NATIONAL EXAMINATIONS

December 2016

04-BS-7 MECHANICS OF FLUIDS

Three (3) hours duration

Notes to Candidates

1. This is a **Closed Book** examination.

2. Exam consists of two Sections. **Section A is Calculative (9 questions)** and **Section B is Analytical (4 questions)**.

3. **Do seven (7) questions from Section A (Calculative) and three (3) questions from Section B (Analytical)**. Note that the Analytical Questions do not require detailed calculations but do require full explanations.

4. **Ten (10) questions constitute a complete paper.** (Total 50 marks).

5. **All questions are of equal value.** (Each 5 marks).

6. If doubt exists as to the interpretation of any question, the candidate is urged to submit, with the answer paper, a clear statement of any assumptions made.

7. **Read the entire question before commencing** the calculations and **take note of hints or recommendations** given.

8. Either one of the approved **Casio** or **Sharp** calculators may be used.

9. **Reference information** for particular questions is given on pages 8 to 13. **All pages of questions attempted are to be returned with the Answer Booklet, showing diagrams generated or where readings were taken and which data was used. Candidates must write their names on these pages.**

10. **Constants** are given on page 14.

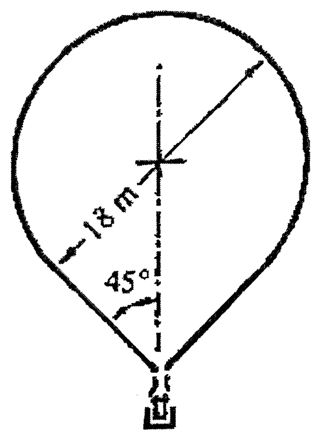
11. **Nomenclature** and **Reference Equations** are given on pages 15 to 18.

SECTION A CALCULATIVE QUESTIONS

Do seven of nine questions. ***Solutions to these questions must be set out logically with all intermediate answers and units given.***

QUESTION 1

A hot air balloon as shown in the adjacent sketch is made in the shape of a sphere with a cone at the bottom. The diameter of the sphere is 18 m and the side of the cone is at an angle of 45° to the vertical axis. This gives the balloon a total volume of 3147 m^3. The total mass is made up as follows:



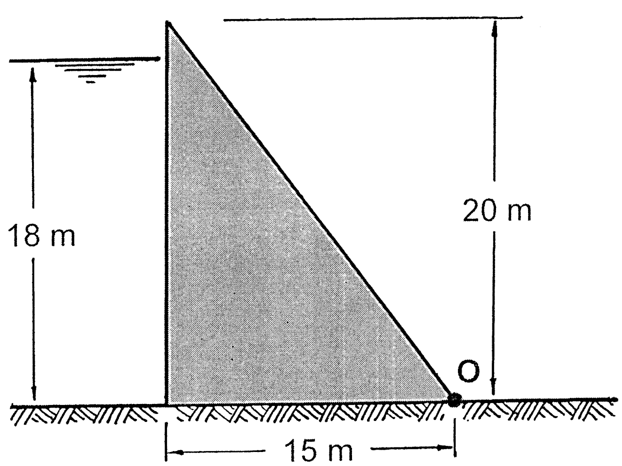
Envelope	100 kg
Basket	60 kg
Fuel Tanks	110 kg
Burners	50 kg
Two People	160 kg

Determine the temperature of the hot air inside the balloon envelope to establish neutral buoyancy (equilibrium) in the atmosphere. The ambient atmospheric conditions are 100 kPa and 15°C.

(5 marks)

QUESTION 2

A gravity dam built of concrete has dimensions as shown in the adjoining sketch. It is filled with water to a depth of 18 m. To prevent cracking of the concrete or separation from the bedrock, the resultant force on the structure must pass through the middle third of the base so as to maintain compressive stress everywhere in the concrete and bedrock. By taking moments about the toe of the dam (Point O), determine the point (horizontal distance from Point O) through which the resultant force passes through the base and hence whether the dam is safe or not.



(5 marks)

QUESTION 3

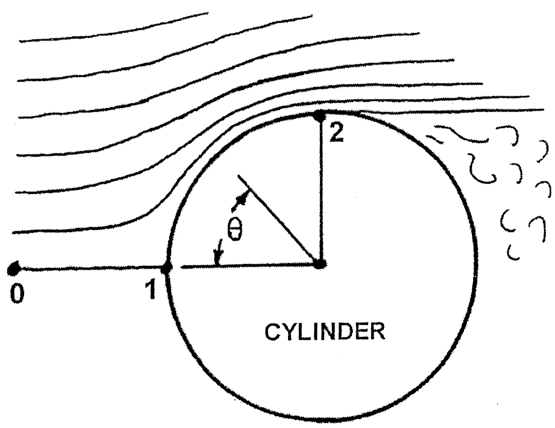
Refer to the Examination Paper Attachments Page 8 **Velocity Variation around Cylinder**.

The diagram shows streamlines around a cylinder with laminar flow from left to right. Consider flow around the top half of the cylinder only. By taking measurements from the diagram determine the variation in velocity V/V_0 around the surface of the cylinder where V_0 is the free stream velocity. Plot this variation of V/V_0 versus angle from the front (0°) to the rear (180°) of the cylinder using the axes below the diagram of the streamlines. Plot at least nine points (approximately 22½° apart). Show where measurements are taken from the diagram and give appropriate calculations. Explain why the plot has the resulting shape.

