National Exams December 2014

04-BS-6: Mechanics of Materials

3 hours duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumptions made.

2. Candidates may use one of two calculators, the Casio or Sharp approved models.

   This is a Closed Book exam. However candidates are permitted to bring the following into the examination room:

   - ONE aid sheet 8.5" x 11" hand-written on both sides containing notes and formulae. **Example problems and solutions to problems are not allowed!**

3. Any FIVE (5) questions (out of 8 given) constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.

4. All questions are of equal value.

5. Information on geometric properties of wide flange or W shape sections is attached at the end of this exam.

NOTE: The aid sheet must be handed in with the exam!

*Your exam will not be marked if you do not hand in an aid sheet,
unless there is a signed statement by the exam invigilator stating
that no aid sheet was used for the exam.*

Question 1: A simply supported beam with a cantilevered overhang supports a uniformly
[20 marks] distributed load acting downward on the overhang as shown. The beam is a
W200 x 22 wide flange section (refer to the attached table for section properties) and
is made of steel having an allowable normal stress of 240 MPa and allowable shear
stress of 60 MPa. The elastic modulus of the steel equals 200 GPa.
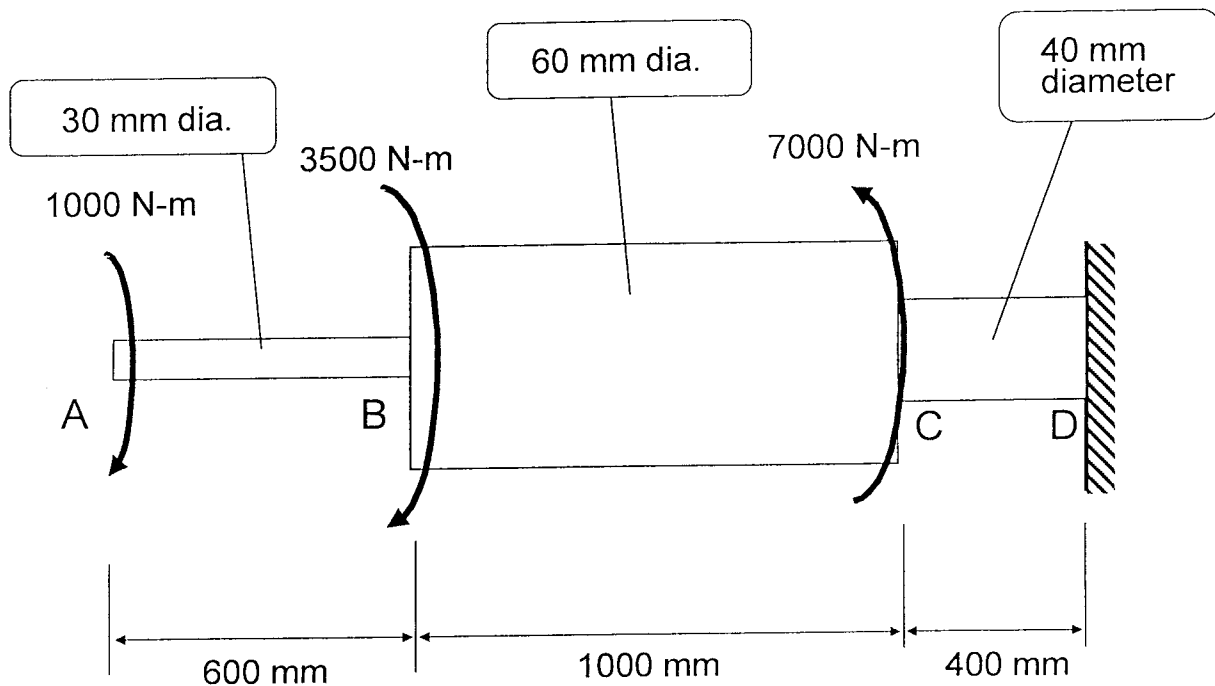
Determine the deflection and slope at the end of the overhang using the method of
integration.
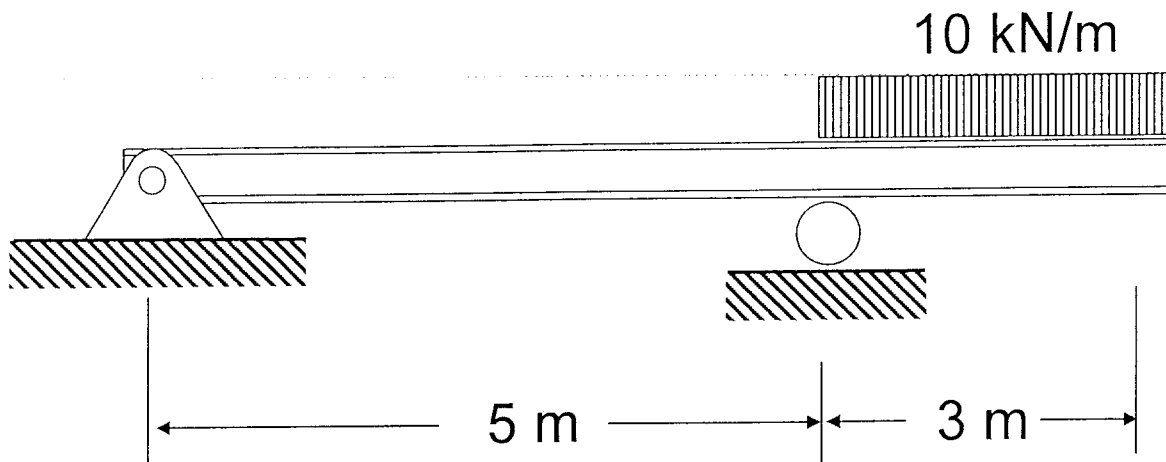
Question 2: A stepped shaft ABCD is subjected to three concentrated torques acting at A, B and C. The shaft is fixed at D. The entire shaft is made of aluminum having a shear modulus G = 25 GPa and a yield stress in shear of 220 MPa. Dimensions (diameter and length) and magnitude of the torques are given in the diagram.

