

National Exams December 2018

17-Ind-A6, Systems Simulation

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. This is a closed book exam. Candidates are permitted to use one of the two permitted calculators (Sharp or Casio models).
3. Candidates are permitted to have an aid sheet consisting of one 8.5" x 11.0" sheet of paper. Writing is permitted on both sides of the paper.
4. This exam consists of three sections (A→C). Within each section, candidates will be given a choice of questions to answer. Please read the instructions for each section carefully. A breakdown of questions and marks is as follows:

Part A:	Do 2 of 3 Questions.	Total marks: 20
Part B:	Do 1 of 2 Questions.	Total marks: 20
Part C:	Do 1 of 2 Questions.	Total marks: 20

Exam:	4 Questions.	Total marks: 60
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4. The value of each question is listed in the exam. Remember to check the instructions for each section. **DO NOT ATTEMPT TO DO ALL QUESTIONS.**
5. Statistical tables are provided. If, for any reason, a value is missing, please note this and use an approximate value.

Part A – Input Modelling Concepts: Complete two of the following three questions. This section is worth a total of 30 marks. Do not attempt all questions.

Robert Baden-Powell, is a Halifax based IE. He volunteers with his local Scout organization and has been tasked with the development of a transport plan to move Scout patrols between Halifax Stanfield International Airport and the camp site in Elderbank, Nova Scotia for Scouts Canada’s next Canadian Jamboree. A jamboree is a large camp. The camp site is 40 km from the airport. About 7,000 people are expected to attend.

- Robert is presently working on a project to estimate the number of cargo trucks needed to transport gear to the campsite. Scout groups travel as patrols. Each patrol has 10 people (2 adults + 8 youth). Each person will bring 1 backpack for their personal gear. They may also bring a second bag (at an additional cost of \$20 for transport). From a survey of patrols in Nova Scotia, the following distribution of extra bags has been estimated.

Number of Additional Bags	Count
0	1
1	1
2	2
3	5
4	12
5	21
6	24
7	19
8	11
9	4
10	0

Given this sample:

- Calculate the average number of extra bags per patrol.
- Calculate the variance of the number of extra bags.
- Hypothesize an appropriate distribution for this data sample and give a brief rationale for selecting that particular distribution (i.e. tell me why that might be a good distribution for this data). Please provide two or three reasons. *Hint:* I am looking for an answer involving logic rather than extensive calculations.
- List the parameter(s) for your hypothesized distribution in (c).

10 Marks

- Robert has hypothesized that the data from Q1 comes from a Poisson distribution with $\lambda = 6$. Evaluate this hypothesis via a χ^2 test using $\alpha = 0.05$. Is Robert’s hypothesis correct? Why or why not?

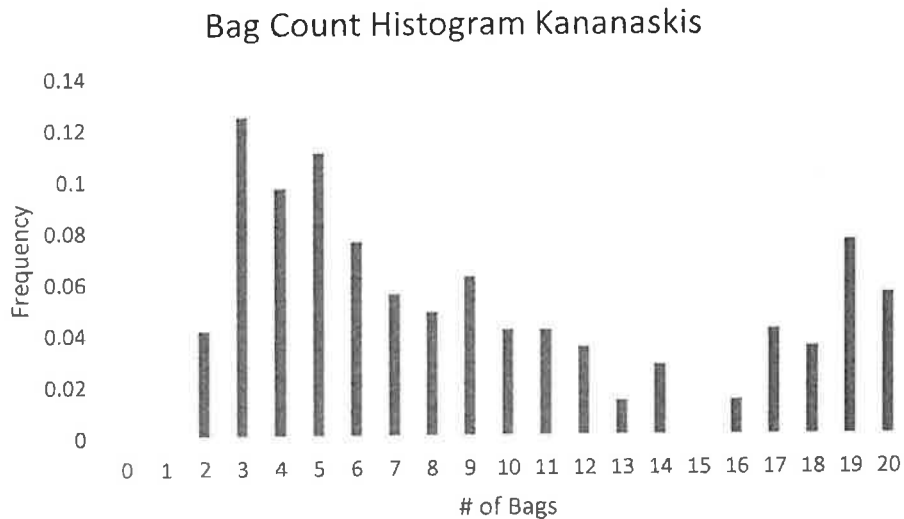
To assist you in your calculations, you may wish to note, for the Poisson distribution, that:

$$f(x) = \frac{\lambda t e^{-\lambda t x}}{x!}$$

Where λ is the expected number of arrivals per unit time.