Hint: Use the continuity equation for the calculations. *(5 marks)*

QUESTION 4

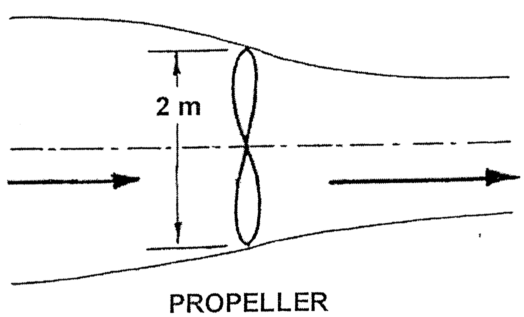
For ideal flow (no friction) the velocity V_1 around the surface of a long cylinder is given by $V_1 = 2\ V_0 \sin\theta$ where V_0 is the free stream velocity and θ the angular location around the cylinder measured from the front (stagnation point). Consider a concrete chimney 20 m in diameter and 275 m in height subject to a wind velocity of 100 km/h. Determine the velocity and pressure of the air on the front (stagnation point – point 1) and sides (90° from stagnation point – point 2) of the chimney. Express the pressure relative to atmospheric pressure (gauge pressure).



(5 marks)

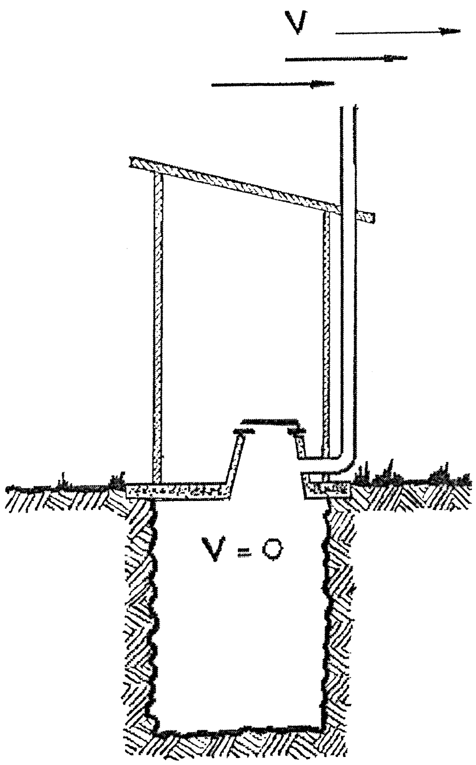
QUESTION 5

A propeller of 2 m in diameter is mounted on an aircraft travelling at 500 km/hr. The airstream leaves the propeller with a velocity of 650 km/hr. Assume that the pressure upstream and downstream of the propeller is equal to atmospheric pressure and that the flow converges as the velocity increases. Determine the thrust developed by the propeller.



(5 marks)

QUESTION 6

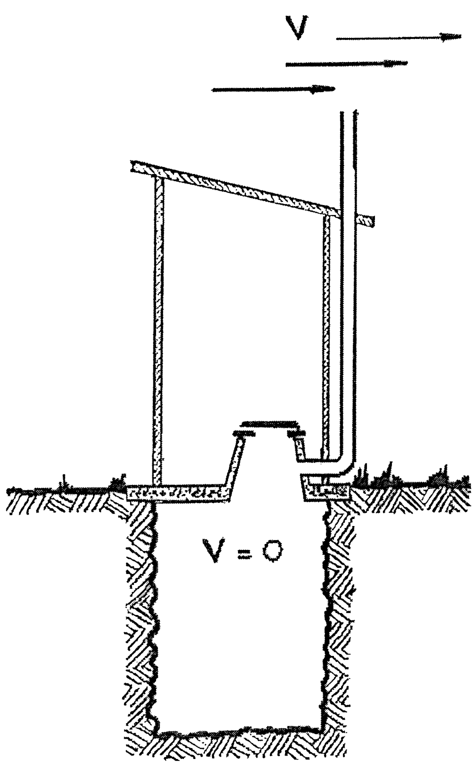


Outhouses as used by our predecessors particularly in rural areas were not much more than a hole in the ground with a seat on top. They were rather unpleasant to use until redesigned by someone with a knowledge of Bernoulli's theorem to extract the odours and are now commonly found in national parks. By installing a simple vent pipe, air can be extracted from the top of the pit thereby minimizing its percolation into the outhouse structure.

For a wind speed of 25 km/hr and an air temperature of 20°C, determine the differential pressure between the outside atmosphere and the pit to promote continuous air removal.

(5 marks)

QUESTION 7



Refer to the Examination Paper Attachments Page 9 and Page 10 **Pipe Flow Data.**

Refer also to the Examination Paper Attachments Page 11 **Moody Diagram.**

An outhouse similar to that described in Question 6 has a pipe to provide natural ventilation to the pit when a light wind is blowing. This pipe is a plastic sewer pipe 100 mm in diameter with a vertical length (height) of 3.0 m and a horizontal length of 0.5 m. It has a short radius elbow, a square edged entrance and a submerged (in atmosphere) discharge.

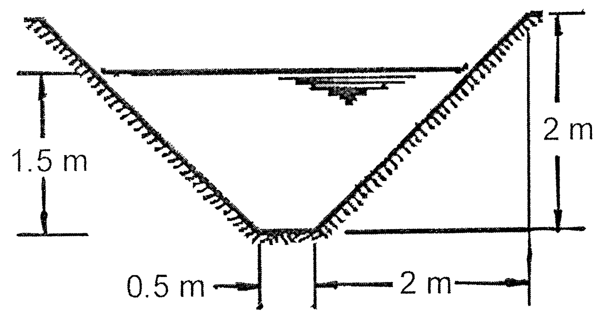
Assuming that it has been designed for a flow rate of 110 m^3/hour at an ambient temperature of 20°C determine the head loss in the pipe expressed as a differential pressure between the pit and the atmosphere.

(5 marks)

QUESTION 8

Refer to the Examination Paper Attachments Page 12 **Moody Diagram.**

Consider a concrete trapezoidal irrigation canal with cross section as shown that is 4.5 m wide by 2 m deep with 45° sides and 0.5 m wide bottom. Water is required to flow 1.5 m deep at a flow rate of 3 m^3/s. This canal is required to deliver water to a location 8 km from the source. Determine from general pipe flow relations the drop in elevation required to maintain the specified flow rate. Assume a roughness of 1 mm for the concrete.



