National Exam May 2018

04-BS-06, **Mechanics of Materials**

Duration: 3 hours

Notes:

1. If doubt exists as to the interpretation of any questions, the candidate is urged to submit with the answer paper a clear statement of any assumptions made.

2. Candidates may use one of two calculators, the Casio or Sharp approved models.

This is a CLOSED BOOK exam. However, candidates are permitted to bring the following into the Examination Room:

- ONE Aid Sheet 8.5" x 11" hand-written on both sides containing notes and formulae. **Example problems and solutions to problems are NOT ALLOWD!**

3. Any FIVE (5) questions (out of 8 given) constitute a complete paper. Only the first five questions as they appear in your Answer Book(s) will be marked.

4. All questions are of equal value.

5. Information on geometric properties of wide flange or W shaped sections is provided in Appendices (at the end of this Question Paper). There are two pages. Note that this information may not be required.

Note: <u>This Question Paper along with the Aid Sheet (if used) must be handed in with the Answer Book(s).</u>
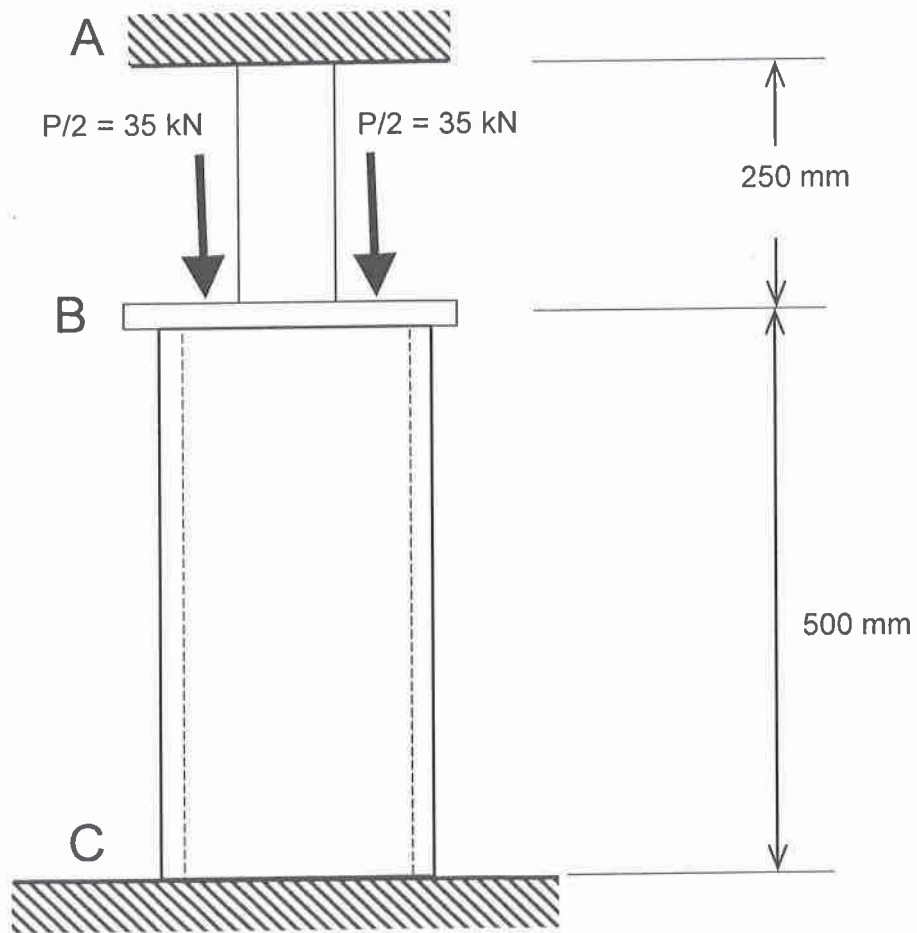
Your Exam will not be marked if you do not hand in the Aid Sheet, unless there is a signed statement by the exam invigilator stating that no Aid Sheet was used in the exam writing.

Questions 1: A rigid plate at B is used to transfer a load $P = 70\ kN$ to a steel rod AB ($E = 200\ GPa$ and $\sigma_Y = 350\ MPa$) and pipe BC made of an aluminum alloy ($E = 70\ GPa$ and $\sigma_Y = 300\ MPa$). The steel rod AB has a cross-sectional area of $500\ mm^2$ and the aluminum pipe BC has a cross-sectional area of $2,000\ mm^2$. The thickness of the rigid plate may be neglected.