10 Marks

3. Robert has concerns about the applicability of the survey data in Q1, since only a small sample of patrols has responded. Accordingly, he has obtained a data sample from the previous Jamboree, held in Kananaskis, Alberta in 2013. The data sample is not precisely the same as the survey data; it provides a count of the number of bags per patrol, rather than the number of bags per person. A histogram of the bags per patrol appears below:



- Explain to Robert what a frequency chart like this means: What does a multi-modal distribution imply about the underlying population?
- What practical difficulties does a multi-modal distribution create for input data modelling?
- Can you suggest any methods for dealing with this data set to improve the theoretical fit? Would, for example, additional data sampling help? Could the data be transformed somehow? Can you describe a mathematical method for converting this to two uni-modal distributions?
- Suppose Robert proposes to simply import the data as an empirical distribution in his model. Discuss the benefits and drawbacks of using

empirical distributions in a simulation as compared to fitting theoretical models

10 Marks

Part B: Modelling Concepts: Complete one of the following two questions. This section is worth a total of 20 marks. Do not attempt all questions.

One of the more pressing planning problems is renting school buses for off-site trips during the jamboree. Each weekday the camp runs, groups have the option of staying on-site for camp activities or going off-site for sight seeing or off-site hikes. Off-site activities can be either a half day or a full day and destinations range from 10 km distant (tidal bore rafting on the Shubenacadie River) to 120 km distant (On-Tree Adventures in Windsor, NS). Morning bus departures from the camp start at 08:00; afternoon bus departures start at 13:00. Groups going out for a ½ day activity in the morning are supposed to be returned to the camp at 12:00. Groups going out for a full day activity, or groups going out for a ½ day activity in the afternoon, are supposed to be returned by 17:00.

1. There is some opportunity for savings if Robert can figure out a schedule to use some buses on multiple routes or to use one bus to drop different groups at different locations. It's a complicated problem, depending on the number of groups that sign up for each of the destinations and their preferred start time, event duration, and event location. Accordingly, Robert has built a simulation model to represent off-site transportation. In his model, Robert has patrols arrive to the transport centre, select a destination and a duration, and line up (FIFO) for a bus. His model is intended to determine how long patrols wait for pickup and return vs. the number of buses rented.

Robert is somewhat confused by the run time settings in his simulation and has arbitrarily selected the following parameters to run under the assumption of batch means. (Batch means is indicated by the tick boxes indicating that statistics are cleared between replications, but not the system):

Run Setup

Run Speed Run Control Reports Project Parameters
Replication Parameters Array Sizes Arena Visual Designer

Number of Replications: Initialize Between Replications
7 Statistics System

Start Date and Time:
 July 10, 2017 2:56:47 PM

Warm-up Period: Time Units:
2.5 Hours

Replication Length: Time Units:
5 Days

Hours Per Day:
24

Base Time Units:
Hours

Terminating Condition:

OK Cancel Apply Help

Robert is, however, experiencing some difficulty with his model. Planning standards require that the average wait is 45 minutes. However, his simulation is presently producing 52 +/- 4.5 minutes wait (at a 95% confidence interval). Robert, being an intelligent industrial engineer, knows that the half width of the CI increases as the number of runs is decreased. He is therefore considering cutting the number of runs from 7 to 5 to ensure that the upper end of his CI will cover the expected mean. However, Robert would really rather have a CI half width that was no more than +/- 2 minutes 19 times out of 20.

- a. What advice can you provide to Robert regarding the number of runs that he should undertake? Is reducing from 7 to 5 a good idea? Why or why not?
- b. Has Robert handled warm-up properly? Can you provide any advice to him about warm-up in this situation?
- c. Should he apply the method of batch means or the deletion/replication method to this problem? Why or why not? Where is batch means appropriate? Where is replication/deletion appropriate? Which, if either, is most appropriate here?
- d. What advice can you give to Robert about determining an appropriate run length for his model in this case?
- e. How many replications should Robert complete if he wants a CI no more the +/- 2 minutes 19 times out of 20?

