National Exams May 2019

16-Mec-B10, Finite Element Analysis

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. This is an OPEN BOOK EXAM.
   Any non-communicating calculator is permitted.

3. FIVE (5) questions constitute a complete exam paper.
   The first five questions as they appear in the answer book will be marked. The questions are to be solved within the context of the finite element method.

4. Each question is of equal value.

5. Some questions require an answer in essay format. Clarity and organization of the answer are important.

Question 1. [20 marks]

A field variable $f(x,y) = x(1+xy)$ is defined over a rectangular domain $\Omega = \{\Re^{2+}: 1 \le x \le 7, 2 \le y \le 6\}$. For the expression

$$g = \int_2^6 \int_1^7 x\,(1 + x^2 y)dxdy$$

where the spatial/geometric variables x and y are discretized using the following bilinear interpolation shape functions:

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta),\ N_2 = \frac{1}{4}(1+\xi)(1-\eta),\ N_3 = \frac{1}{4}(1+\xi)(1+\eta), N_4 = \frac{1}{4}(1-\xi)(1+\eta)$$

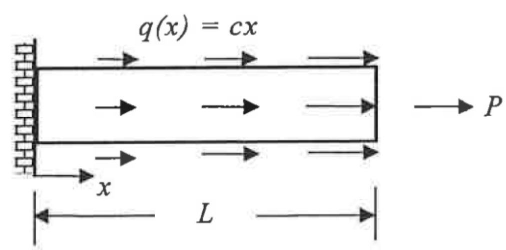
where $-1 \le \xi, \eta \le 1$ for the local coordinates ξ, η,

(a) [15 marks] Use the Gauss quadrature numerical integration method, using the appropriate number of gauss points, to evaluate the expression *g*. (*The Jacobian must be provided and the entire evaluation must be cast in vector-matrix format as commonly employed in finite element analysis.*)

(b) [5 marks] Explain any similarity or difference between your answer in part (a) and the exact solution *g* = 9696.

Question 2. [20 marks]

A cantilevered bar is loaded by a constant axial load *P* and a linearly varying distributed load $q(x) = cx$ as shown in the figure - *note that c is a constant*. The cross-sectional area and length of the bar are denoted by A and L, respectively. Assuming the bar is made of a material with Young's modulus of elasticity E, the system governing equation can be written as



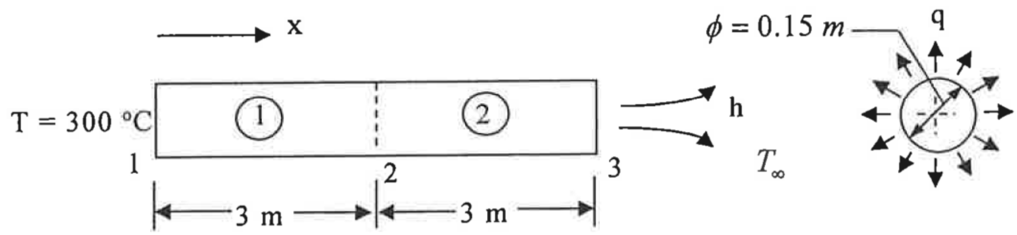
$$EA\frac{d^2u(x)}{dx^2} + cx = 0 \quad 0 < x < L$$

$$\textit{subject to}: \quad u(0) = 0 \quad \textit{and} \quad EA\frac{du(x)}{dx}\bigg|_{x=L} = P$$

Use the Bubnov-Garlerkin method (i.e., Garlekin method) to determine an a cubic polynomial solution of the axial displacement $u(x,t)$.

Question 3. [20 marks]

The figure below depicts a 6 m long rod with diameter ϕ = 0.15 m. The rod is discretized into two elements of equal length numbered 1 and 2 as shown.



