NATIONAL EXAMS December 2014
07-Elec-B2 Advanced Control Systems

3 hours duration

NOTES:

1. If a doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Candidates may use one of two calculators, **a Casio or Sharp approved model.**

3. This is a closed-book examination. Tables of Laplace and z-transforms are attached as page 4 and page 5.

4. Any four questions constitute a complete paper. Only the first four questions as they appear in your answer book will be marked.

5. All questions are of equal value.

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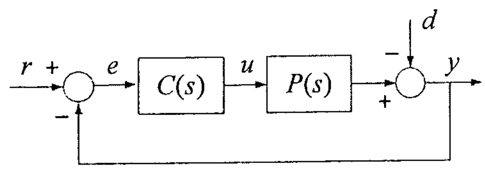
1. Consider the control system below with, $P(s) = \frac{16}{(s+2)^2}$, $C(s) = \frac{sK_p + K_i}{s}$

(a) Design a proportional-integral control system such that the phase margin is at least 45 degrees and the gain crossover is as large as feasible.

(b) Sketch the magnitude of the Bode plots associated with the closed loop transfer function that relates y to r, and the closed loop transfer function that relates y to d.



(c) Determine the steady state output, y, for constant inputs, $r = r_0$ and $d = d_0$.

2. Consider the open loop dynamics of a satellite attitude control system,

$$\dot{\theta}(t) = \omega(t)$$
$$\ddot{\theta}(t) = -3\theta(t) - u(t)$$
$$\dot{h}(t) = u(t)$$

Define the state vector, $x(t) = [\theta(t) \quad \omega(t) \quad h(t)]^T$, the control input, $u(t)$, and the output, $\omega(t)$.

(a) Determine a state space model for the open loop system.

(b) Is the system controllable and observable? Justify your answer.

(c) For the open loop system, let $\theta(0) = 1$, $\dot{\theta}(0) = 0$, $h(0) = 0$, $u(t) = 0$. Determine $\omega(t)$.

(d) Find a state feedback controller, if it exists, such that the closed loop poles are all located at s = -2 + j, -2 - j, -4.

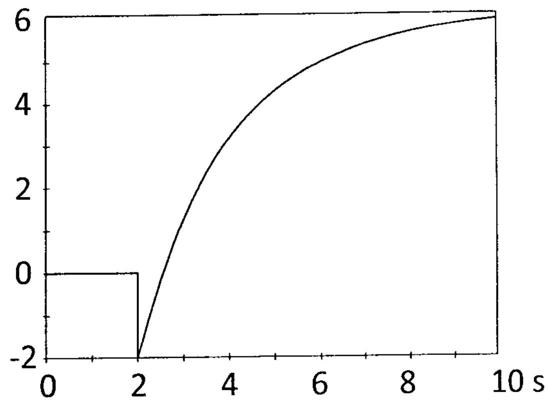
3. The discrete model for a system has the form, $Y(z) = P(z)U(z)$, where, $P(z) = \frac{b}{z-a}$.

(a) Measurements of $u(k)$ and $y(k)$ are taken at time instants, k, as listed in the Table. Find a least squares estimate for a and b.

k	$y(k)$	$u(k)$
0	300	75
1	387	30
2	375	15
3	324	0

(b) If $u(k) = 2$, what is the steady state output as predicted by the identified model?

4. Determine the transfer functions, $P(s)$ and $G(s)$ below.

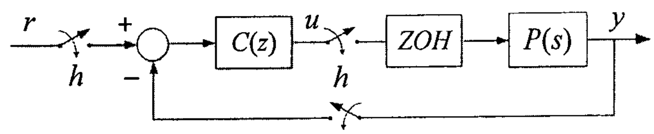


(a) A unit step is applied at the input of an open loop plant, $P(s)$, at time $t = 0$. The measured response is shown on the right. Determine the transfer function, $P(s)$.

(b) When a step of magnitude 2 is applied to the input of a plant, $G(s)$, the steady state output is 10. When a sinusoid of amplitude 2 and frequency 8 rad/sec is applied, the phase lag at the output is 90° and the output amplitude is 15. Assume the system is second order system and has no finite zeros. Find the transfer function, $G(s)$.