(a) Determine the forces carried by the steel rod AB and aluminum pipe BC.
(b) Determine the displacement of the plate at B.

20 marks

A

P/2 = 35 kN   P/2 = 35 kN

250 mm

B

500 mm

C

04-BS-06: Mechanics of Materials   National Exam May 2018

Questions 2: A simply supported beam with an overhang supports a uniformly-distributed load of 5 kN/m acting on the main span and a concentrated load of 100 kN at the end of the overhang.

Determine the shear and moment throughout the beam as functions of x. This means that you need to give formulas for $V(x)$ and $M(x)$ along the length of the beam.

Next, construct the shear force and bending moment diagrams. Remember to label points of maximum and negative bending moment, as well as any inflection points. Show your work by indicating exactly how you obtained your answer.

20 marks
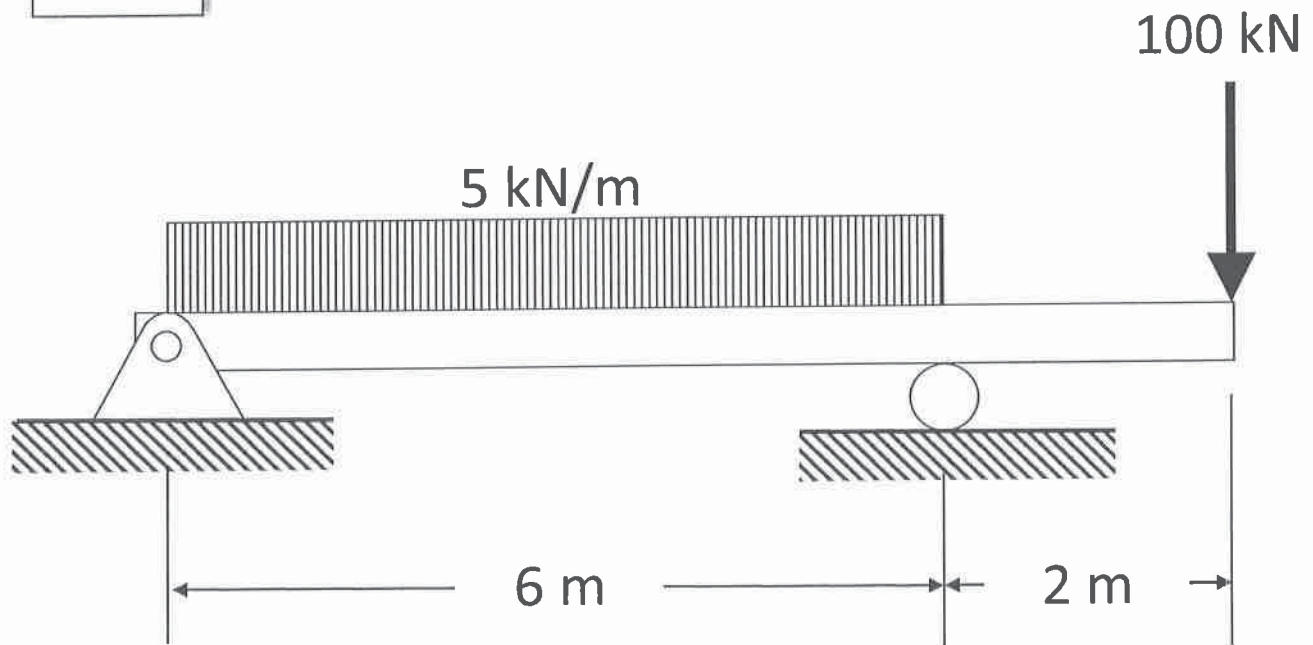
Remember that it is important you set this problem up correctly by calculating the correct reaction forces at the supports.

No credit will be given for a solution using the Principle of Superposition, when combinations of existing solutions are used to find an answer.

Questions 3: A simply supported beam is subjected to a triangular distributed load as shown below. The beam has the cross-section given and is made from steel having an allowable normal stress of $220\ MPa$ and elastic modulus of $200\ GPa$.

Determine the location and value of maximum deflection for the beam using the Method of Integration.

20 marks

6 kN/m

9 m

100

10

300 mm
(typ.)