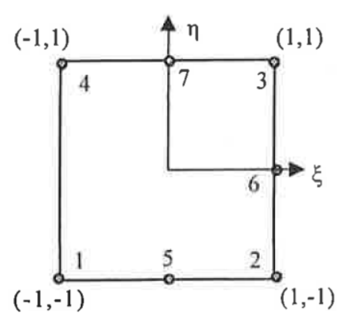
The thermal conductivity in the x-direction of the first and second element is $K_{xx}^{(1)} = 150$ W/(m°C) and $K_{xx}^{(2)} = 400$ W/(m°C), respectively. The rod has a uniform internal heat source of 400 W/m^3. Further, a uniform outgoing heat flux q_{out} = 12 W/m^2 acts over the whole cylindrical surface of the rod. The temperature at the left-hand end of the rod is maintained constant at 300 °C. Heat convection arises **only** at right-hand end of the rod with convection heat flux h = 25 W/(m^2. °C) and the downstream temperature T_∞ = 120 °C. Determine

(a) [14 marks] the temperature at nodes 2 and 3,

(b) [3 marks] the heat flux over element #1,

(c) [3 marks] the heat flow rate and the direction at node1.

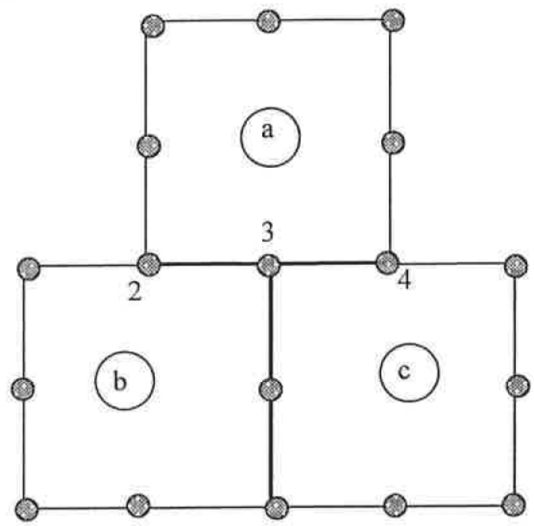
Question 4. [20 marks]



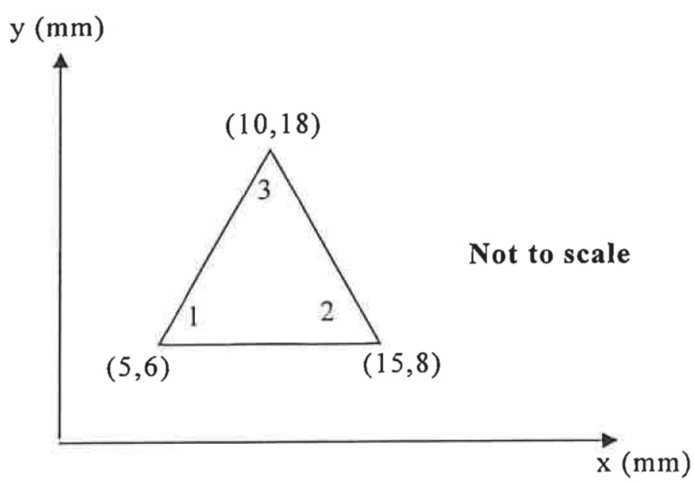
(a) [14 marks] Determine the shape functions (N_i, i = 1 to 7) of the seven-node transition element in natural/local coordinates (ξ,η) such that $-1 \le \xi,\eta \le 1$.

(b) [3 marks] Does the shape function $N_1(\xi,\eta)$ satisfy C^0 continuity along the sides that contain node 1? Explain.

(c) [3 marks] A finite element mesh is generated using eight-noded Serendipity elements, namely a, b, and c, as shown in the figure. What is the major problem with the 2-3-4 interface?



Question 5. [20 marks]



The nodal displacements for the plane strain element shown in the figure above are: $u_1 = 0.003$ mm and $v_1 = 0$ mm; $u_2 = v_2 = 0$ mm; $u_3 = 0.005$ mm and $v_3 = 0.003$ mm. The plate thickness $t = 0.6$ mm, and it is made from a material with Young's modulus of elasticity $E = 210$ MPa and Poisson's ratio $\nu = 0.3$.

(a) [15 marks] Determine the element stresses σ_x, σ_y, and τ_{xy}.