5. Consider the sampled data and digital control system below. The input to the ZOH and (continuous) output, y, are uniformly sampled with a sample period of h. $C(z)$ and $P(s)$ *are* given by,

$$C(z) = Kz^{-1}, \quad P(s) = \frac{1}{3s}$$



(a) Determine the discrete closed loop transfer function, $T(z)$, that relates $Y(z)$ to $R(z)$.

(b) Sketch and annotate the root locus as K varies from zero to infinity.

(c) Is the closed loop system stable for all values of K? If not determine the limiting value of K for stability.

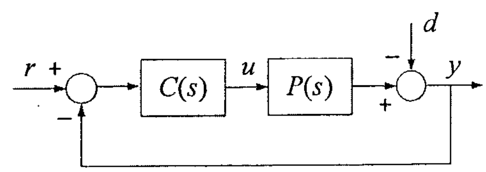
(d) Assume $Kh = 1.5$, and r is initially zero up until $t = 0$, with all initial conditions zero. Suddenly r changes as indicated below.

$$r(0) = 1,\ r(h) = 0,\ r(2h) = 0,\ r(3h) = 0,\ r(4h) = 0$$

Sketch and carefully annotate the transient response of the continuous output, $y(t)$, for $0 \le t \le 4h$.

6. Consider the feedback system below with,

$$C(s) = K, \quad P(z) = \frac{e^{-s}}{8s}.$$



(a) Construct (approximately to scale) the polar or Nyquist plot for the (open) loop transfer function with $K = 1$.

(b) Determine the gain and phase margin.

(c) Determine the value of K that results in a phase margin of 50 degrees.

(d) Determine the steady state value of y when r is a step of magnitude r_0 and d is ramp of slope d_0.

Inverse Laplace Transforms	
$F(s)$	$f(t)$
$\dfrac{A}{s+\alpha}$	$Ae^{-\alpha t}$
$\dfrac{C+jD}{s+\alpha+j\beta}+\dfrac{C-jD}{s+\alpha-j\beta}$	$2e^{-\alpha t}\left(C\cos\beta t+D\sin\beta t\right)$
$\dfrac{A}{(s+\alpha)^{n+1}}$	$\dfrac{At^{n}e^{-\alpha t}}{n!}$
$\dfrac{C+jD}{(s+\alpha+j\beta)^{n+1}}+\dfrac{C-jD}{(s+\alpha-j\beta)^{n+1}}$	$\dfrac{2t^{n}e^{-\alpha t}}{n!}\left(C\cos\beta t+D\sin\beta t\right)$

Inverse z-Transforms	
$F(z)$	$f(nT)$
$\dfrac{Kz}{z-a}$	Ka^{n}
$\dfrac{(C+jD)z}{z-re^{j\varphi}}+\dfrac{(C-jD)z}{z-re^{-j\varphi}}$	$2r^{n}\left(C\cos n\varphi-D\sin n\varphi\right)$
$\dfrac{Kz}{(z-a)^{r}}$, $r=2,3\ldots$	$\dfrac{Kn(n-1)\ldots(n-r+2)}{(r-1)!a^{r-1}}a^{n}$

Table of Laplace and z-Transforms (h denotes the sample period)		
$f(t)$	$F(s)$	$F(z)$
unit impulse	1	1
unit step	$\frac{1}{s}$	$\frac{z}{z-1}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$	$\frac{z}{z-e^{\alpha h}}$
t	$\frac{1}{s^2}$	$\frac{hz}{(z-1)^2}$
$\cos \beta t$	$\frac{s}{s^2+\beta^2}$	$\frac{z(z-\cos\beta h)}{z^2-2z\cos\beta h+1}$
$\sin \beta t$	$\frac{\beta}{s^2+\beta^2}$	$\frac{z\sin\beta h}{z^2-2z\cos\beta h+1}$
$e^{-\alpha t}\cos \beta t$	$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$	$\frac{z(z-e^{-\alpha h}\cos\beta h)}{z^2-2ze^{-\alpha h}\cos\beta h+e^{-2\alpha h}}$
$e^{-\alpha t}\sin \beta t$	$\frac{\beta}{(s+\alpha)^2+\beta^2}$	$\frac{ze^{-\alpha h}\sin\beta h}{z^2-2ze^{-\alpha h}\cos\beta h+e^{-2\alpha h}}$
$t\,f(t)$	$-\frac{dF(s)}{ds}$	$-zh\frac{dF(z)}{dz}$
$e^{-\alpha t}f(t)$	$F(s+\alpha)$	$F(ze^{\alpha h})$