[12 marks] (a) determine the maximum shear stress in the shaft and sketch the corresponding variation of shear stress along the shaft radius at this point (where the stress is maximum).

[8 marks] (b) find the angle of twist at the end of the shaft (point A) and give your answer in degrees.

30 mm dia.

60 mm dia.

40 mm diameter

1000 N-m

3500 N-m

7000 N-m

A   B   C   D

600 mm   1000 mm   400 mm

Question 3: The truss below is made from steel bars, each of which has a circular cross section
[20 marks] with a diameter of 40 mm. Determine the largest load P that can be applied at joints
2 and 4 without causing the inclined member 4-6 to buckle.

The truss members are pin-supported at their ends. Consider in-plane buckling only
and use a safety factor of 2 for the Euler buckling load. Do not use a safety factor for
yielding of the steel. The steel used in the truss members has an allowable yield
strength equal to 240 MPa and elastic modulus of 200 GPa.

Note: $A_{circle} = \pi r^2$ and $I_{circle} = \pi r^4 / 4$
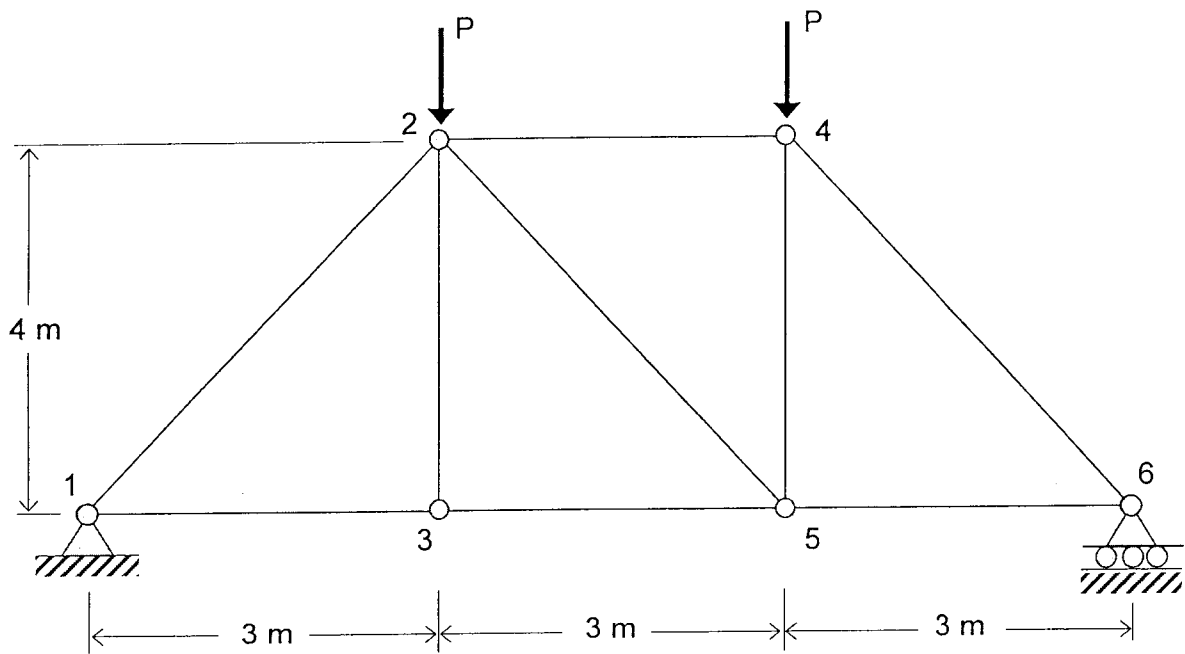
Question 4: An element in a state of plane stress is subjected to the normal and shear stresses shown below.

[20 marks] Use the Mohr's circle solution (*not* the transformation equations) to determine the following:

    (a) the stress components acting on an inclined plane (orientated $20°$ from the horizontal as shown). Show your answer on a properly oriented element.

    (b) the maximum in-plane shear stress (and associated normal stresses) and orientation of the corresponding planes. Once again, show your answer on a sketch of a properly oriented element.
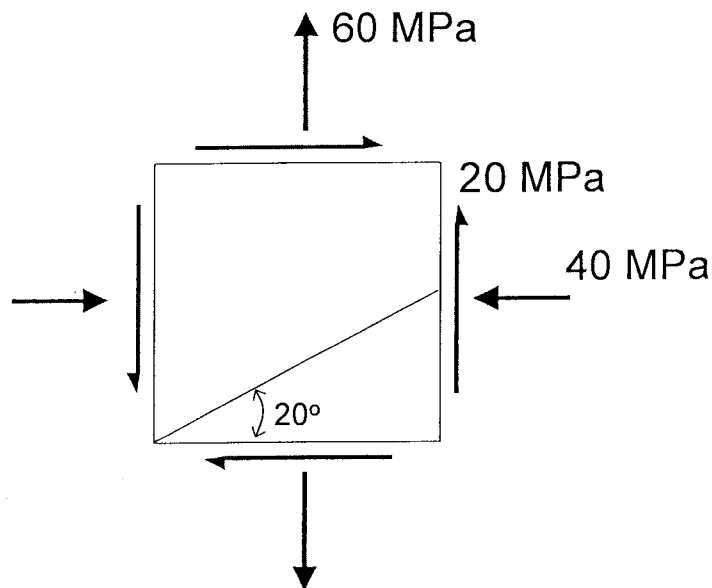

WARNING! Credit will **only** be given for a **solution using Mohr's circle**. Not the stress transformation equations. This means that you need to draw a Mohr's circle based on the stress components from this problem. Remember to show numbers on your circle. Your **calculations** must be based on the geometry of your circle. So use your calculator. In other words, you are expected to use trigonometry to construct your Mohr's circle. Do not give a graphical solution that is scaled off!

The stress transformation equations can only be used to check your answer.

Question 5: A rigid triangular frame is pivoted at C with a 16 mm diameter pin and supported by two 4 mm diameter cables at A and B as shown below. The cables are not of equal length, and are made of steel with an elastic modulus E of 200 GPa and yield stress of 240 MPa. A vertical load of P = 1200 N is applied to the triangular frame at D.