10

10
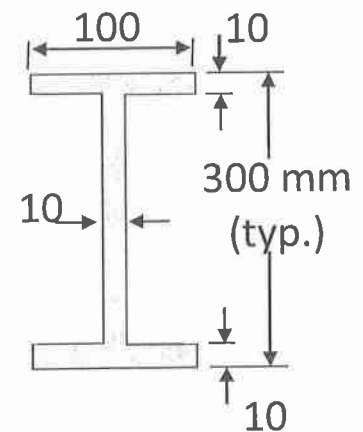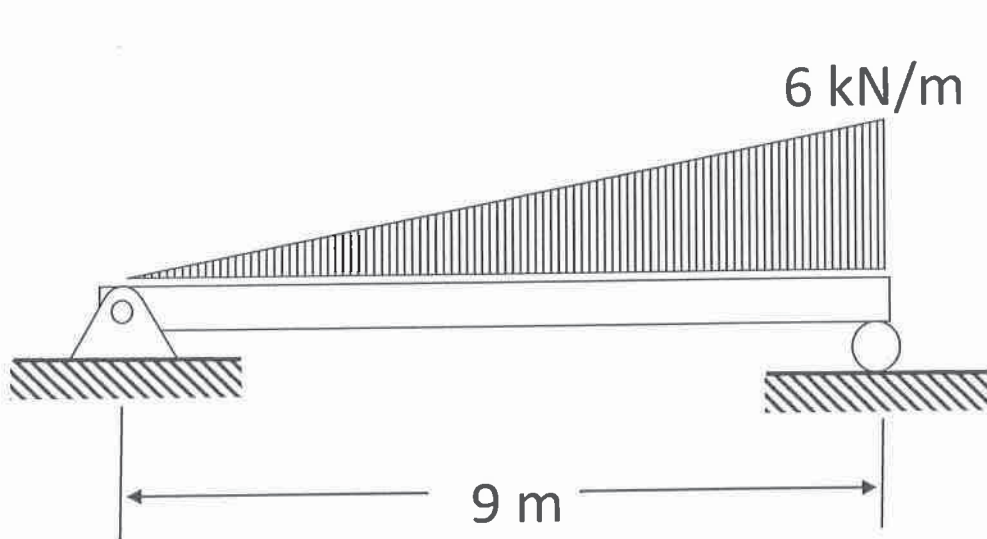
Beam cross-section
(All dimensions in mm)

Questions 4: Use the Mohr's circle solution (*not* the transformation equations) to:

Determine the principal stresses and orientation of the principal plane. Show your answer by sketch on a properly oriented element.

Determine the maximum in-plane shear stress (and associated normal stresses) and orientation of the corresponding planes. Again, show your answer by sketch on a properly oriented element.

