16-CHEM-B1, TRANSPORT PHENOMENA

DECEMBER 2019

3 hours duration

NOTES

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. The examination is an **open book exam.** One textbook of your choice with notations listed on the margins etc., but no loose notes are permitted into the exam.
3. Candidates may use any **non-communicating** calculator.
4. All problems are worth 25 points. **One problem** from **each** of sections A, B, and C must be attempted. A **fourth** problem from **any section** must also be attempted.
5. **Only the first four** questions as they appear in the answer book will be marked.
6. State all assumptions clearly.

SECTION A: Fluid Mechanics

A1. An incompressible fluid flows through a tube of circular cross section, for which the tube radius changes linearly from R_0 at the tube entrance to a slightly smaller value R_L at the tube exit. Assume that the Hagen-Poiseuille equation is approximately valid over a differential length, dz, of the tube so that the mass flow rate (w) is

$$w = (\pi\ \rho\ [R(z)]^4/8\mu)\ (-dP/dz)$$

This is a differential equation for P as a function of z, but, when the explicit expression for R(z) is inserted, it is not easily solved.

a) **[2 points]** Write down an expression for R as a function of z.

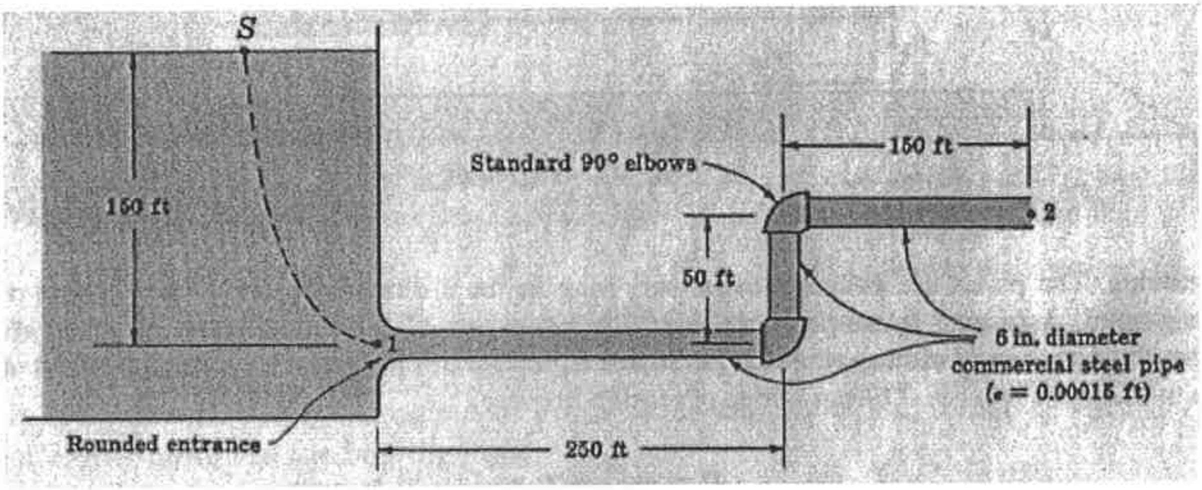
b) **[3 points]** Change the independent variable to R in the above equation and obtain the following expression for mass flow rate.

$$w = \frac{\pi R^4 \rho}{8\mu}\left(-\frac{d\mathcal{P}}{dR}\right)\left(\frac{R_L - R_0}{L}\right)$$

c) **[15 points]** Use the answer from (b) to obtain the following expression for mass flow rate.

$$w = \frac{\pi(\mathcal{P}_0 - \mathcal{P}_L)R_0^4\rho}{8\mu}\left[1 - \frac{1 + (R_L/R_0) + (R_L/R_0)^2 - 3(R_L/R_0)^3}{1 + (R_L/R_0) + (R_L/R_0)^2}\right]$$

A2. Water flows from a large reservoir and discharges into the atmosphere at Point 2 as shown in the figure below:



Determine the volumetric flow rate of water discharged.