(5 marks)

QUESTION 9

Refer to the Examination Paper Attachments Page 13 **Drag Diagram for Solid Bodies.**

Consider a slurry of fine magnetite and water as is used for coal separation from rock. The magnetite is kept in suspension by agitation of the mixture and the resultant specific gravity of the slurry (about 1.5) is substantially higher than that of water. Coal will float in such a slurry but rock will sink. If agitation is stopped determine the rate of settling of the magnetite in the water. The magnetite particles are 50 μm in diameter and have a specific gravity of 5.0.

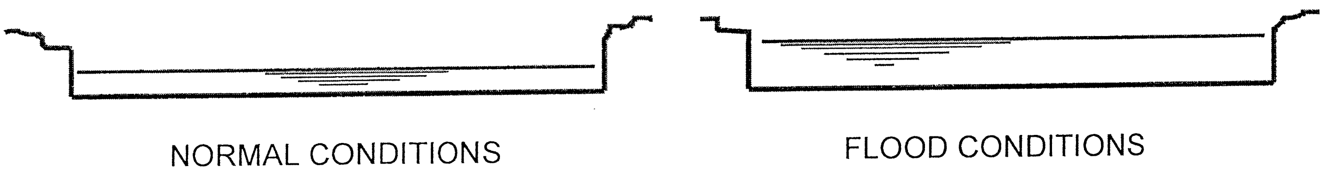
Since the particles are very small, assume Stokes' Law but determine, from the empirical curve, whether using Stoke's Law was a good assumption. Justify your answer by plotting the settling rate conditions on the diagram.

(5 marks)

SECTION B GRAPHICAL AND ANALYTICAL QUESTIONS

Do three of four questions. These questions do not require detailed calculations but complete written explanations must be given to support the answers where descriptive answers are required.

QUESTION 10



A wide shallow river of uniform depth and constant slope caries water at a certain flow rate. Under flood conditions the water depth is twice (two times) the normal depth. Assume that the river banks are high enough to prevent the river from increasing in width as the level rises.

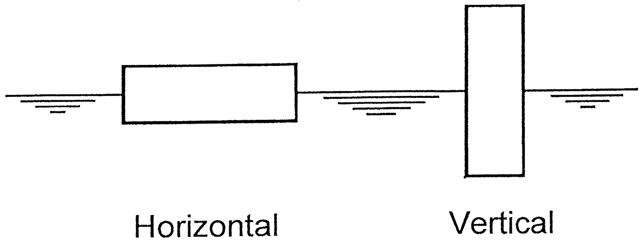
(a) State whether the velocity under flood conditions will be ***less than / the same as / greater than*** the normal flow velocity.

(b) State whether the flow rate will be ***less than / the same as / between one and two times / two times / greater than two times*** the normal flow rate.

(c) Justify your answers with reference to the applicable theoretical relations.

(5 marks)

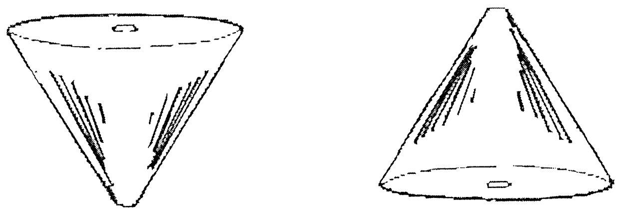
QUESTION 11



A long bar of square cross section with density half that of water will float half submerged as shown in the sketch above. State which orientation will be stable - horizontal or vertical. Explain why the chosen orientation is stable and how this can be proven.

(5 marks)

QUESTION 12

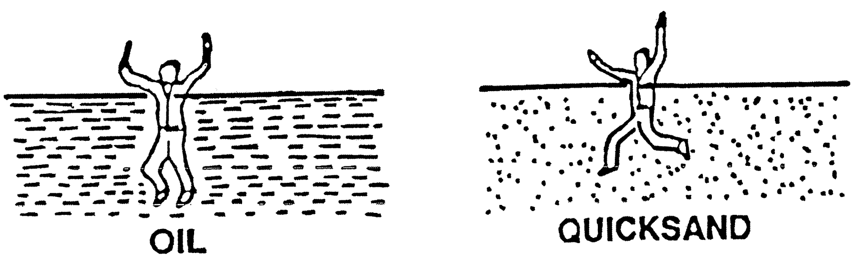


Upright Tank **A** Inverted Tank **B**

The sketch above shows two conical tanks A and B of the same geometry and dimensions but one is inverted. Both are filled with water and adequately vented at the top. Assuming each has a similar hole at the bottom to discharge this water, determine which will be emptied faster. Assume no change in the discharge coefficient due to the shape of the tank. Fully justify the answer with appropriate equations.

(5 marks)