20 Marks

2. Assume that Robert has completed his model to his satisfaction. He obtains a sample of data from the transport company showing average route times (one way) for a test run between the airport and the camp. Robert then runs his simulation model for a total of 5 replications. The data is given below. To aid you in the ca

Sample	Actual	Simulation
1	46.7	45.6
2	54.9	50.5
3	40.4	40.1
4	53.6	40.8
5	51.5	41.5

- a. Using a value of $\alpha = 0.1$, determine if the model accurately predicts the throughput of the real system. State any assumptions that you make to answer this question.
- b. Describe the difference between verification of a simulation model and validation. Is the test in (a) verification or validation? Can you describe what it means to build model credibility?
- c. Autocorrelation is an issue in simulation results. What is autocorrelation and why does it tend to exist in simulation output? Do Robert's results exhibit significant autocorrelation? If, autocorrelation exists in an output sample, what advice would you have about detecting it and extending the run length.

Hint: Please provide an algorithmic approach to setting run length with respect to minimizing autocorrelation (i.e. provide a recipe for what to do if autocorrelation is detected in simulation output).

- d. Robert has been focusing on validating the mean response of the simulation against the mean value of the sample data from the bus company. However, should he be concerned about model variability as well? Do the data samples have the same variance? What would it mean *if* the simulation model exhibited more (or less) variability than the actual data *even if* the means were the same. Would a difference in variability for the model be a concern for this project?

20 Marks

Part C: Output Analysis: Complete one of the following two questions. This section is worth a total of 20 marks. Do not attempt all questions.

- Robert has finished his simulation model and has run three different routing scenarios for transporting groups. These are labelled 1, 2, and 3. Each involves a slightly different number of buses and different routing options. He runs his model for 5 different replications under each scenario. The results appear below. Please note that the overall mean across the three scenarios is 41.7 and the variance is 22.94.

Replication	Scenario 1	Scenario 2	Scenario 3
1	47.6	40.6	37.5
2	46.6	42.5	38.8
3	47.2	46.4	33.3
4	44.9	39.4	36.0
5	47.7	41.0	36.5
Sum	234	209.9	182.1
Average	46.8	42.0	36.4

While there appear to be differences in the scenarios, are any of the responses statistically different from any other? Which one(s)? How do we know this? Support your argument with an appropriate analysis.

Hint: There are several possible ways to determine which of these scenarios is best. Select one method and describe why it is appropriate here. Complete the analysis and answer the questions above.

20 Marks

- Assume Robert is to conduct a series of screening runs, using a design of experiments (DOE) approach, to test bus scheduling and loading policies. The screening experiment includes two factors: the size of buses used (large or small) and the loading priority (near destinations first vs. far destinations first). Both controls are binary: they can be implemented or not. You may assume that implemented is the high setting and not implemented is the low setting.

The partial design matrix for this problem is listed below:

Run	Factors	
	A	B
1	-1	-1
2	1	-1
3	-1	1
4	1	1

- a. Complete the design matrix for this experiment and interpret the meaning of the settings (i.e. tell me what the signs mean in terms of the control factor settings, the main effects, and the interaction effects). You may assume that Factor A refers to the bus size, while Factor B refers to the loading policy.
- b. Assume that Robert has completed five replications of each run listed in the design matrix above. The following output gives the average wait time for a patrol to be loaded:

Run	Rep 1	Rep 2	Rep 3	Avg
1	40	42	38	40.00
2	90	88	94	90.67
3	60	64	58	60.67
4	110	113	108	110.33

Complete an analysis of variance (ANOVA) on this output and determine which, if any, factors are significant. You may assume that $SST = 8829$ and an alpha value of 0.05. Given this data, can you suggest appropriate settings for Factor A and Factor B? Can they be set independently, or must they be set together? Why?

20 Marks

Bonus

Robert Baden Powell was a real person and (though not an IE, nor did he ever live in Nova Scotia as best I know). His picture is below:



What is his claim to fame?