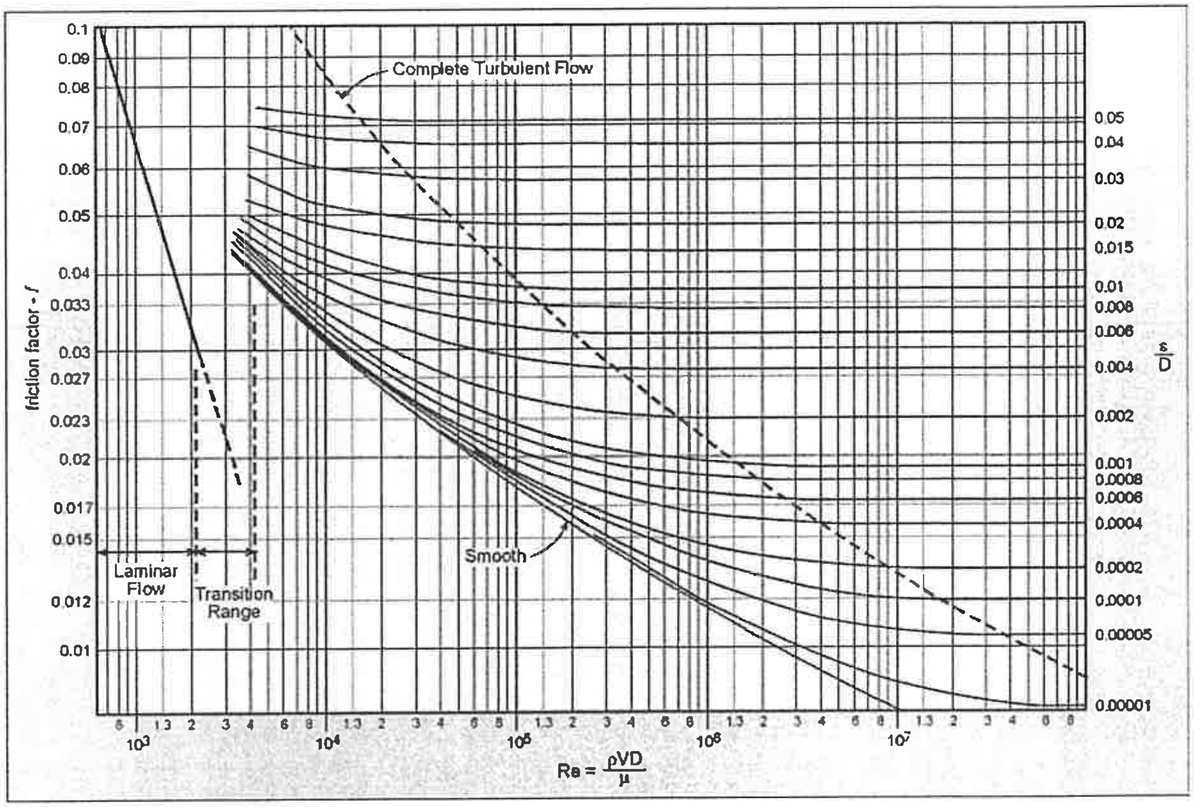
DATA:

Kinematic viscosity of water = 1×10^{-5} ft^2/sec

Loss coefficient for rounded entrance to a tube/pipe = 0.25

Loss coefficient for a 90° elbow = 0.90

Average roughness height (ε) of the commercial steel pipe = 0.00015 ft



Moody friction factor (f) vs. Reynolds number (Re) for pipes

SECTION B: Heat Transfer

B1. Consider an air heater consisting of semicircular tube (radius = 2 cm) for which the plane surface is maintained at 1000 K and the other surface is well insulated. Both surfaces have an emissivity of 0.8. If the atmospheric air flows through the tube at 0.01 kg/s and $T_m = 400$ K,

