

# National Exams May 2019

17-Ind-A1

Operations Research

3 hours duration

## NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. Any non-communicating calculator is permitted. This is an Open Book exam. Note to candidates: You must indicate the type of calculator being used. i.e. write the name and model designation of the calculator, on the first left hand sheet of the exam workbook.
3. There may be more questions than you are able to answer in the allotted time. Although the total value of the questions is 175, any marks achieved will be considered toward the 100 total requirements.

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1. A company has one item in its inventory that requires special storage. The company estimates the storage cost at \$2.00 per unit per year. The demand for the item is 40,000 units per year, and the ordering cost is \$16.00 per order.
    - a. Assuming shortages are not allowed, write an expression for the total yearly cost in terms of the order quantity  $Q$ , and determine the optimal order quantity.
    - b. Assuming shortages are allowed and cost \$4.00 per unit per year, write an expression for the total yearly cost in terms of the order quantity  $Q$  and the maximum shortage level  $s$ , and determine the optimal order quantity.
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2. A company that conducts surveys, is going to bid on a job that has the following requirements
    - a. There must be at least 300 personal interviews
    - b. There must be at least 500 interviews (personal or telephone) conducted at night.
    - c. Of the day interviews, at least 60% of them must be by telephone.
    - d. There must be at least 1000 total interviews ((personal or telephone)

The cost of doing an interview is as follows:

	Personal	Telephone
Day	\$2.00	\$1.00
Night	\$2.10	\$1.20

In preparing the bid the company wants to know the minimum cost of meeting the requirements.

Define the decision variables and set up the linear programming model.  
Do not solve.

- 10 3. Use a graphical procedure to solve the problem

$$\text{Maximize } 6x_1 + 4x_2$$

Subject to

$$2x_1 + x_2 \leq 10$$

$$x_1 + x_2 \leq 8$$

$$x_2 \leq 7$$

$$x_1 \geq 0$$

$$x_2 \geq 0$$

- 15 4. Consider the following problem

$$\text{Maximize } z = x_1 + 2x_2$$

Subject to

$$x_1 + 3x_2 \leq 16 \quad \text{constraint 1}$$

$$x_1 + x_2 \leq 7 \quad \text{constraint 2}$$

$$x_1 \geq 0$$

$$x_2 \geq 0$$

The simplex method yields the following final set of equations

$$z + (1/2)x_3 + (1/2)x_4 = (23/2)$$

$$x_1 - (1/2)x_3 + (3/2)x_4 = (5/2)$$

$$x_2 + (1/2)x_3 - (1/2)x_4 = (9/2)$$

where  $x_3$  is the slack variable for constraint 1, and  $x_4$  is the slack variable for constraint 2.

- What is the optimal solution?
- Which variables are basic variables, and which are non-basic in the optimal solution?
- How much can the coefficient of  $x_2$  in the objective function vary without affecting the optimal solution?
- How much can the right-hand side of constraint 1 vary, before there would be a change in answer (b)?

- 15 5. The sales manager of a publisher of university textbooks has six travelling sales staff to assign to three different regions. She has decided that each region should be assigned one or more dedicated sales staff. The estimated sales per region varies with the number of staff assigned as follows:

No. of sales staff	Region 1	Region 2	Region 3
1	35	21	28
2	48	42	41
3	60	56	53
4	69	70	65

Use dynamic programming to determine how the six travelling sales staff should be assigned to the three regions to maximize the total sales.

- 15 6. A single overhead crane attends to re-load ten machines. When a machine finishes its load, the overhead crane is called to unload the machine and to provide it with a new load from an adjacent storage area. The machine time per load is assumed exponential with mean 30 minutes. The time from the moment the crane moves to service a machine until a new load is installed is also exponential with mean 10 minutes.

- What percentage of time is the crane idle?
- What is the expected number of machines waiting for crane service?

- 20 7. An oil company is installing an oil pipeline from an oil field to a refinery. The pipeline requires the welding of 1000 seams, to be carried out by the company's own welders. Defective seams result in leaks, which must be reworked at a cost of \$1,200 per seam. It is estimated from past experience that 5% of the seams will be defective with probability 0.30, or 10% will be defective with probability 0.50, or 20% will be defective with probability 0.20. The company can also hire an expert cleanup team of welders at a one-time cost of \$130,000, who would check all of the welds done by the company welders and repair them as required.

- Based on an expected value criterion, should the company bring in the expert clean-up team to check and rework the welds, or repair the welds as they occur?
- The company can also improve its information about the quality of its own welders on this job, by x-ray inspection of a randomly selected completed weld at a cost of \$2,000. Is it worthwhile to carry out this inspection?

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8. Every time a machine breaks down it requires 1, 2, or 3 hours to fix it according to the following probability distribution.

Repair time (hours)	Probability
1	0.30
2	0.50
3	0.20

The number of machine breakdowns per day is given by

Machine breakdowns per day	Probability
0	0.30
1	0.30
2	0.40

- a) Draw a flowchart for a procedure that will estimate the average amount of work required in a day to fix machines.
- b) Using the following random number list, carry out the simulation for two days operation, and compare the simulated average with the theoretical average amount of repair work per day.

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9. A company is introducing a new product into the market. If the sales are high, there is a 0.5 probability that they will remain so next month. If they are low, the probability that they will become high next month is only 0.2. The company has the option of launching an advertisement campaign before releasing the product to market. If it does and the sales are high, the probability that they will remain high next month will increase to 0.8. On the other hand, an advertising campaign while the sales are low will raise the probability to only 0.4.

If no advertisement is used and the sales are high, the returns are expected to be 10 if the sales remain high next month, and 4 if they do not. If the sales are low at first and then rise to high the next month the expected return is 7. If they remain low the next month there is a return of -2. Using advertisement will result in returns of 7 if the product starts with high sales and continues to be so, and 6 if it does not. If the sales start low, the returns are -5 and 3, depending on whether or not they remain low.

Determine the company's optimal policy over the next 2 months.

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10. A particular project consists of nine tasks. The crash times, normal times,  $b_{i,j}$  values giving the cost increase associated with a unit time saving for activity (i,j), and precedence relationships are given below:

Task	Min Task Time	Normal Task Time	$b_{i,j}$	Immediate Predecessors
A	1	3	4	none
B	2	4	1	none
C	0.5	2	1	A
D	2	5	1	A
E	1	6	3	B, C
F	1	2	7	D, E
G	3	4	9	D, E
H	2	3	5	F
I	4	5	8	G

- a) Draw an appropriate project diagram and identify the tasks on the critical path.
- b) Write, but do not solve, an LP model for the problem of determining optimal activity times for completing the project by a specified deadline.