20 marks

70 MPa

30 MPa

10 MPa
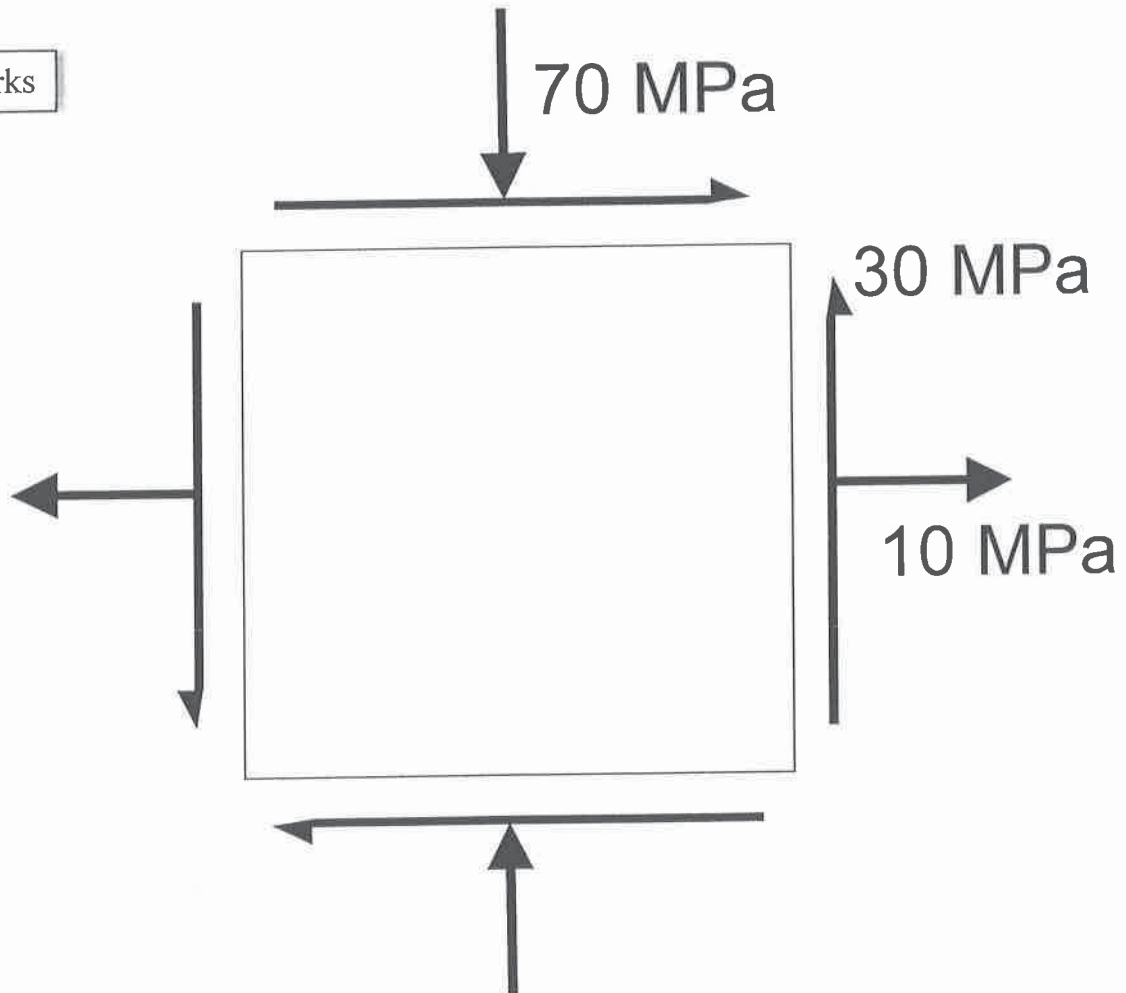
Note that credit will only be **given** for a **solution using Mohr's circle**. This means that you need to draw a Mohr's circle based on the stress components given in this problem. Remember to show numbers on your circle. Your **calculations** must be based on the geometry of your circle. So use your calculator. In other words, you are expected to use trigonometry to construct your Mohr's circle. Do not give a graphical solution that is scaled off! The stress transformation equations can only be used to check your answer.

Questions 5: A 150 mm x 150 mm timber cross-section is reinforced with a 20 mm thick steel plate at the bottom and 20 mm thick aluminum plate at the top as shown. The beam supports a maximum positive moment of 20 $kN.m$ (bending about the horizontal axis). The elastic moduli of the materials used in the beam are given below.

$$E_{wood} = 10\ GPa, E_{steel} = 200\ GPa, E_{aluminum} = 70\ GPa$$

Determine:

i) Maximum stress in wood

ii) Maximum stress in steel

iii) Maximum stress in aluminum

20 marks
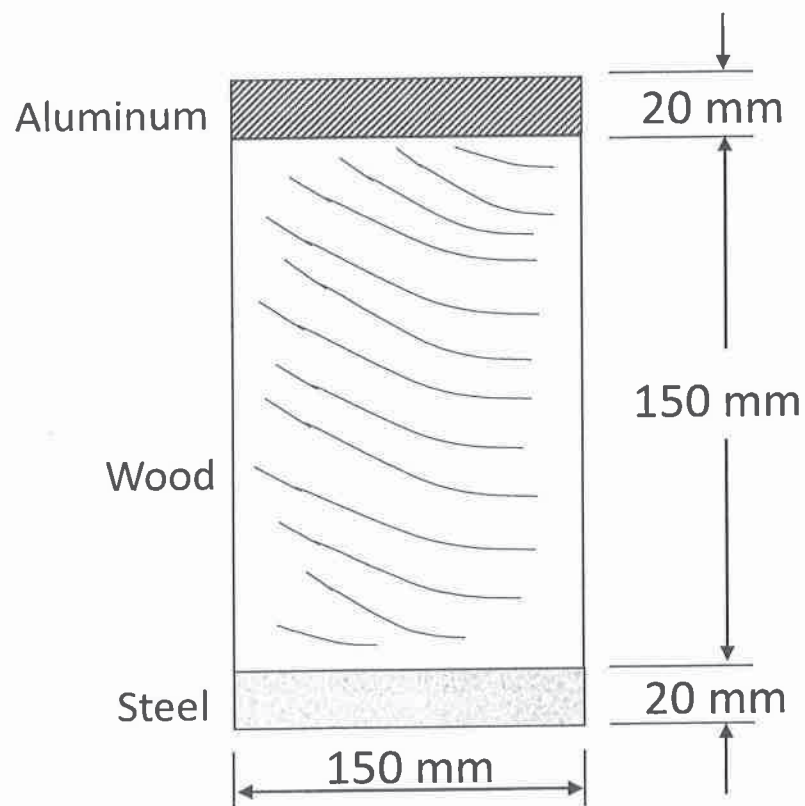
Questions 6: A circular stepped shaft $ABCD$ is fixed at A and subjected to a concentrated torque of $200\ N.m$ clockwise at D, and another concentrated torque of $2,000\ N.m$ counterclockwise at C.