1 Marks

Areas Under the Normal Curve

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.40	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.30	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.20	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.10	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.00	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.90	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.80	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.70	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.60	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.50	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.40	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.30	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.20	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.10	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.00	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.90	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.80	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.70	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.60	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.50	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.40	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.30	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.20	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.10	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.00	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.90	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.80	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.70	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.60	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.50	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.40	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.30	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.20	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.10	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.00	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Areas Under the Normal Curve

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.10	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.20	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.30	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.40	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.50	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.60	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.70	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.80	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.90	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.00	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.10	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.20	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.30	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.40	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.50	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.60	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.70	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.80	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.90	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.00	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.10	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.20	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.30	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.40	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.50	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.60	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.70	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.80	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.90	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.00	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.10	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.20	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.30	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.40	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Critical Values of the t-Distribution

v	α						
	0.40	0.30	0.20	0.15	0.10	0.05	0.025
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
50	0.255	0.528	0.849	1.047	1.299	1.676	2.009
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
70	0.254	0.527	0.847	1.044	1.294	1.667	1.994
80	0.254	0.526	0.846	1.043	1.292	1.664	1.990
90	0.254	0.526	0.846	1.042	1.291	1.662	1.987
100	0.254	0.526	0.845	1.042	1.290	1.660	1.984
110	0.254	0.526	0.845	1.041	1.289	1.659	1.982
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
130	0.254	0.526	0.844	1.041	1.288	1.657	1.978
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

Critical Values of the t-Distribution

v	α						
	0.02	0.015	0.01	0.0075	0.005	0.0025	0.0005
1	15.895	21.205	31.821	42.433	63.657	127.321	636.619
2	4.849	5.643	6.965	8.073	9.925	14.089	31.599
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	2.197	2.336	2.528	2.661	2.845	3.153	3.850
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	2.158	2.291	2.473	2.598	2.771	3.057	3.690
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	2.150	2.282	2.462	2.586	2.756	3.038	3.659
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551
50	2.109	2.234	2.403	2.519	2.678	2.937	3.496
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
70	2.093	2.215	2.381	2.494	2.648	2.899	3.435
80	2.088	2.209	2.374	2.486	2.639	2.887	3.416
90	2.084	2.205	2.368	2.480	2.632	2.878	3.402
100	2.081	2.201	2.364	2.475	2.626	2.871	3.390
110	2.078	2.199	2.361	2.471	2.621	2.865	3.381
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373
130	2.075	2.194	2.355	2.465	2.614	2.856	3.367
∞	2.054	2.170	2.327	2.433	2.576	2.808	3.291

Critical Values of the Chi-Squared Distribution

v	α									
	0.995	0.99	0.98	0.975	0.95	0.90	0.80	0.75	0.70	0.50
1	0.000	0.000	0.001	0.001	0.004	0.016	0.064	0.102	0.148	0.455
2	0.010	0.020	0.040	0.051	0.103	0.211	0.446	0.575	0.713	1.386
3	0.072	0.115	0.185	0.216	0.352	0.584	1.005	1.213	1.424	2.366
4	0.207	0.297	0.429	0.484	0.711	1.064	1.649	1.923	2.195	3.357
5	0.412	0.554	0.752	0.831	1.145	1.610	2.343	2.675	3.000	4.351
6	0.676	0.872	1.134	1.237	1.635	2.204	3.070	3.455	3.828	5.348
7	0.989	1.239	1.564	1.690	2.167	2.833	3.822	4.255	4.671	6.346
8	1.344	1.646	2.032	2.180	2.733	3.490	4.594	5.071	5.527	7.344
9	1.735	2.088	2.532	2.700	3.325	4.168	5.380	5.899	6.393	8.343
10	2.156	2.558	3.059	3.247	3.940	4.865	6.179	6.737	7.267	9.342
11	2.603	3.053	3.609	3.816	4.575	5.578	6.989	7.584	8.148	10.341
12	3.074	3.571	4.178	4.404	5.226	6.304	7.807	8.438	9.034	11.340
13	3.565	4.107	4.765	5.009	5.892	7.042	8.634	9.299	9.926	12.340
14	4.075	4.660	5.368	5.629	6.571	7.790	9.467	10.165	10.821	13.339
15	4.601	5.229	5.985	6.262	7.261	8.547	10.307	11.037	11.721	14.339
16	5.142	5.812	6.614	6.908	7.962	9.312	11.152	11.912	12.624	15.338
17	5.697	6.408	7.255	7.564	8.672	10.085	12.002	12.792	13.531	16.338
18	6.265	7.015	7.906	8.231	9.390	10.865	12.857	13.675	14.440	17.338
19	6.844	7.633	8.567	8.907	10.117	11.651	13.716	14.562	15.352	18.338
20	7.434	8.260	9.237	9.591	10.851	12.443	14.578	15.452	16.266	19.337
21	8.034	8.897	9.915	10.283	11.591	13.240	15.445	16.344	17.182	20.337
22	8.643	9.542	10.600	10.982	12.338	14.041	16.314	17.240	18.101	21.337
23	9.260	10.196	11.293	11.689	13.091	14.848	17.187	18.137	19.021	22.337
24	9.886	10.856	11.992	12.401	13.848	15.659	18.062	19.037	19.943	23.337
25	10.520	11.524	12.697	13.120	14.611	16.473	18.940	19.939	20.867	24.337
26	11.160	12.198	13.409	13.844	15.379	17.292	19.820	20.843	21.792	25.336
27	11.808	12.879	14.125	14.573	16.151	18.114	20.703	21.749	22.719	26.336
28	12.461	13.565	14.847	15.308	16.928	18.939	21.588	22.657	23.647	27.336
29	13.121	14.256	15.574	16.047	17.708	19.768	22.475	23.567	24.577	28.336
30	13.787	14.953	16.306	16.791	18.493	20.599	23.364	24.478	25.508	29.336