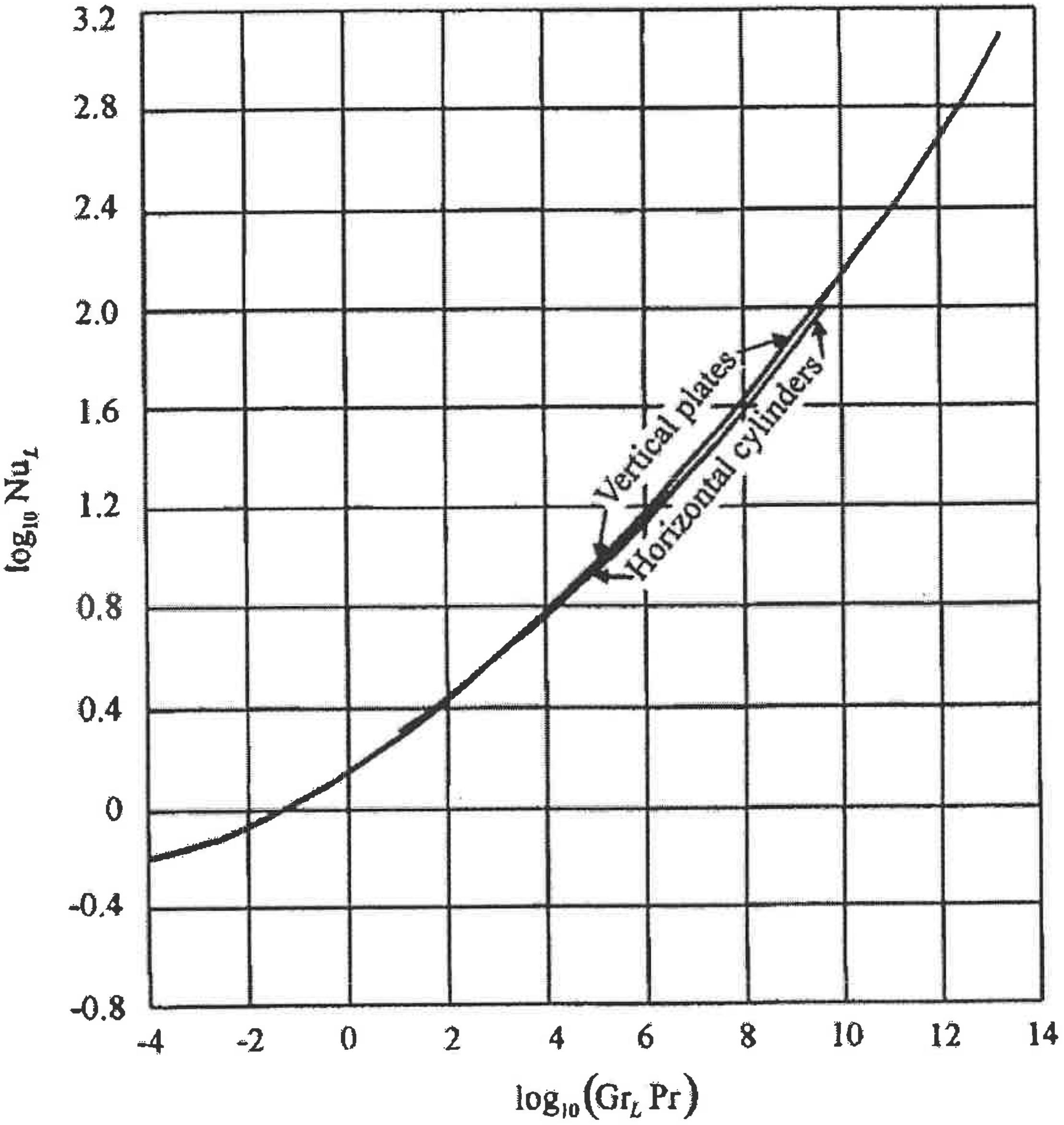
a) **[20 points]** What is the temperature of the insulated surface?

b) **[5 points]** What is the rate at which heat must be supplied per unit length to maintain the plane surface at 1000 K?

B2. An aluminum plate (1.2 m x 1.2 m x 1 cm) is heated uniformly to 370 K. Calculate the initial rate of energy lost for the following conditions:

a) **[12 points]** The plate is cooled in a horizontal position by a stream of air at 290 K flowing with a velocity of 2 m/s.

b) **[13 points]** The plate is suspended vertically in stagnant air at 290 K.