Part of the shaft is made hollow by having a $44\ mm$ diameter hole drilled in the portion AB as shown below. All other dimensions are given in the figure below as well.

The shaft is made of steel with a shear modulus $G = 80\ GPa$, elastic modulus $E = 200\ GPa$, and assumed yield stress (in shear) $\tau_Y = 160\ MPa$.

(a) Determine the maximum shear stress in the stepped shaft, and sketch the corresponding variation of shear stress along the shaft radius at this location.

(b) Find the angle of twist at the end of the shaft (point D).

20 marks

04-BS-06: Mechanics of Materials   National Exam May 2018
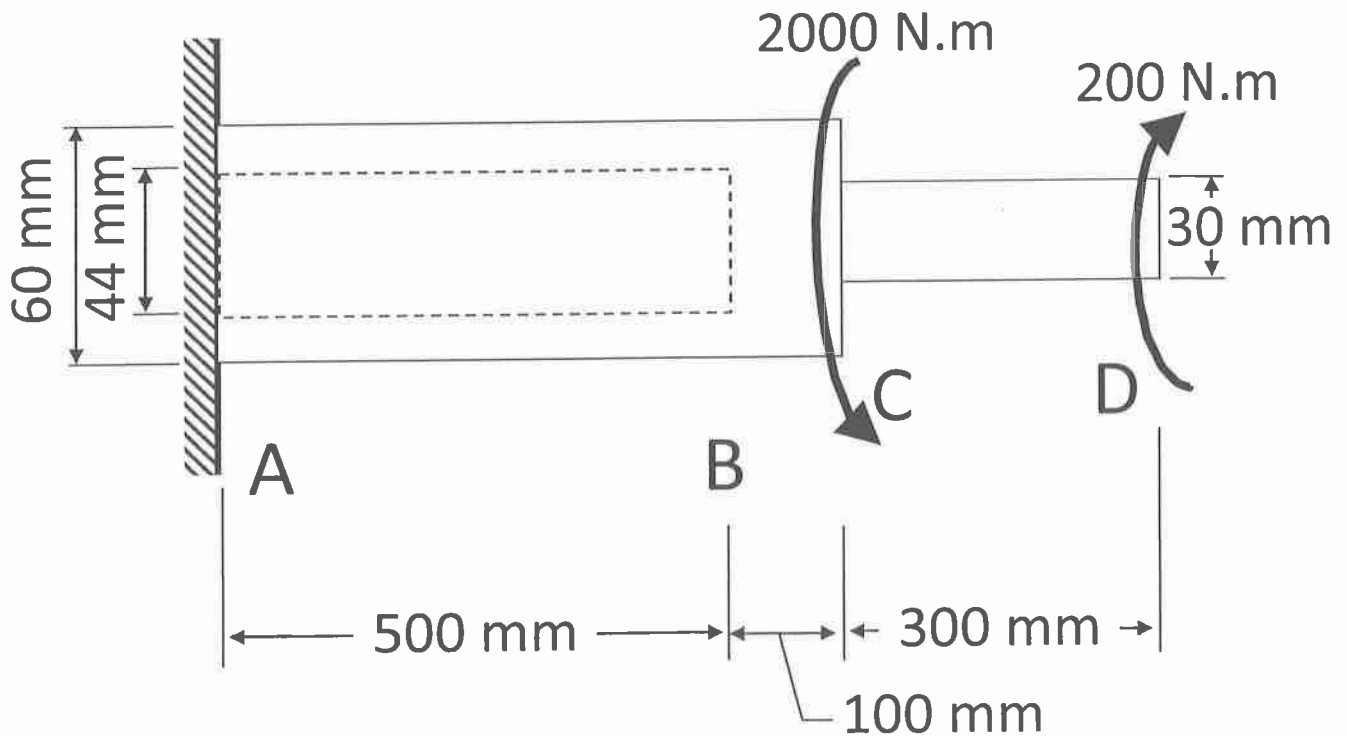
Questions 7: The structural system below uses a 100 mm x 100 mm square bar and 25 mm diameter cable to support a load P. The square bar BC is made of steel with a yield strength of 300 MPa and elastic modulus of 200 GPa, while the cable AC is made of high-strength steel with a yield strength of 1,800 MPa and elastic modulus of 190 GPa.