QUESTION 13



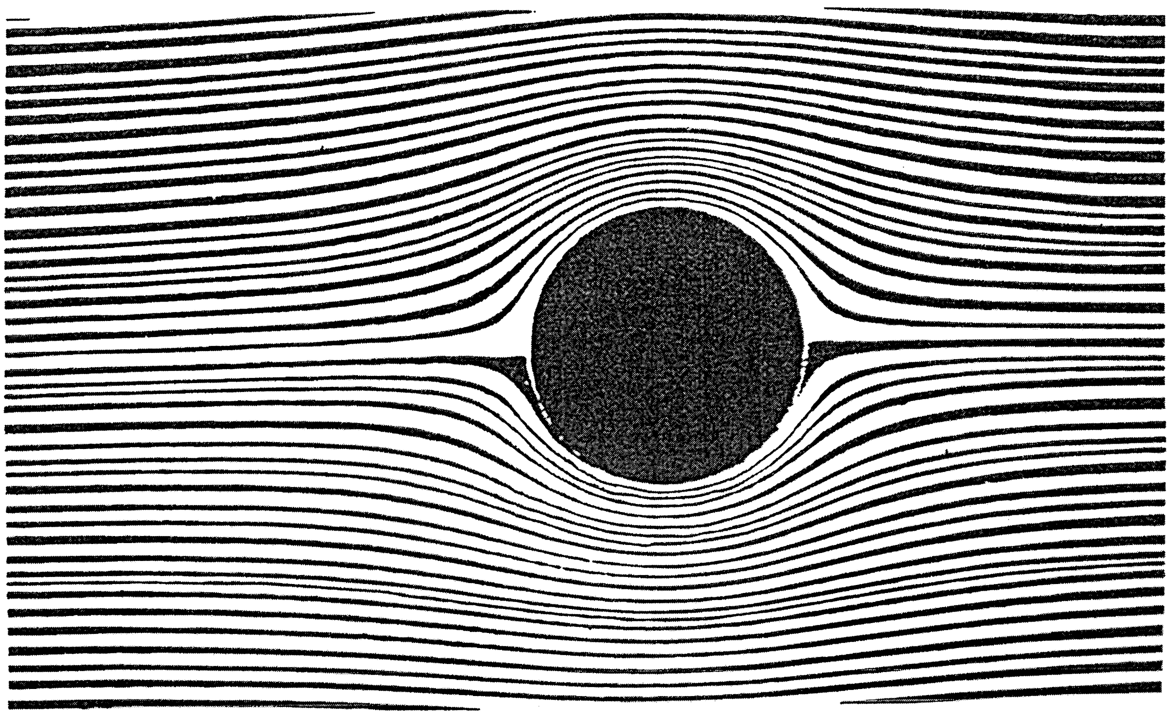
Two ponds contain light oil and quicksand (a slurry of fine sand and water in suspension) as shown above. State in which humans are more likely to sink more deeply. Give a full explanation for your answer.

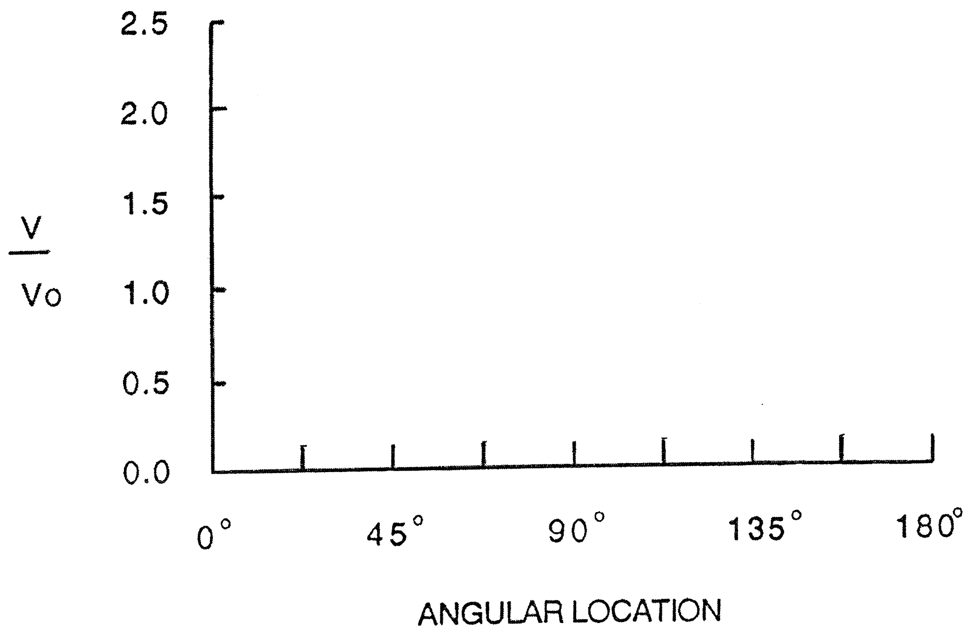
(5 marks)

EXAMINATION PAPER ATTACHMENTS

NAME ....

QUESTION 3 VELOCITY VARIATION AROUND CYLINDER





QUESTION 7 PIPE FLOW DATA

Values of loss factors for pipe fittings

Fitting	k	L/D
Globe valve, wide open	10	350
Angle valve, wide open	5	175
Close-return bend	2.2	75
T, through side outlet	1.8	67
Short-radius elbow	0.9	32
Medium-radius elbow	0.75	27
Long-radius elbow	0.60	20
45° elbow	0.42	15
Gate valve, wide open	0.19	7
half open	2.06	72

Loss coefficients for sudden contraction

D_2/D_1	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
k_c	0.50	0.45	0.42	0.39	0.36	0.33	0.28	0.22	0.15	0.06	0.00

Values of absolute roughness e for new commercial pipes

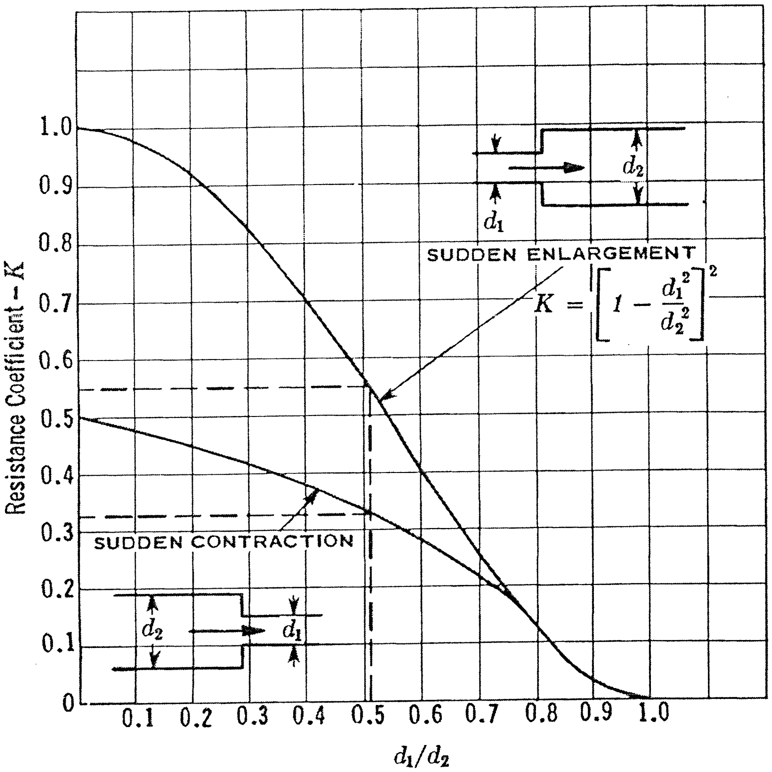
	Feet	Millimeters
Glass, plastic (smooth)	0.0	0.0
Drawn tubing, brass, lead, copper, centrifugally spun cement, bituminous lining, transite	0.000005	0.0015
Commercial steel, wrought iron, welded-steel pipe	0.00015	0.046
Asphalt-dipped cast iron	0.0004	0.12
Galvanized iron	0.0005	0.15
Cast iron, average	0.00085	0.25
Wood stave	0.0006–0.003	0.18–0.9
Concrete	0.001–0.01	0.3–3
Riveted steel	0.003–0.03	0.9–9