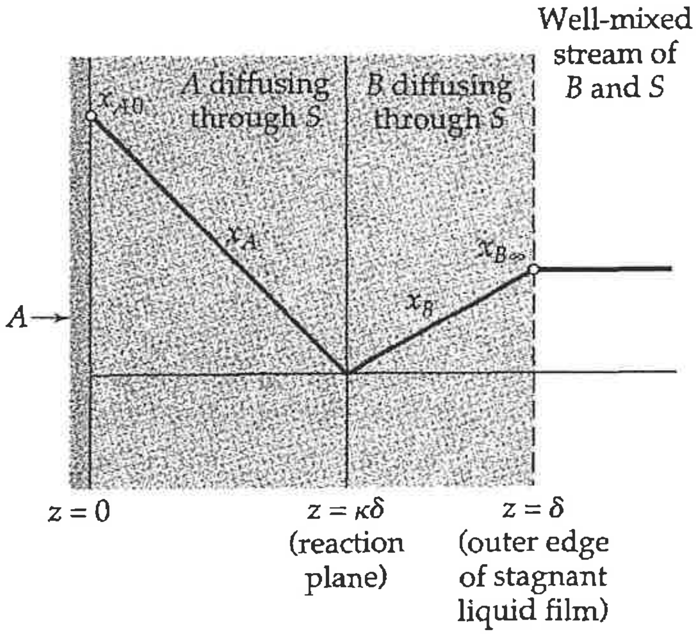


Heat Transfer Coefficients for Natural Convection

"Heat Transmission" by M.H. McAdams, Int. McGraw Hill, New York, 1954.

SECTION C: Mass Transfer

C1. A solid A is dissolving in a flowing liquid stream S in a steady state, isothermal flow system. Assume in accordance with the film model that the surface of A is covered with a stagnant liquid film of thickness δ and that the liquid outside the film is well mixed as shown below.



a) **[7 points]** Develop an expression for the rate of dissolution of A into the liquid if the concentrate of A in the main liquid stream is negligible.

b) **[18 points]** Develop a corresponding expression for the rate of dissolution if the liquid contains a substance B, which, at the plane $z = \kappa\delta$, reacts instantaneously and irreversibly with A. The reaction equation is A + B → Product. The main liquid stream consists primarily of B and S, with B at a mole fraction of x_B.

C2. Show that the general equation for molecular diffusion of a sphere in a stationary medium and in the absence of a chemical reaction is given by:

$$\frac{1}{D}\frac{\partial C_A}{\partial t} = \left(\frac{\partial^2 C_A}{\partial r^2}\right) + \frac{1}{r^2}\left(\frac{\partial^2 C_A}{\partial \theta^2}\right) + \frac{2}{r}\left(\frac{\partial C_A}{\partial r}\right)$$
$$+ \frac{1}{r^2 \sin^2\theta}\left(\frac{\partial^2 C_A}{\partial \varphi^2}\right) + \frac{\cot\theta}{r^2}\left(\frac{\partial C_A}{\partial \theta}\right)$$

where C_A is the concentration of the diffusing substance, D is the molecular diffusivity, t is the time, and r, θ and φ and β are spherical polar coordinates.

APPENDIX

Summary of the Conservation Equations

Table A.1 The Continuity Equation

$\frac{\partial \rho}{\partial t}+(\nabla \cdot \rho\vec{u})=0$	(1.1)
Rectangular coordinates (*x*, *y*, *z*) $\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u_x)+\frac{\partial}{\partial y}(\rho u_y)+\frac{\partial}{\partial z}(\rho u_z)=0$	(1.1a)
Cylindrical coordinates (*r*, *θ*, *z*) $\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r u_r)+\frac{1}{r}\frac{\partial}{\partial \theta}(\rho u_\theta)+\frac{\partial}{\partial z}(\rho u_z)=0$	(1.1b)
Spherical coordinates (*r*, *θ*, *ϕ*) $\frac{\partial \rho}{\partial t}+\frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 u_r)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\rho u_\theta \sin\theta)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(\rho u_\phi)=0$	(1.1c)