Critical Values of the Chi-Squared Distribution

v	α									
	0.3	0.25	0.2	0.1	0.05	0.025	0.02	0.01	0.005	0.001
1	1.074	1.323	1.642	2.706	3.841	5.024	5.412	6.635	7.879	10.828
2	2.408	2.773	3.219	4.605	5.991	7.378	7.824	9.210	10.597	13.816
3	3.665	4.108	4.642	6.251	7.815	9.348	9.837	11.345	12.838	16.266
4	4.878	5.385	5.989	7.779	9.488	11.143	11.668	13.277	14.860	18.467
5	6.064	6.626	7.289	9.236	11.070	12.833	13.388	15.086	16.750	20.515
6	7.231	7.841	8.558	10.645	12.592	14.449	15.033	16.812	18.548	22.458
7	8.383	9.037	9.803	12.017	14.067	16.013	16.622	18.475	20.278	24.322
8	9.524	10.219	11.030	13.362	15.507	17.535	18.168	20.090	21.955	26.124
9	10.656	11.389	12.242	14.684	16.919	19.023	19.679	21.666	23.589	27.877
10	11.781	12.549	13.442	15.987	18.307	20.483	21.161	23.209	25.188	29.588
11	12.899	13.701	14.631	17.275	19.675	21.920	22.618	24.725	26.757	31.264
12	14.011	14.845	15.812	18.549	21.026	23.337	24.054	26.217	28.300	32.909
13	15.119	15.984	16.985	19.812	22.362	24.736	25.472	27.688	29.819	34.528
14	16.222	17.117	18.151	21.064	23.685	26.119	26.873	29.141	31.319	36.123
15	17.322	18.245	19.311	22.307	24.996	27.488	28.259	30.578	32.801	37.697
16	18.418	19.369	20.465	23.542	26.296	28.845	29.633	32.000	34.267	39.252
17	19.511	20.489	21.615	24.769	27.587	30.191	30.995	33.409	35.718	40.790
18	20.601	21.605	22.760	25.989	28.869	31.526	32.346	34.805	37.156	42.312
19	21.689	22.718	23.900	27.204	30.144	32.852	33.687	36.191	38.582	43.820
20	22.775	23.828	25.038	28.412	31.410	34.170	35.020	37.566	39.997	45.315
21	23.858	24.935	26.171	29.615	32.671	35.479	36.343	38.932	41.401	46.797
22	24.939	26.039	27.301	30.813	33.924	36.781	37.659	40.289	42.796	48.268
23	26.018	27.141	28.429	32.007	35.172	38.076	38.968	41.638	44.181	49.728
24	27.096	28.241	29.553	33.196	36.415	39.364	40.270	42.980	45.559	51.179
25	28.172	29.339	30.675	34.382	37.652	40.646	41.566	44.314	46.928	52.620
26	29.246	30.435	31.795	35.563	38.885	41.923	42.856	45.642	48.290	54.052
27	30.319	31.528	32.912	36.741	40.113	43.195	44.140	46.963	49.645	55.476
28	31.391	32.620	34.027	37.916	41.337	44.461	45.419	48.278	50.993	56.892
29	32.461	33.711	35.139	39.087	42.557	45.722	46.693	49.588	52.336	58.301
30	33.530	34.800	36.250	40.256	43.773	46.979	47.962	50.892	53.672	59.703