Determine whether the structure can support a load P of 250 kN as shown.

Consider in-plane buckling only and use a factor of safety of 2 for the Euler buckling load. The safety factor for yielding of steel is 1.5.

20 marks

$P = 250$ kN
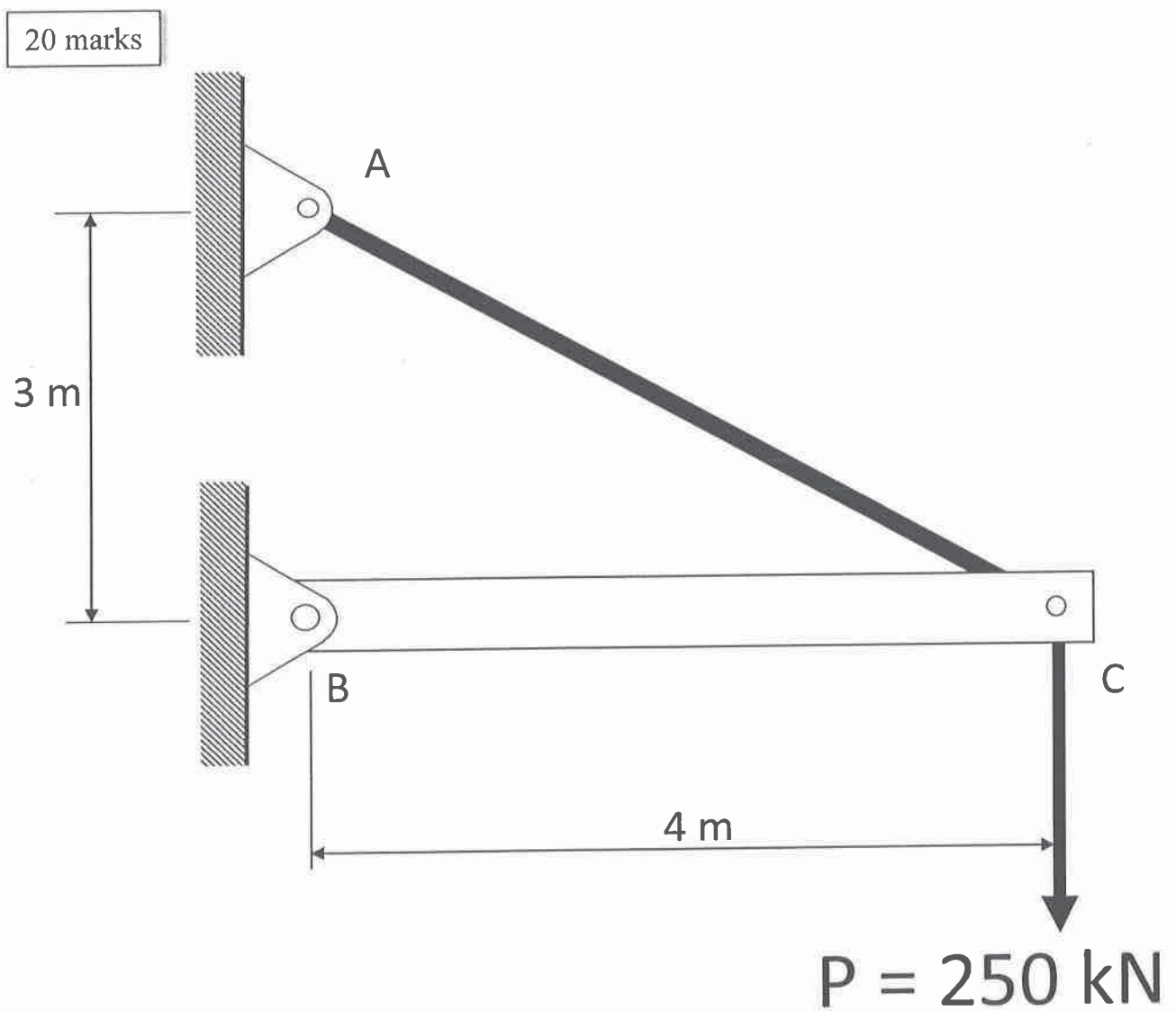
Questions 8: A cantilevered beam (fixed at base) is loaded with a uniformly-distributed load over part of the span in addition to an inclined load at the end of the member as shown below.