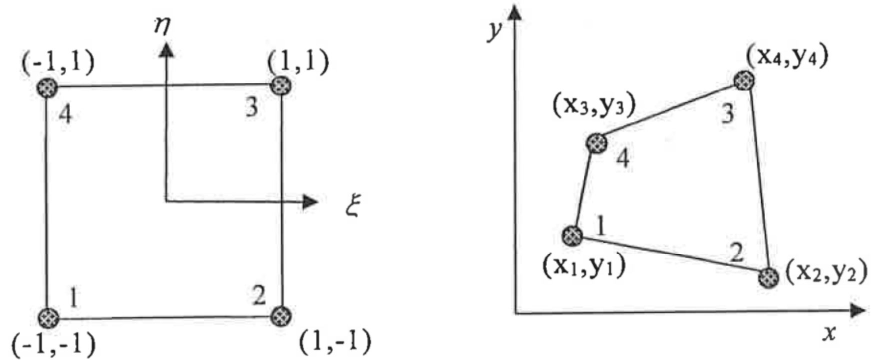
(b) [3 marks] Determine the principal stresses σ_1 and σ_2 and principal angle θ_p.

(c) [2 marks] Describe a major disadvantage of the constant-strain triangular element in bending problems and explain how it can be overcome?

Question 6. [20 marks]

(a) [2 marks] What is an isoparametric element?

(b) [10 marks] The four-node isoparametric quadrilateral element shown below is used to map a region in the parent domain into that in the global domain.

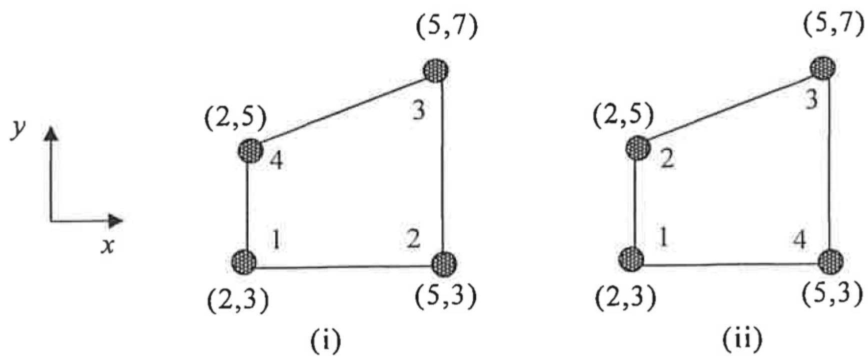


The shape functions of the element are given as

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta),\ N_2 = \frac{1}{4}(1+\xi)(1-\eta),\ N_3 = \frac{1}{4}(1+\xi)(1+\eta),\ N_4 = \frac{1}{4}(1-\xi)(1+\eta)$$

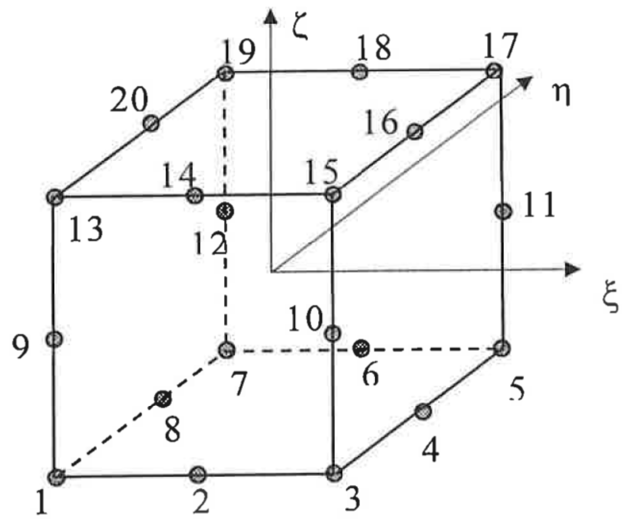
Determine the Jacobian matrix of the element.

(c) [4 marks] Use the Jacobian matrix obtained in part (b) to evaluate the Jacobian of the following elements.



(d) [4 marks] What can be inferred or concluded from the Jacobian expressions obtained in part (c) with regard to the mapping of the coordinates?

Question 7. [20 marks]



(a) [14 marks] Determine the shape functions (N_i, i = 1 to 20) of the above 20-noded quadratic Serendipity prism in natural/local coordinates (ξ, η, ζ) such that $-1 \leq \xi, \eta, \zeta \leq 1$.

(b) [4 marks] Verify that the shape functions determined in part (a) satisfy the two common properties of acceptable set of shape functions.

(c) [2 marks] Why will a finite element method practitioner choose the 20-noded quadratic Serendipity prism over a quadratic Lagrange prism?