Critical Values of the F Distribution

$f_{0.05(v1, v2)}$										
v2	v1									
	1	2	3	4	5	6	7	8	9	10
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83

Critical Values of the F Distribution

		$f_{0.05(v1, v2)}$									
v2	v1										
	10	12	15	20	24	30	40	60	120	∞	
1	241.88	243.91	245.95	248.01	249.05	250.10	251.14	252.20	253.25	254.30	
2	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50	
3	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53	
4	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63	
5	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37	
6	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67	
7	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23	
8	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93	
9	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71	
10	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54	
11	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.41	
12	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30	
13	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21	
14	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13	
15	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07	
16	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01	
17	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96	
18	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92	
19	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88	
20	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84	
21	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81	
22	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78	
23	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76	
24	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73	
25	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71	
26	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69	
27	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67	
28	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65	
29	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64	
30	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62	
40	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51	
50	2.03	1.95	1.87	1.78	1.74	1.69	1.63	1.58	1.51	1.44	
60	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39	
120	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.26	
∞	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.01	

Critical Values of the F Distribution

$$f_{0.01(v1, v2)}$$

v2	v1									
	1	2	3	4	5	6	7	8	9	10
1	4052.18	4999.50	5403.35	5624.58	5763.65	5858.99	5928.36	5981.07	6022.47	6055.85
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80
50	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78	2.70
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32

Critical Values of the F Distribution

$f_{0.01(v1, v2)}$

v2	v1									
	10	12	15	20	24	30	40	60	120	∞
1	6055.85	6106.32	6157.28	6208.73	6234.63	6260.65	6286.78	6313.03	6339.39	6365.55
2	99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.49	99.50
3	27.23	27.05	26.87	26.69	26.60	26.50	26.41	26.32	26.22	26.13
4	14.55	14.37	14.20	14.02	13.93	13.84	13.75	13.65	13.56	13.46
5	10.05	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
6	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
7	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65
8	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86
9	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31
10	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
11	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60
12	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
13	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17
14	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.01
15	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87
16	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75
17	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65
18	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57
19	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49
20	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42
21	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36
22	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31
23	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26
24	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
25	3.13	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27	2.17
26	3.09	2.96	2.81	2.66	2.58	2.50	2.42	2.33	2.23	2.13
27	3.06	2.93	2.78	2.63	2.55	2.47	2.38	2.29	2.20	2.10
28	3.03	2.90	2.75	2.60	2.52	2.44	2.35	2.26	2.17	2.07
29	3.00	2.87	2.73	2.57	2.49	2.41	2.33	2.23	2.14	2.04
30	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01
40	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.81
50	2.70	2.56	2.42	2.27	2.18	2.10	2.01	1.91	1.80	1.68
60	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60
120	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38
∞	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.01