(a) Determine the distribution of normal stress at the base of the cantilever and sketch the distribution of stress over the cross-section of the member.

(b) Determine the maximum shear stress in the cross-section at the base of the cantilever.

(c) Determine the shear stress at point E on the cross-section at the base of the cantilever.

20 marks

beam cross-section
(all dimensions in mm)

04-BS-06: Mechanics of Materials   National Exam May 2018

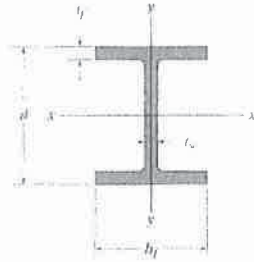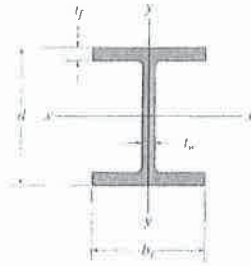| Wide-Flange Sections or W Shapes SI Units											
			Web thickness	Flange		x–x axis			y–y axis		
Designation	Area A	Depth d		width b_f	thickness t_f	I	S	r	I	S	r
mm ⨉ kg/m	mm²	mm	mm	mm	mm	10⁶ mm⁴	10³ mm³	mm	10⁶ mm⁴	10³ mm³	mm
W310 × 129	16 500	318	13.10	308.0	20.6	308	1940	137	100	649	77.8
W310 × 74	9 480	310	9.40	205.0	16.3	165	1060	132	23.4	228	49.7
W310 × 67	8 530	306	8.51	204.0	14.6	145	948	130	20.7	203	49.3
W310 × 39	4 930	310	5.84	165.0	9.7	84.8	547	131	7.23	87.6	38.3
W310 × 33	4 180	313	6.60	102.0	10.8	65.0	415	125	1.92	37.6	21.4
W310 × 24	3 040	305	5.59	101.0	6.7	42.8	281	119	1.16	23.0	19.5
W310 × 21	2 680	303	5.08	101.0	5.7	37.0	244	117	0.986	19.5	19.2
W250 × 149	19 000	282	17.30	263.0	28.4	259	1840	117	86.2	656	67.4
W250 × 80	10 200	256	9.40	255.0	15.6	126	984	111	43.1	338	65.0
W250 × 67	8 560	257	8.89	204.0	15.7	104	809	110	22.2	218	50.9
W250 × 58	7 400	252	8.00	203.0	13.5	87.3	693	109	18.8	185	50.4
W250 × 45	5 700	266	7.62	148.0	13.0	71.1	535	112	7.03	95	35.1
W250 × 28	3 620	260	6.35	102.0	10.0	39.9	307	105	1.78	34.9	22.2
W250 × 22	2 850	254	5.84	102.0	6.9	28.8	227	101	1.22	23.9	20.7
W250 × 18	2 280	251	4.83	101.0	5.3	22.5	179	99.3	0.919	18.2	20.1
W200 × 100	12 700	229	14.50	210.0	23.7	113	987	94.3	36.6	349	53.7
W200 × 86	11 000	222	13.00	209.0	20.6	94.7	853	92.8	31.4	300	53.4
W200 × 71	9 100	216	10.20	206.0	17.4	76.6	709	91.7	25.4	247	52.8
W200 × 59	7 580	210	9.14	205.0	14.2	61.2	583	89.9	20.4	199	51.9
W200 × 46	5 890	203	7.24	203.0	11.0	45.5	448	87.9	15.3	151	51.0
W200 × 36	4 570	201	6.22	165.0	10.2	34.4	342	86.8	7.64	92.6	40.9
W200 × 22	2 860	206	6.22	102.0	8.0	20.0	194	83.6	1.42	27.8	22.3
W150 × 37	4 730	162	8.13	154.0	11.6	22.2	274	68.5	7.07	91.8	38.7
W150 × 30	3 790	157	6.60	153.0	9.3	17.1	218	67.2	5.54	72.4	38.2
W150 × 22	2 860	152	5.84	152.0	6.6	12.1	159	65.0	3.87	50.9	36.8
W150 × 24	3 060	160	6.60	102.0	10.3	13.4	168	66.2	1.83	35.9	24.5
W150 × 18	2 290	153	5.84	102.0	7.1	9.19	120	63.3	1.26	24.7	23.5
W150 × 14	1 730	150	4.32	100.0	5.5	6.84	91.2	62.9	0.912	18.2	23.0

