &= 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)
\end{aligned}$$

$$v_p = \frac{\omega}{k} \qquad v_g = \frac{d\omega}{dk}$$

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The translation, refraction at a spherical interface, thin lens, and spherical mirror matrices are listed below.

$$\begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ \dfrac{n_1 - n_2}{Rn_2} & \dfrac{n_1}{n_2} \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ \dfrac{-1}{f} & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ \dfrac{-2}{R} & 1 \end{bmatrix}$$

THE END