Critical Values for Bartlett's Test

		$b_k(0.01;n)$								
		Number of populations, k								
n	2	3	4	5	6	7	8	9	10	
3	0.1411	0.1672								
4	0.2843	0.3165	0.3475	0.3729	0.3937	0.4110				
5	0.3984	0.4304	0.4607	0.4850	0.5046	0.5207	0.5343	0.5458	0.5558	
6	0.4850	0.5149	0.5430	0.5653	0.5832	0.5975	0.6100	0.6204	0.6293	
7	0.5512	0.5787	0.6045	0.6248	0.6410	0.6542	0.6652	0.6744	0.6824	
8	0.6031	0.6282	0.6518	0.6704	0.6851	0.6970	0.7069	0.7153	0.7225	
9	0.6445	0.6676	0.6892	0.7062	0.7197	0.7305	0.7395	0.7471	0.7536	
10	0.6783	0.6996	0.7195	0.7352	0.7475	0.7575	0.7657	0.7726	0.7786	
11	0.7063	0.7260	0.7445	0.7590	0.7703	0.7795	0.7871	0.7935	0.7990	
12	0.7299	0.7483	0.7654	0.7789	0.7894	0.7980	0.8050	0.8109	0.8160	
13	0.7501	0.7672	0.7832	0.7958	0.8056	0.8135	0.8201	0.8256	0.8303	
14	0.7674	0.7835	0.7985	0.8103	0.8195	0.8269	0.8330	0.8382	0.8426	
15	0.7825	0.7977	0.8118	0.8229	0.8315	0.8385	0.8443	0.8491	0.8532	
16	0.7958	0.8101	0.8235	0.8339	0.8421	0.8486	0.8541	0.8586	0.8625	
17	0.8076	0.8211	0.8338	0.8436	0.8514	0.8576	0.8627	0.8670	0.8707	
18	0.8181	0.8309	0.8429	0.8523	0.8596	0.8655	0.8704	0.8745	0.8780	
19	0.8275	0.8397	0.8512	0.8601	0.8670	0.8727	0.8773	0.8811	0.8845	
20	0.8360	0.8476	0.8586	0.8671	0.8737	0.8791	0.8835	0.8871	0.8903	

Values for Tukey's Test
Upper Percentage Points of the Studentized Range Distribution: Values of $q(0.05, k, \nu)$

Degrees of Freedom ν	Number of Treatments k									
	2	3	4	5	6	7	8	9	10	
1	18.00	27.00	32.80	37.20	40.50	43.10	45.10	47.10	49.10	
2	6.09	5.33	9.80	10.89	11.73	12.43	13.03	13.54	13.99	
3	4.50	5.91	6.83	7.51	8.04	8.47	8.85	9.18	9.46	
4	3.93	5.04	5.76	6.29	6.71	7.06	7.35	7.60	7.83	
5	3.64	4.60	5.22	5.67	6.09	6.80	6.58	6.33	6.03	
6	3.46	4.34	4.90	5.30	6.49	6.32	6.12	5.90	5.63	
7	3.34	4.16	4.68	5.06	6.16	6.00	5.82	5.61	5.36	
8	3.26	4.04	4.53	4.89	5.92	5.77	5.60	5.40	5.17	
9	3.20	3.95	4.41	4.76	5.74	5.59	5.43	5.24	5.02	
10	3.15	3.88	4.33	4.65	5.60	5.46	5.30	5.12	4.91	
11	3.11	3.82	4.26	4.57	5.49	5.35	5.20	5.03	4.82	
12	3.08	3.77	4.20	4.51	5.39	5.27	5.12	4.95	4.75	
13	3.06	3.73	4.15	4.45	5.32	5.19	5.05	4.88	4.69	
14	3.03	3.70	4.11	4.41	5.25	5.13	4.99	4.83	4.64	
15	3.01	3.67	4.08	4.37	5.20	5.08	4.94	4.78	4.59	
16	3.00	3.65	4.05	4.33	5.15	5.03	4.90	4.74	4.56	
17	2.98	3.63	4.02	4.30	5.11	4.99	4.86	4.70	4.52	
18	2.97	3.61	4.00	4.28	5.07	4.96	4.82	4.67	4.49	
19	2.96	3.59	3.98	4.25	5.04	4.92	4.79	4.65	4.47	
20	2.95	3.58	3.96	4.23	5.01	4.90	4.77	4.62	4.45	
24	2.92	3.53	3.90	4.17	4.92	4.81	4.68	4.54	4.37	
30	2.89	3.49	3.85	4.10	4.82	4.72	4.60	4.46	4.30	
40	2.86	3.44	3.79	4.04	4.73	4.63	4.52	4.39	4.23	
60	2.83	3.40	3.74	3.98	4.65	4.55	4.44	4.31	4.16	
120	2.80	3.36	3.68	3.92	4.56	4.47	4.36	4.24	4.10	
∞	2.77	3.31	3.63	3.86	4.47	4.39	4.29	4.17	4.03	

