

**National Exams December 2019**

**16-Chem-A6, Process Dynamics and Control**

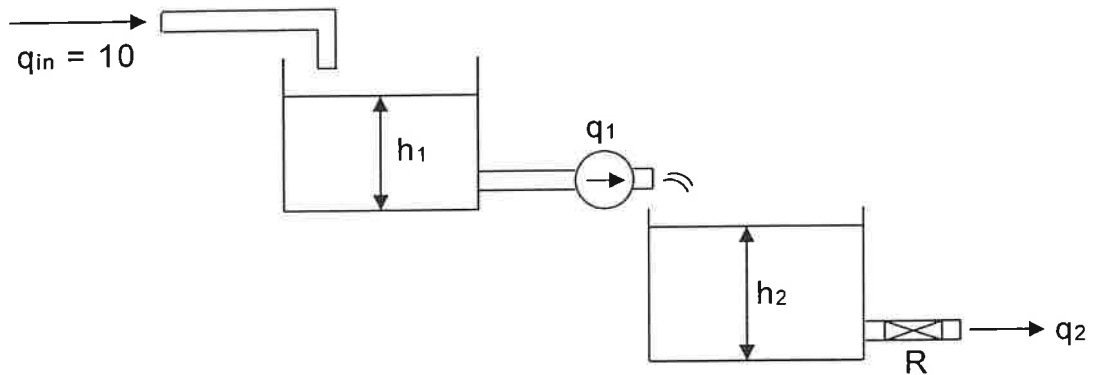
3 hours duration

**NOTES:**

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. This is an OPEN BOOK EXAM  
Any non-communicating calculator is permitted.
3. FIVE (5) questions constitute a complete exam paper.  
The first five questions as they appear in the answer book will be marked.
4. Each question is of equal value.
5. Most questions require an answer in essay format. Clarity and organization of the answer are important.

Note: If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made

**Problem #1** (20% total)



Two tanks are connected in series in a non-interacting fashion as shown in the figure.

Assume:  $\rho = 1$   $A = 1$  ( $A$ -cross-section of each tank)

$$q_2 = \frac{1}{R} \sqrt{\frac{\Delta P}{\rho g}} \text{ and } q_1 \text{ is determined by a pump.}$$

The initial value of the inlet flowrate is  $q_{in} = 10$  and remains constant. The initial level in tank 1 is  $h_1(t=0) = 10$ .  $q_1$  is the manipulated variable. All  $q$ 's are volumetric flow rates.  $R = 2$ .

- (10%) (a) Show the differential equations that describe the behaviour of  $h_1(t)$  and  $h_2(t)$ .
- (10%) (b) Compute transfer functions between  $h_1$  to  $q_{in}$  and  $h_2$  to  $q_{in}$ .

Note: If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made

**Problem #2** (20% total)

A process is described by the following transfer function:

$$G_p = \frac{10(1-s)e^{-10s}}{100s+1}$$

- (10%) (a) Design an IMC (Internal Model Controller) for this process. Show your design with a block diagram.
- (10%) (b) Assuming a perfect model of the process, compute the closed loop response for a unit step in the set point if the desired closed loop time constant is equal to 10.

Note: If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made

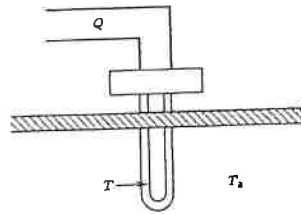
**Problem #3** (20% total)

The heating element shown in the drawing below transfers heat largely by a radiation mechanism. If the rate of electrical energy input to the heater is  $Q$  and the rod temperature and ambient temperatures are, respectively,  $T$  and  $T_a$ , then an appropriate unsteady-state model for the system is

$$mC \frac{dT}{dt} = Q - k(T^4 - T_a^4)$$

$m$  is the mass of the heater,  $C$  is specific heat and  $k$  is radiation coefficient.

- (15%) a) Linearize this model and then find the transfer functions relating  $\delta T$  to  $\delta Q$  and  $\delta T$  to  $\delta T_a$ . (Be sure they are both in standard form, i.e. show gain and time constant.)



- (5%) b) If you were to design a proportional feedback controller to control  $T$  by manipulating  $Q$ , what should be the sign of the controller to guarantee stability? Justify your answer.

Note: If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made

**PROBLEM 4 (20%)**

A thermometer with a time constant of 0.2 min is immersed in a temperature bath and after the thermometer comes to equilibrium with the bath, the bath temperature is increased linearly with time at the rate of 1 °C / min.

10% (a) what is the difference between the indicated temperature and bath temperature (i) 0.1 min (ii) 10 min after the change in temperature is applied?

5% (b) What is the maximum deviation between the indicated temperature and bath temperature and when does it occurs?

5% (c) Plot the forcing function and the response on the same graph. After a long enough time by how many minutes the response will lag after the input?

**PROBLEM 5 (20%)**

The input ( $e$ , where  $e$  is the feedback error) to a PI controller is as follows:

$$\text{for } 0 \leq t < 1 \quad e = 0.5,$$

$$\text{for } 1 \leq t < 2 \quad e = 0$$

$$\text{for } 2 \leq t < 3 \quad e = -0.5$$

$$\text{for } t \geq 3 \quad e = 0$$

10% a) Find the Laplace transform  $E(s)$  (transform of  $e(t)$ ).

10% b) Find and plot the output of the controller with proportional gain

$K_c = 2$  and reset time (integration constant)  $\tau_I = 0.5$  min.

Note: If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made

**PROBLEM 6** (20%)

The characteristic equation of a closed loop system is given by:

$$C(s) = s^4 + 4s^3 + 6s^2 + 4s + (1 + K_c)$$

10% a) Find the range of values of the gain  $K_c$  for which the closed loop is stable.

10% b) Determine the values of  $K_c$  for which the closed loop is at the limit of stability.

**PROBLEM 7** (20%)

Find the inverse transform for the following functions:

(10%) a)  $Y(s) = \frac{s-3}{s(s^2-6s+18)}$  , is the corresponding time response stable?

(10%) b)  $Y(s) = \frac{s-3}{s^2(s^2-6s+18)}$  , is the corresponding time response stable?

Note: If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made

**PROBLEM 8** (20%)

Consider a closed loop system composed of the following elements: a- proportional controller with gain  $K_c$ , b-process with transfer function  $G_p = \frac{1}{(s+1)^3}$  and c-sensor with transfer function  $H$ .

(5%) a) Find the largest gain  $K_c$  for which the closed loop system is stable for the following two cases: i)  $H=1$  and ii)  $H = e^{-0.7s}$ . Do not use any approximation for the delay.

(5%) b) Plot the Bode plots (amplitude ratio normalized and phase) for case ii in item 1 above corresponding to the frequency response of the product  $K_c \cdot G_p \cdot H$ . Indicate clearly asymptotes, corner frequency, value of slopes of asymptotes and extreme values of the phase angle for very small and very large values of frequencies.

(10%) c) If  $K_c=1$ , calculate the gain and phase margins for case i and ii in item a) above.