Professional Engineers Ontario

National Exams - May 2015

07-Str-B3

Applications of the Finite Element Method

3 hours duration

Notes:

1. There are 4 pages in this examination, including the front page.
2. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
3. This is a closed book examination, with two 8½×11 in. pages of hand written notes.
4. Candidates may use one of the approved non-communicating calculators.
5. **Attempt to answer all three problems.**
6. All problems are of equal value.

(May 2015)

Problem 1:
(You have to answer questions *1.* through *10*)

1. The solution of an elasticity problem obtained by the Finite Element Method does not verify equilibrium within the domain, explain why?

2. Select the correct continuation to the following statement:

·The strain energy of an elastic structure calculated from a finite element solution is:

- ☐ higher than the exact value
- ☐ equal to the exact value
- ☐ lower than the exact value

3. Draw the approximate shape functions for the two truss elements shown in figure 1-a and 1-b.

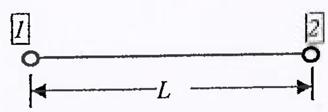


Figure 1-a

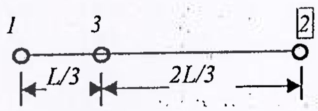


Figure 1-b

4. What does the term "hybrid finite element" mean?

5. What does the term "reduced integration" mean?

6. Identify the defects associated with connecting four-node and eight-node elements in Figure 1-c.

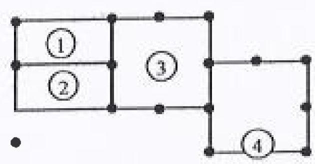


Figure 1-c

7. How many zero eigenvalues has a square bilinear element in plain strain condition? What is the physical significance of these modes?

8. In the context of dynamic analysis, explain the difference between "consistent" and "diagonal" mass matrices. What are the advantages and disadvantages of using each one of them?

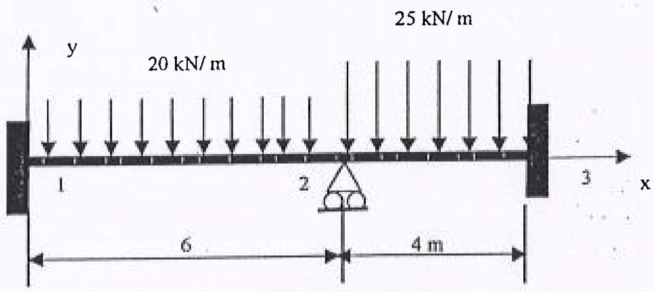
9. How do you proceed to analyse a reinforced concrete beam strengthened with a steel plate on the bottom face using the finite element method?

10. What is the difference between "Euler-Bernouilli" and "Timoshenko" beam elements? When do you use each one of them?

(May 2015)

Problem 2

Using two beam elements, calculate the reactions and draw the shear and moment diagrams for the structure shown below (Figure 2). All members have the same rigidity. $E = 200\ GPa,\ I = 40 \times 10^6\ mm^4$.



(Figure. 2)

The stiffness matrix of the beam element is shown below.

$$[k] = \frac{EI}{L}\begin{bmatrix} \frac{12}{L^2} & \frac{6}{L} & -\frac{12}{L^2} & \frac{6}{L} \\ \frac{6}{L} & 4 & -\frac{6}{L} & 2 \\ -\frac{12}{L^2} & -\frac{6}{L} & \frac{12}{L^2} & -\frac{6}{L} \\ \frac{6}{L} & 2 & -\frac{6}{L} & 4 \end{bmatrix}$$

(May 2015)

Problem 3

3.1 For the linear triangular element shown in Figure 3-a, the strain-displacement matrix, $[B]$,is constant and given by the following equation:

$$\{\varepsilon\} = \frac{1}{2A_T}\begin{bmatrix} \beta_1 & 0 & | & \beta_2 & 0 & | & \beta_3 & 0 \\ 0 & \gamma_1 & | & 0 & \gamma_2 & | & 0 & \gamma_3 \\ \gamma_1 & \beta_1 & | & \gamma_2 & \beta_2 & | & \gamma_3 & \beta_3 \end{bmatrix}\begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix} = \begin{bmatrix} B_1 & | & B_2 & | & B_3 \end{bmatrix}\begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix} = [B]\{d^e\}$$

where A_T is the triangle area, the six coefficients β_i and γ_i $(i = 1..3)$ are given by :

$$\beta_1 = y_2 - y_3, \quad \beta_2 = y_3 - y_1, \quad \beta_3 = y_1 - y_2$$
$$\gamma_1 = x_3 - x_2, \quad \gamma_2 = x_1 - x_3, \quad \gamma_3 = x_2 - x_1$$

According to this notation the stiffness matrix of the element can be written in the following form:

$$[k] = \begin{bmatrix} [k_{11}] & [k_{12}] & [k_{13}] \\ & [k_{22}] & [k_{23}] \\ sym & & [k_{33}] \end{bmatrix}$$

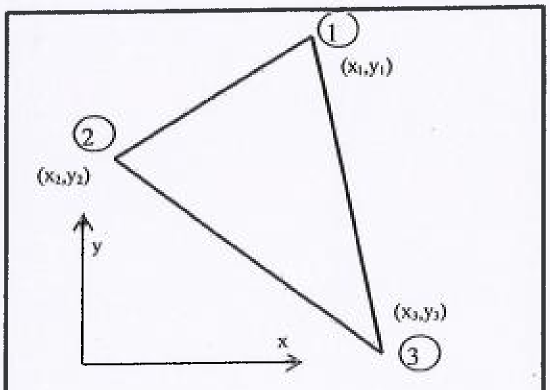


Figure 3-a

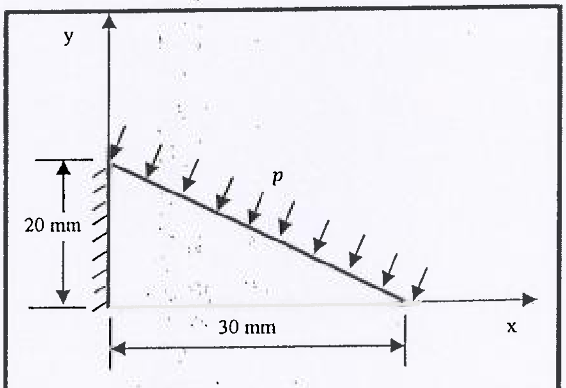


Figure 3-b

Show that for the plane stress condition, the 2×2 matrix $[k_{33}]$ is given by:

$$[k_{33}] = \frac{Et}{4A_T(1-\nu^2)}\begin{bmatrix} \beta_3^2 + \frac{1-\nu}{2}\gamma_3^2 & \left(\frac{1+\nu}{2}\right)\gamma_3\beta_3 \\ \left(\frac{1+\nu}{2}\right)\gamma_3\beta_3 & \gamma_3^2 + \frac{1-\nu}{2}\beta_3^2 \end{bmatrix}$$

with t denotes the element thickness, E and ν are the modulus of elasticity and Poisson ratio, respectively..

3.2 A triangular plate of 30 mm length, 20 mm height and 10 mm thickness is loaded as shown in Figure 3-b. Using only one triangular linear element, determine an approximation of the stress distribution $\sigma_x, \sigma_y, \tau_{xy}$ in the plate. Use $E = 70\ GPa$, $\nu = 0.3$ and $p = 100\ MPa$.

3.3 Comment on the local equilibrium of the approximated stress field within the plate.