Determine the following:

[12 marks] (a) forces carried by each cable

[4 marks] (b) corresponding displacement at the point D where the load P is applied

[4 marks] (c) shear stress in the 16 mm diameter pin at C. Assume the pin is loaded in double shear.

4 mm dia. cables

200 mm

300 mm

A

B

C

D

600 mm

800 mm

800 mm

P=1200 N
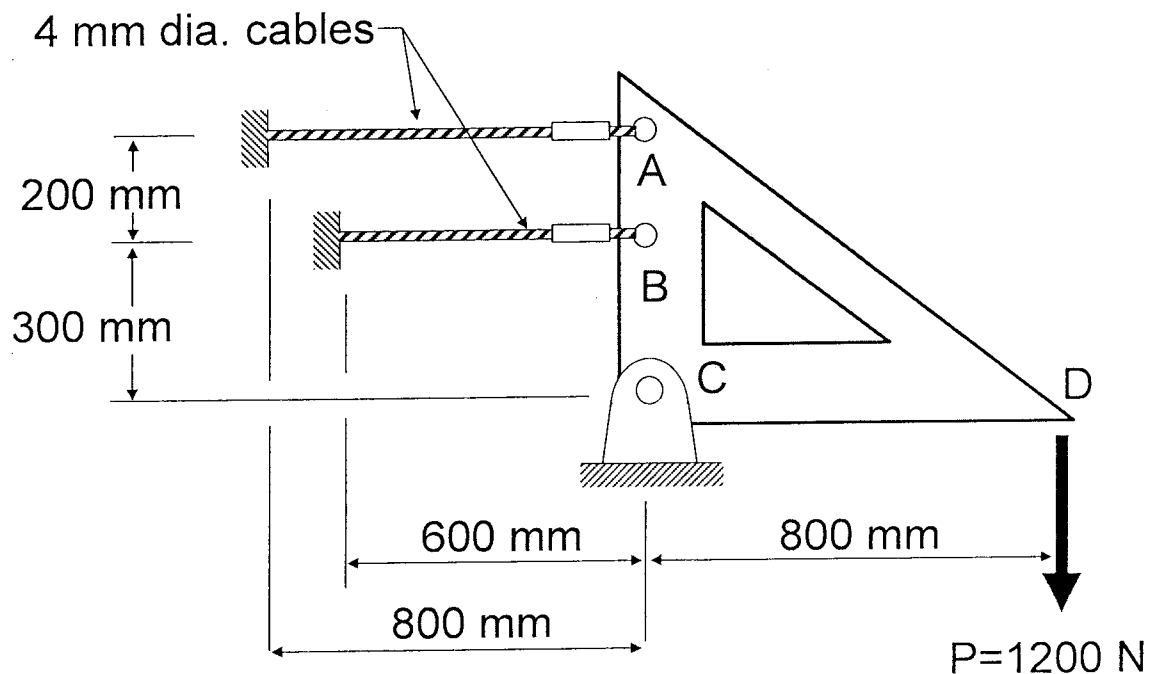
Question 6: The rectangular column below is subjected to an eccentric axial load of 600 kN and
[20 marks] an inclined load of 80 kN acting on a collar attached to the column as shown. The
column is made of steel with a yield strength of 400 MPa and elastic modulus of
200 GPa.

(a) compute the normal stress distribution in the rectangular column section at
section a-a which is located 300 mm below the collar. Show your answers on a
sketch. Give maximum and minimum values. Ignore any potential for buckling of
the column.