Note: $\frac{e}{D} = \frac{e \text{ in feet}}{D \text{ in feet}} = 12 \times \frac{e \text{ in feet}}{D \text{ in inches}} = \frac{e \text{ in mm}}{D \text{ in mm}}$

QUESTION 7 PIPE FLOW DATA

Resistance in Pipe

Resistance Due to Sudden Enlargements and Contractions[20]



Sudden enlargement: The resistance coefficient K for a sudden enlargement from 6-inch Schedule 40 pipe to 12-inch Schedule 40 pipe is **0.55**, based on the 6-inch pipe size.

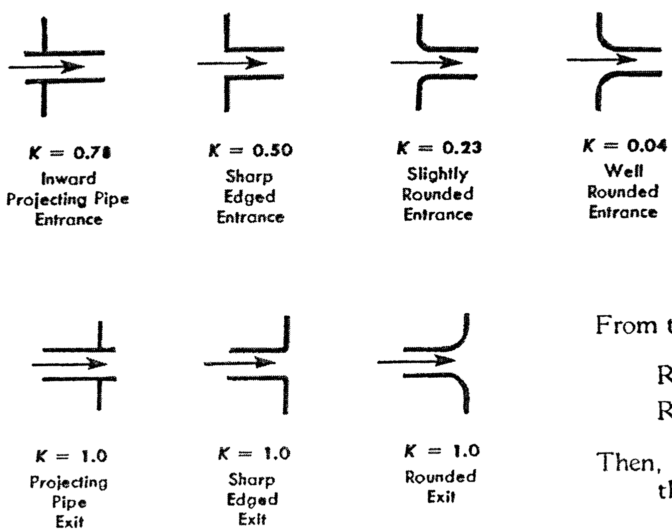
$$\frac{d_1}{d_2} = \frac{6.065}{11.938} = 0.51$$

Sudden contraction: The resistance coefficient K for a sudden contraction from 12-inch Schedule 40 pipe to 6-inch Schedule 40 pipe is **0.33**, based on the 6-inch pipe size.

$$\frac{d_1}{d_2} = \frac{6.065}{11.938} = 0.51$$

Note: The values for the resistance coefficient, K, are based on velocity in the small pipe. To determine K values in terms of the greater diameter, multiply the chart values by $(d_2/d_1)^4$.

Resistance Due to Pipe Entrance and Exit



Problem: Determine the total resistance coefficient for a pipe one diameter long having a sharp edged entrance and a sharp edged exit.

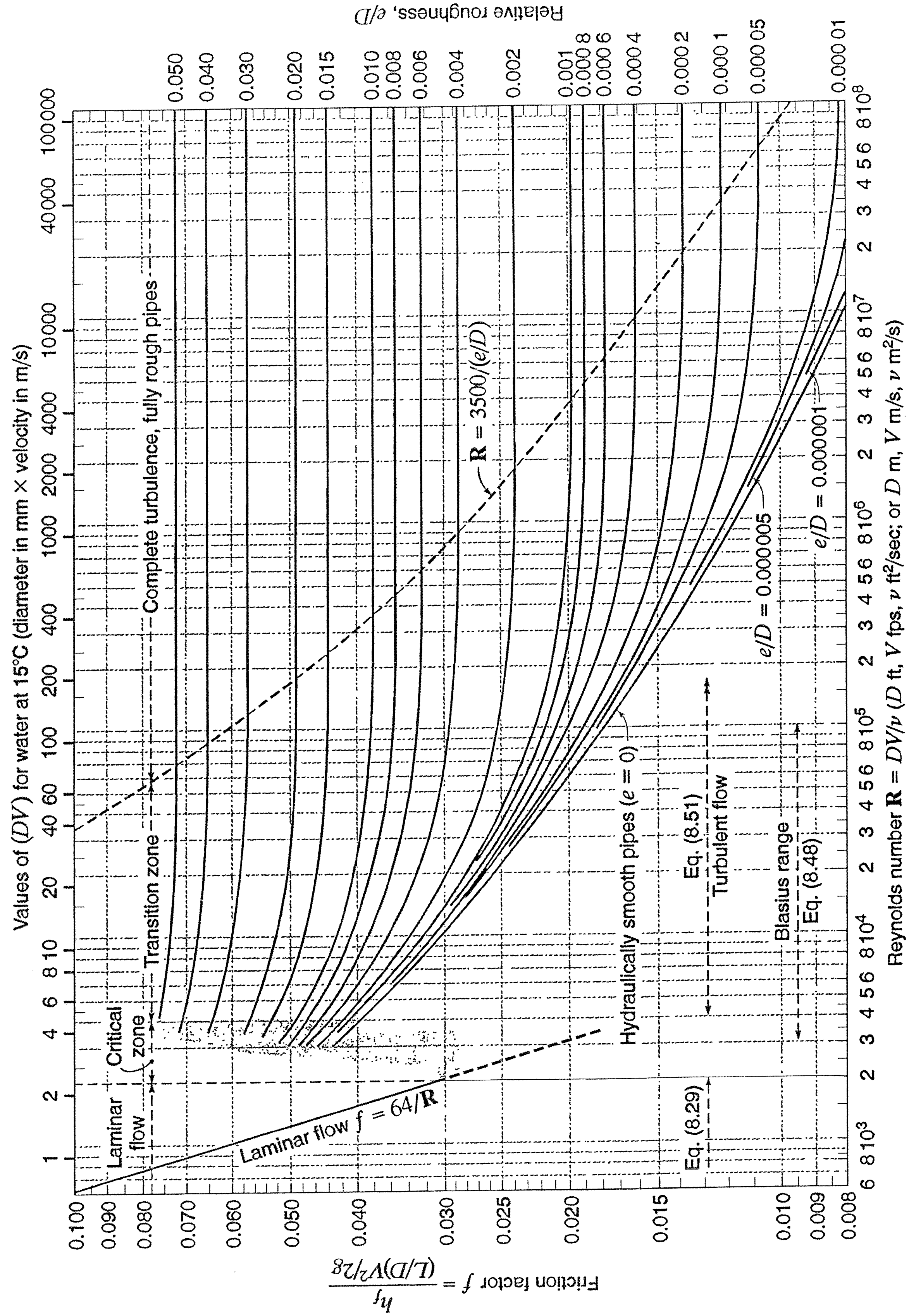
Solution: The resistance of pipe one diameter long is small and can be neglected ($K = f\,L/D$).