Table A.2 The Navier-Stokes equations for Newtonian fluids of constant ρ and μ

	$\frac{\partial \vec{u}}{\partial t}+(\vec{u}\cdot\nabla)\vec{u}=-\frac{1}{\rho}\nabla P+\vec{g}+\nu(\nabla^2\vec{u})$	(A2)
Rectangular coordinates (*x*, *y*, *z*)		
x-component	$\frac{\partial u_x}{\partial t}+u_x\frac{\partial u_x}{\partial x}+u_y\frac{\partial u_x}{\partial y}+u_z\frac{\partial u_x}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+g_x+\nu\left(\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_x}{\partial y^2}+\frac{\partial^2 u_x}{\partial z^2}\right)$	(A2a)
y-component	$\frac{\partial u_y}{\partial t}+u_x\frac{\partial u_y}{\partial x}+u_y\frac{\partial u_y}{\partial y}+u_z\frac{\partial u_y}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial y}+g_y+\nu\left(\frac{\partial^2 u_y}{\partial x^2}+\frac{\partial^2 u_y}{\partial y^2}+\frac{\partial^2 u_y}{\partial z^2}\right)$	(A2b)
z-component	$\frac{\partial u_z}{\partial t}+u_x\frac{\partial u_z}{\partial x}+u_y\frac{\partial u_z}{\partial y}+u_z\frac{\partial u_z}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial z}+g_z+\nu\left(\frac{\partial^2 u_z}{\partial x^2}+\frac{\partial^2 u_z}{\partial y^2}+\frac{\partial^2 u_z}{\partial z^2}\right)$	(A2c)

Cylindrical coordinates (r, θ, z)

r-component

$$\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}+u_z\frac{\partial u_r}{\partial z}-\frac{u_\theta^2}{r}=-\frac{1}{\rho}\frac{\partial P}{\partial r}+g_r+\nu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(ru_r)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial^2 u_r}{\partial z^2}\right] \tag{A2d}$$

θ-component

$$\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+u_z\frac{\partial u_\theta}{\partial z}+\frac{u_r u_\theta}{r}=-\frac{1}{\rho r}\frac{\partial P}{\partial \theta}+g_\theta+\nu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(ru_\theta)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}+\frac{\partial^2 u_\theta}{\partial z^2}\right] \tag{A2e}$$

z-component

$$\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}+u_z\frac{\partial u_z}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial z}+g_z+\nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}+\frac{\partial^2 u_z}{\partial z^2}\right] \tag{A2f}$$

Spherical coordinates (r, θ, ϕ)

r-component

$$\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}+\left(\frac{u_\phi}{r\sin\theta}\right)\frac{\partial u_r}{\partial \phi}-\frac{u_\theta^2}{r}-\frac{u_\phi^2}{r}=-\frac{1}{\rho}\frac{\partial P}{\partial r}+g_r+\nu\left[\frac{1}{r^2}\frac{\partial^2}{\partial r^2}\left(r^2u_r\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial u_r}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_r}{\partial \phi^2}\right] \tag{A2g}$$

θ-component

$$\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+\left(\frac{u_\phi}{r\sin\theta}\right)\frac{\partial u_\theta}{\partial \phi}+\frac{u_r u_\theta}{r}-\frac{u_\phi^2}{r}\cot\theta=-\frac{1}{\rho r}\frac{\partial P}{\partial \theta}+g_\theta+\nu\left[\begin{array}{l}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u_\theta}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(u_\theta\sin\theta\right)\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_\theta}{\partial \phi^2}\\+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}-\frac{2\cot\theta}{r^2\sin\theta}\frac{\partial u_\phi}{\partial \phi}\end{array}\right] \tag{A2h}$$

ϕ-component

$$\frac{\partial u_\phi}{\partial t}+u_r\frac{\partial u_\phi}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\phi}{\partial \theta}+\frac{u_\phi}{r\sin\theta}\frac{\partial u_\phi}{\partial \phi}+\frac{u_r u_\phi}{r}+\frac{u_\theta u_\phi}{r}\cot\theta=-\frac{1}{\rho r\sin\theta}\frac{\partial P}{\partial \phi}+g_\varphi+\nu\left[\begin{array}{l}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u_\phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(u_\phi\sin\theta\right)\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_\phi}{\partial \phi^2}\\+\frac{2}{r^2\sin\theta}\frac{\partial u_r}{\partial \phi}+\frac{2\cot\theta}{r^2\sin\theta}\frac{\partial u_\theta}{\partial \phi}\end{array}\right] \tag{A2i}$$