(b) compute the shear stress distribution at the same location. Show your answers on
a sketch. Give maximum and minimum values.
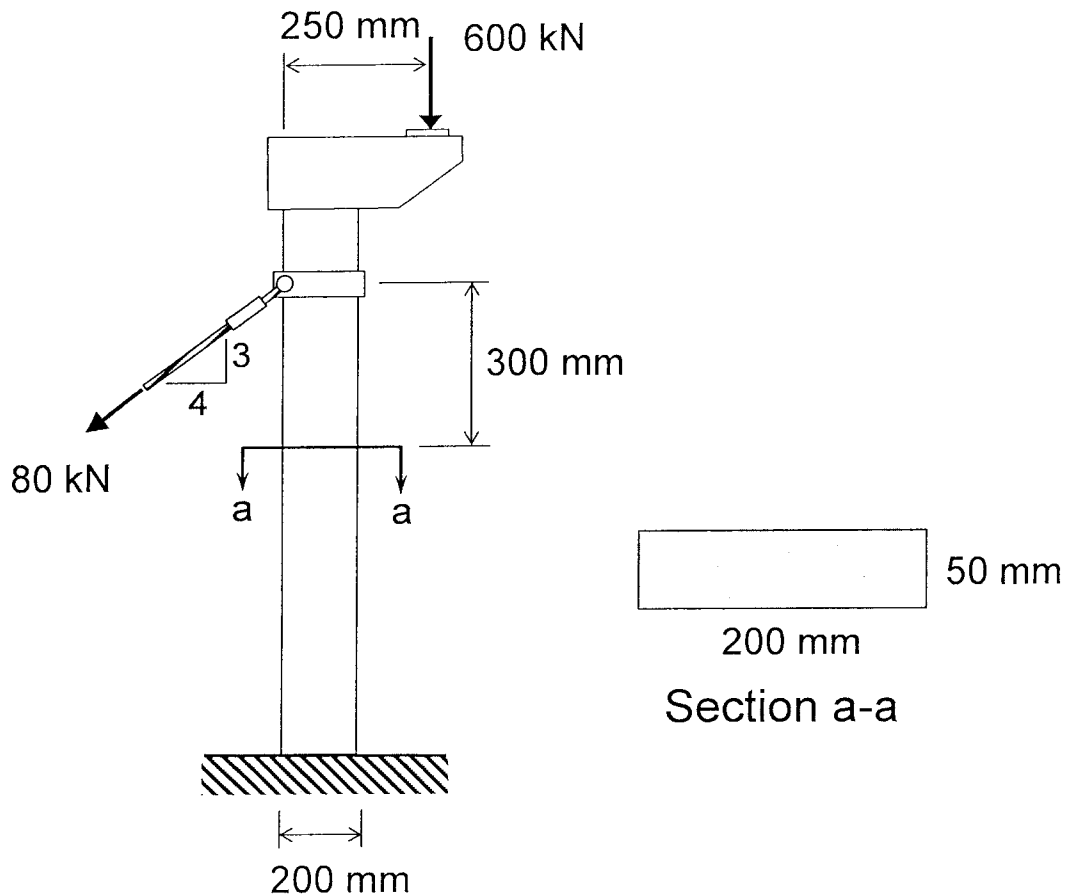
Question 7: A simply supported beam supports a uniformly distributed load of 20 kN/m acting over the simple span in addition to a concentrated couple applied at the left hand support and a concentrated load of 40 kN acting at the end of the overhang.

The beam is a wide flange section using steel with an allowable normal stress of 260 MPa and allowable shear stress of 75 MPa. The elastic modulus of the steel is 200 GPa.

[20 marks] Determine the shear and moment throughout the beam as functions of x. This means that you need to give formula(s) for **V(x)** and **M(x)** along the length of the beam. Next construct the shear force and bending moment diagrams. Remember to label points of extreme positive and negative bending moment, as well as any inflection points. Show your work by indicating exactly how you obtained your answer.

Remember that it is important you set this problem up correctly by calculating the correct reaction forces at the support(s).