Wide-Flange Sections or W Shapes SI Units

Designation	Area A	Depth d	Web thickness t_w	Flange width b_f	Flange thickness t_f	x–x axis I	x–x axis S	x–x axis r	y–y axis I	y–y axis S	y–y axis r
mm × kg/m	mm²	mm	mm	mm	mm	10^6 mm⁴	10^3 mm³	mm	10^6 mm⁴	10^3 mm³	mm
W610 × 155	19 800	611	12.70	324.0	19.0	1 290	4 220	255	108	667	73.9
W610 × 140	17 900	617	13.10	230.0	22.2	1 120	3 630	250	45.1	392	50.2
W610 × 125	15 900	612	11.90	229.0	19.6	985	3 220	249	39.3	343	49.7
W610 × 113	14 400	608	11.20	228.0	17.3	875	2 880	247	34.3	301	48.8
W610 × 101	12 900	603	10.50	228.0	14.9	764	2 530	243	29.5	259	47.8
W610 × 92	11 800	603	10.90	179.0	15.0	646	2 140	234	14.4	161	34.9
W610 × 82	10 500	599	10.00	178.0	12.8	560	1 870	231	12.1	136	33.9
W460 × 97	12 300	466	11.40	193.0	19.0	445	1 910	190	22.8	236	43.1
W460 × 89	11 400	463	10.50	192.0	17.7	410	1 770	190	20.9	218	42.8
W460 × 82	10 400	460	9.91	191.0	16.0	370	1 610	189	18.6	195	42.3
W460 × 74	9 460	457	9.02	190.0	14.5	333	1 460	188	16.6	175	41.9
W460 × 68	8 730	459	9.14	154.0	15.4	297	1 290	184	9.41	122	32.8
W460 × 60	7 590	455	8.00	153.0	13.3	255	1 120	183	7.96	104	32.4
W460 × 52	6 640	450	7.62	152.0	10.8	212	942	179	6.34	83.4	30.9
W410 × 85	10 800	417	10.90	181.0	18.2	315	1 510	171	18.0	199	40.8
W410 × 74	9 510	413	9.65	180.0	16.0	275	1 330	170	15.6	173	40.5
W410 × 67	8 560	410	8.76	179.0	14.4	245	1 200	169	13.8	154	40.2
W410 × 53	6 820	403	7.49	177.0	10.9	186	923	165	10.1	114	38.5
W410 × 46	5 890	403	6.99	140.0	11.2	156	774	163	5.14	73.4	29.5
W410 × 39	4 960	399	6.35	140.0	8.8	126	632	159	4.02	57.4	28.5
W360 × 79	10 100	354	9.40	205.0	16.8	227	1 280	150	24.2	236	48.9
W360 × 64	8 150	347	7.75	203.0	13.5	179	1 030	148	18.8	185	48.0
W360 × 57	7 200	358	7.87	172.0	13.1	160	894	149	11.1	129	39.3
W360 × 51	6 450	355	7.24	171.0	11.6	141	794	148	9.68	113	38.7
W360 × 45	5 710	352	6.86	171.0	9.8	121	688	146	8.16	95.4	37.8
W360 × 39	4 960	353	6.48	128.0	10.7	102	578	143	3.75	58.6	27.5
W360 × 33	4 190	349	5.84	127.0	8.5	82.9	475	141	2.91	45.8	26.4

04-BS-06: Mechanics of Materials   National Exam May 2018