Table A.3 The Energy Equation for Incompressible Media

$\rho c_P\left[\frac{\partial T}{\partial t}+(\vec{u}\cdot\nabla)(T)\right]=\left[\nabla\cdot k\nabla T\right]+\dot{T}_G$	(A3)
Rectangular coordinates (*x*, *y*, *z*) $\rho c_P\left[\frac{\partial T}{\partial t}+u_x\frac{\partial T}{\partial x}+u_y\frac{\partial T}{\partial y}+u_z\frac{\partial T}{\partial z}\right]=\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{T}_G$	(A3a)
Cylindrical coordinates (*r*, *θ*, *z*) $\rho c_P\left[\frac{\partial T}{\partial t}+u_r\frac{\partial T}{\partial r}+\frac{u_\theta}{r}\frac{\partial T}{\partial \theta}+u_z\frac{\partial T}{\partial z}\right]=\frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k\frac{\partial T}{\partial \theta}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{T}_G$	(A3b)
Spherical coordinates (*r*, *θ*, *ϕ*) $\rho c_P\left[\frac{\partial T}{\partial t}+u_r\frac{\partial T}{\partial r}+\frac{u_\theta}{r}\frac{\partial T}{\partial \theta}+\frac{u_\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}\right]=$ $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2k\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\dot{T}_G$	(A3c)

Table A4: The continuity equation for species *A* in terms of the molar flux

$\frac{\partial C_A}{\partial t}=-\left(\nabla\cdot\vec{N}_A\right)+\dot{R}_{A,G}$	(4.)
Rectangular coordinates (*x*, *y*, *z*) $\frac{\partial C_A}{\partial t}=-\left(\frac{\partial[N_A]_z}{\partial x}+\frac{\partial[N_A]_y}{\partial y}+\frac{\partial[N_A]_z}{\partial z}\right)+\dot{R}_{A,G}$	(4a)
Cylindrical coordinates (*r*, *θ*, *z*) $\frac{\partial C_A}{\partial t}=-\left\{\frac{1}{r}\frac{\partial}{\partial r}[rN_A]_r+\frac{1}{r}\frac{\partial}{\partial \theta}[N_A]_\theta+\frac{\partial}{\partial z}[N_A]_z\right\}+\dot{R}_{A,G}$	(4b)
Spherical coordinates (*r*, *θ*, *ϕ*) $\frac{\partial C_A}{\partial t}=-\left\{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2[N_A]_r\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left([N_A]_\theta\sin\theta\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}[N_A]_\phi\right\}+\dot{R}_{A,G}$	(4c)

Table A.5: The continuity equation for species *A*

$\frac{\partial C_A}{\partial t} + (\bar{u} \cdot \nabla) C_A = D_A \nabla^2 C_A + \dot{R}_{A,G}$	(5)
Rectangular coordinates (*x*, *y*, *z*) $\frac{\partial C_A}{\partial t} + u_x \frac{\partial C_A}{\partial x} + u_y \frac{\partial C_A}{\partial y} + u_z \frac{\partial C_A}{\partial z} = \frac{\partial}{\partial x}\left(D \frac{\partial C_A}{\partial x}\right) + \frac{\partial}{\partial y}\left(D \frac{\partial C_A}{\partial y}\right) + \frac{\partial}{\partial z}\left(D \frac{\partial C_A}{\partial z}\right) + \dot{R}_{A,G}$	(5a)
Cylindrical coordinates (*r*, *θ*, *z*) $\frac{\partial C_A}{\partial t} + u_r \frac{\partial C_A}{\partial r} + \frac{u_\theta}{r} \frac{\partial C_A}{\partial \theta} + u_z \frac{\partial C_A}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r}\left(rD \frac{\partial C_A}{\partial r}\right) + \frac{1}{r^2} \frac{\partial}{\partial \theta}\left(D \frac{\partial C_A}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(D \frac{\partial C_A}{\partial z}\right) + \dot{R}_{A,G}$	(5b)
Spherical coordinates (*r*, *θ*, *ϕ*) $\frac{\partial C_A}{\partial t} + u_r \frac{\partial C_A}{\partial r} + \frac{u_\theta}{r} \frac{\partial C_A}{\partial \theta} + \frac{u_\phi}{r \sin\theta} \frac{\partial C_A}{\partial \phi} = \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 D \frac{\partial C_A}{\partial r}\right)$ $+ \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta}\left(D \sin\theta \frac{\partial C_A}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial}{\partial \phi}\left(D \frac{\partial C_A}{\partial \phi}\right) + \dot{R}_{A,G}$	(5c)