40 kN

30 kN-m

20 kN/m

A

B   C

8 m   3 m
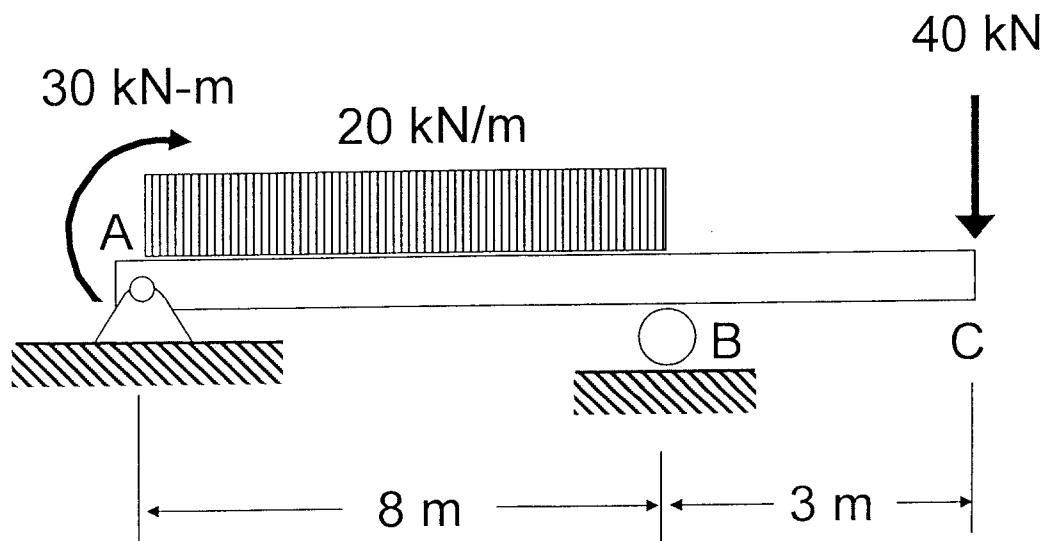
No credit will be given for a solution using the principle of superposition, when combinations of existing solutions are used to find an answer.

Question 8:
[20 marks]
A simply supported T-beam is subjected to a concentrated load as shown. The beam has the cross-section shown below and is made from steel having an allowable normal stress of 260 MPa and allowable shear stress of 60 MPa. The elastic modulus of the steel equals 200 GPa.

Determine the following:

(a) maximum (absolute) normal stress in the beam

(b) maximum shear stress in the beam

(c) shear stress at the tip of the flange (point E) at a section located 2 m from the left support. To receive marks you must give reasons to justify your answer.

30 kN

3 m

4 m

40

150

30

E

150

beam cross-section
(all dimensions in mm)
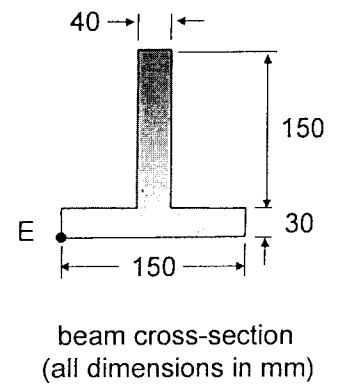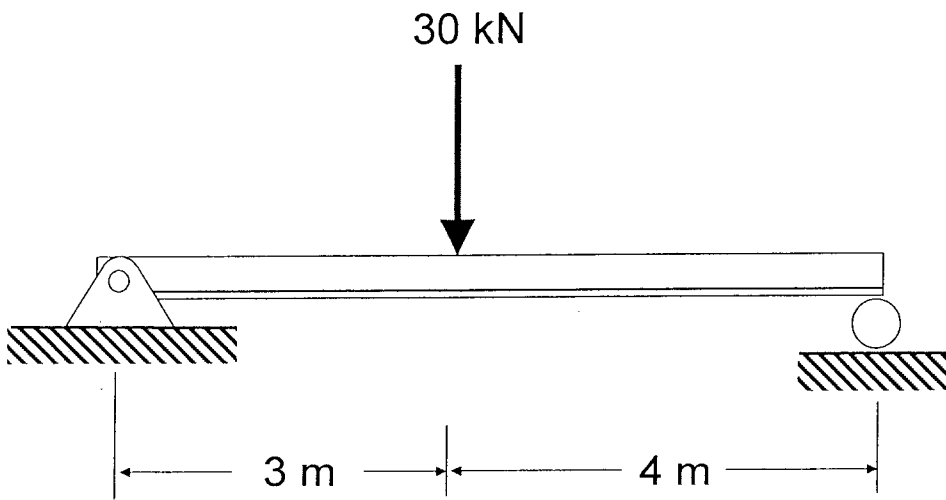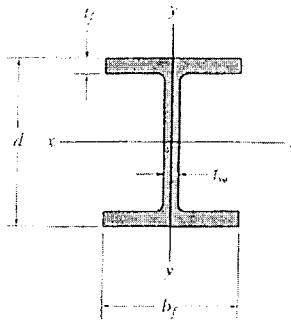
Wide-Flange Sections or W Shapes: SI Units