From the diagrams, note:

Resistance for a sharp edged entrance	= 0.5
Resistance for a sharp edged exit	= 1.0

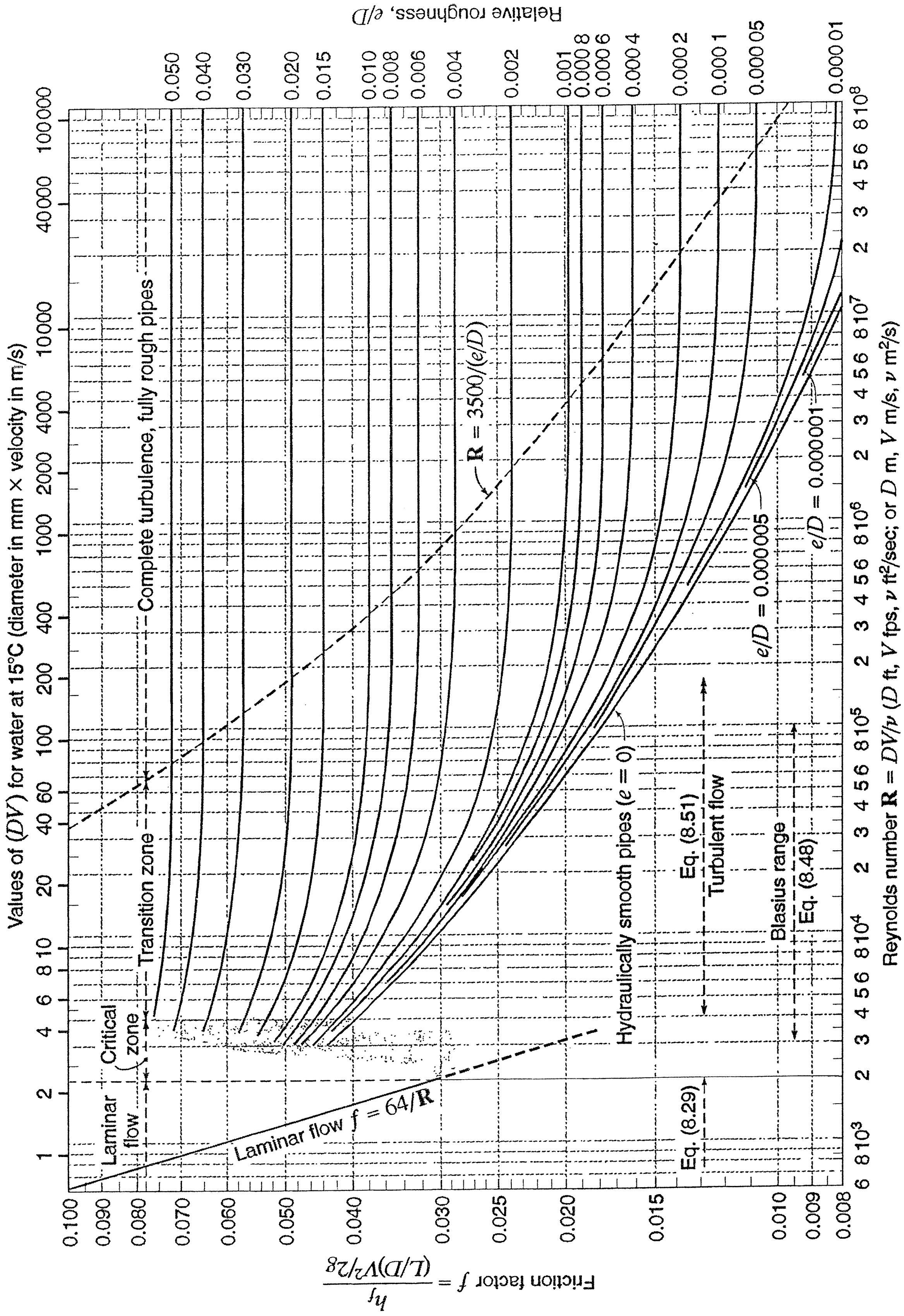
Then,
the total resistance, K, for the pipe = **1.5**

QUESTION 7 MOODY DIAGRAM NAME ....



Moody chart for pipe friction factor (Stanton diagram).

QUESTION 8 MOODY DIAGRAM NAME ....



Moody chart for pipe friction factor (Stanton diagram).

QUESTION 9 DRAG DIAGRAM FOR SOLID BODIES NAME ....