Designation	Area A	Depth d	Web thickness t_w	Flange width b_f	Flange thickness t_f	x–x axis I	x–x axis S	x–x axis r	y–y axis I	y–y axis S	y–y axis r
mm × kg/m	mm²	mm	mm	mm	mm	10⁶ mm⁴	10³ mm³	mm	10⁶ mm⁴	10³ mm³	mm
W310 × 129	16 500	318	13.10	308.0	20.6	308	1940	137	100	649	77.8
W310 × 74	9 480	310	9.40	205.0	16.3	165	1060	132	23.4	228	49.7
W310 × 67	8 530	306	8.51	204.0	14.6	145	948	130	20.7	203	49.3
W310 × 39	4 930	310	5.84	165.0	9.7	84.8	547	131	7.23	87.6	38.3
W310 × 33	4 180	313	6.60	102.0	10.8	65.0	415	125	1.92	37.6	21.4
W310 × 24	3 040	305	5.59	101.0	6.7	42.8	281	119	1.16	23.0	19.5
W310 × 21	2 680	303	5.08	101.0	5.7	37.0	244	117	0.986	19.5	19.2
W250 × 149	19 000	282	17.30	263.0	28.4	259	1840	117	86.2	656	67.4
W250 × 80	10 200	256	9.40	255.0	15.6	126	984	111	43.1	338	65.0
W250 × 67	8 560	257	8.89	204.0	15.7	104	809	110	22.2	218	50.9
W250 × 58	7 400	252	8.00	203.0	13.5	87.3	693	109	18.8	185	50.4
W250 × 45	5 700	266	7.62	148.0	13.0	71.1	535	112	7.03	95	35.1
W250 × 28	3 620	260	6.35	102.0	10.0	39.9	307	105	1.78	34.9	22.2
W250 × 22	2 850	254	5.84	102.0	6.9	28.8	227	101	1.22	23.9	20.7
W250 × 18	2 280	251	4.83	101.0	5.3	22.5	179	99.3	0.919	18.2	20.1
W200 × 100	12 700	229	14.50	210.0	23.7	113	987	94.3	36.6	349	53.7
W200 × 86	11 000	222	13.00	209.0	20.6	94.7	853	92.8	31.4	300	53.4
W200 × 71	9 100	216	10.20	206.0	17.4	76.6	709	91.7	25.4	247	52.8
W200 × 59	7 580	210	9.14	205.0	14.2	61.2	583	89.9	20.4	199	51.9
W200 × 46	5 890	203	7.24	203.0	11.0	45.5	448	87.9	15.3	151	51.0
W200 × 36	4 570	201	6.22	165.0	10.2	34.4	342	86.8	7.64	92.6	40.9
W200 × 22	2 860	206	6.22	102.0	8.0	20.0	194	83.6	1.42	27.8	22.3
W150 × 37	4 730	162	8.13	154.0	11.6	22.2	274	68.5	7.07	91.8	38.7
W150 × 30	3 790	157	6.60	153.0	9.3	17.1	218	67.2	5.54	72.4	38.2
W150 × 22	2 860	152	5.84	152.0	6.6	12.1	159	65.0	3.87	50.9	36.8
W150 × 24	3 060	160	6.60	102.0	10.3	13.4	168	66.2	1.83	35.9	24.5
W150 × 18	2 290	153	5.84	102.0	7.1	9.19	120	63.3	1.26	24.7	23.5
W150 × 14	1 730	150	4.32	100.0	5.5	6.84	91.2	62.9	0.912	18.2	23.0