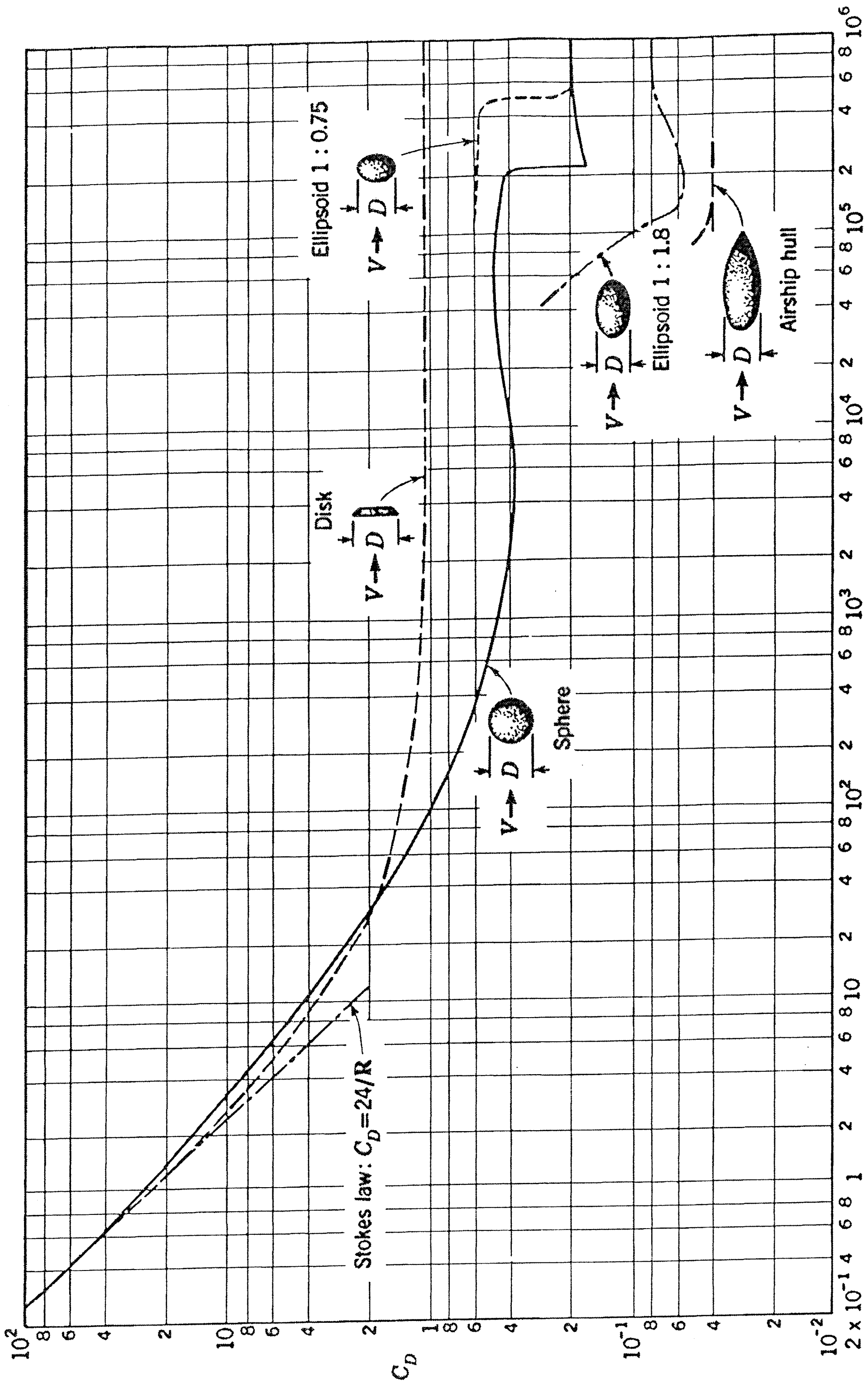


Figure 10.10 Drag coefficient for bodies of revolution. (Adapted from L. Prandtl, "Ergebnisse der aerodynamischen Versuchsanstalt zu Göttingen," p. 29, R. Oldenbourg, Munich and Berlin, 1923; and F. Eisner, "Das Widerstandsproblem," *Proc. 3d Internatn. Congr. Appl. Mech., p. 32, 1930.*)

04-BS-7 MECHANICS OF FLUIDS

GENERAL REFERENCE INFORMATION

CONSTANTS

In engineering calculations a high degree of accuracy is seldom attained due to the neglect of minor influences or the inaccuracy of available data. For consistency in calculations however the following reasonably accurate constants should be used:

Atmospheric Pressure p_o = 100 kPa
Gravitational Acceleration g = 9.81 m/s^2
Specific Gravity of Water = 1.00
Specific Gravity of Glycerine = 1.26
Specific Gravity of Mercury = 13.56
Specific Gravity of Benzene = 0.90
Specific Gravity of Carbon Tetrachloride = 1.59
Density of Water ρ = 1000 kg/m^3
Density of Sea Water ρ = 1025 kg/m^3
Density of Concrete ρ = 2400 kg/m^3
Density of Air ρ = 1.19 kg/m^3 (at 20°C), ρ = 1.21 kg/m^3 (at 15°C)
Absolute Viscosity of Water μ = 1.0 x 10^{-3} Ns/m^2
Absolute Viscosity of Air μ = 1.8 x 10^{-5} Ns/m^2
Surface Tension of Water σ = 0.0728 N/m (at 20°C)
Specific Heat of Water c_p = 4.19 kJ/kg°C
Specific Heat of Air c_p = 1005 J/kg°C
Specific Heat of Air c_v = 718 J/kg°C
Gas Constant for Air R = 287 J/kg°K
Gas Constant for Helium R = 2077 J/kg°K
Gas Constant for Hydrogen R = 4120 J/kg°K

NOMENCLATURE FOR REFERENCE EQUATIONS (SI UNITS)

a	Width	m
A	Flow area, Surface area	m^2
CV	Calorific value	J/kg
c_p	Specific heat at constant pressure	J/kg°C
b	Width	m
D	Diameter	m
E	Energy	J
F	Force	N
g	Gravitational acceleration	m/s^2
h	System head	m
h_L	Head loss	m
H	Pump or turbine head	m
I	Moment of inertia	m^4
k	Ratio of specific heats	
k	Loss coefficient	
K	Constant	
L	Length	m
m	Mass	kg
M	Mass flow rate	kg/s
N	Rotational speed	rev/s
p	Pressure	Pa (N/m^2)
P	Power	W (J/s)
q	Specific heat	J/kg
Q	Flow rate	m^3/s
r	Radius	m
R	Specific gas constant	J/kg K
T	Temperature	K
U	Blade velocity	m/s
v	Specific volume	m^3/kg
V	Velocity	m/s
V	Volume	m^3
w	Specific work	J/kg
W	Work	J
y	Depth	m
z	Elevation	m
η	Efficiency	
μ	Dynamic viscosity	Ns/m^2
ν	Kinematic viscosity	m^2/s
ρ	Density	kg/m^3
σ	Surface tension	N/m
τ	Thrust	N
τ	Shear stress	N/m^2

REFERENCE EQUATIONS

Equation of State

$$p\ v = R\ T$$
$$p = \rho\ RT$$

Universal Gas Law

$$p\ v^n = \text{constant}$$

Compressibility

$$\beta = -\Delta / V\ \Delta p$$

Viscous Force and Viscosity

$$F = \mu\ A\ du/dy$$
$$\mu = \tau / (du/dy)$$
$$\nu = \mu / \rho$$

Capillary Rise and Internal Pressure due to Surface Tension

$$h = (\sigma \cos\theta / \rho\ g) \times (\text{perimeter} / \text{area})$$
$$p = 2\ \sigma / r$$

Pressure at a Point

$$p = \rho\ g\ h$$

Forces on Plane Areas and Centre of Pressure

$$F = \rho\ g\ y_c\ A$$
$$y_p = y_c + I_c / y_c\ A$$

Moments of Inertia

Rectangle: $I_c = b\ h^3 / 12$
Triangle: $I_c = b\ h^3 / 36$
Circle: $I_c = \pi\ D^4 / 64$

Surface Area of Solids

Sphere: $A = \pi\ D^2$

Volumes of Solids

Sphere: $V = \pi D^3 / 6$
Cone: $V = \pi D^2 h / 12$
Spherical Segment: $V = (3a^2 + 3b^2 + 4h^2)\,\pi h / 2g$

Continuity Equation

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 = M$$

General Energy Equation

$$p_1 / \rho_1 g + z_1 + V_1^2 / 2g + q_{in} / g + w_{in} / g$$
$$= p_2 / \rho_2 g + z_2 + V_2^2 / 2g + h_L + q_{out} / g + w_{out} / g$$

Bernoulli Equation

$$p_1 / \rho g + z_1 + V_1^2 / 2g = p_2 / \rho g + z_2 + V_2^2 / 2g$$

Momentum Equation

Conduit: $F_R = p_1 A - p_2 A - M (V_2 - V_1)$
Free Jet: $F_R = -\rho Q (V_2 - V_1)$

Flow Measurement

Venturi Tube: $Q = [C A_2 / \{1 - (D_2 / D_1)^4\}^{1/2}][2 g \Delta h]^{1/2}$
Flow Nozzle: $Q = K A_2 [2 g \Delta h]^{1/2}$
Orifice Meter: $Q = K A_o [2 g \Delta h]^{1/2}$

Flow over Weirs

Rectangular Weir: $Q = C_d (2/3) [2g]^{1/2} L H^{3/2}$

Power

Turbomachine: $P = \rho g Q H$
Free Jet: $P = \frac{1}{2} \rho Q V^2$
Moving Blades: $P = M \Delta V U$

Aircraft Propulsion

$F_{thrust} = M (V_{jet} - V_{aircraft})$
$P_{thrust} = M (V_{jet} - V_{aircraft}) V_{aircraft}$
$E_{jet} = \frac{1}{2} (V_{jet}^2 - V_{aircraft}^2)$
$P_{jet} = \frac{1}{2} M (V_{jet}^2 - V_{aircraft}^2)$

$E_{fuel} = CV_{fuel}$
$P_{fuel} = M_{fuel}\ CV_{fuel}$
$\eta_{thermal} = P_{jet} / P_{fuel}$
$\eta_{propulsion} = P_{thrust} / P_{jet} = 2\ V_{aircraft} / (V_{jet} + V_{aircraft})$
$\eta_{overall} = \eta_{thermal} \times \eta_{propulsion}$

Wind Power

$P_{total} = ½\ \rho\ A_T\ V_1^3$
$P_{max} = 8/27\ \rho\ A_T\ V_1^3$
$H_{max} = P_{max} / P_{total} = 16/27$

Reynolds Number

$Re = d\ V\ \rho / \mu$

Flow in Pipes

$h_L = f\ (L / D)\ (V^2 / 2\ g)$
$D_e = 4$ (flow area) / (wetted perimeter)
$D = D_e$ for non-circular pipes
$L = L_{total} + L_e$ for non-linear pipes
$(L / D) = 35\ k$ for $Re \sim 10^4$

Drag on Immersed Bodies

Friction Drag: $F_f = C_f\ ½\ \rho\ V^2\ B\ L\ \ (B = \pi\ D)$
Pressure Drag: $F_p = C_p\ ½\ \rho\ V^2\ A$
Total Drag: $F_D = C_D\ ½\ \rho\ V^2\ \mathbf{A}$

Aircraft Wing: $F_L = C_L\ ½\ \rho\ V^2\ A_{wing}$
Aircraft Wing: $F_D = C_D\ ½\ \rho\ V^2\ A_{wing}$

Karmen Vortex Frequency

$f \approx 0.20\ (V / D)\ (1 - 20 / Re)$