16-CHEM-B1, TRANSPORT PHENOMENA

MAY 2017

3 hours duration

NOTES

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. The examination is an **open book exam.** One textbook of your choice with notations listed on the margins etc., but no loose notes are permitted into the exam.

3. Candidates may use any **non-communicating** calculator.

4. All problems are worth 25 points. **One problem** from **each** of sections A, B, and C must be attempted. A **fourth** problem from **any section** must also be attempted.

5. **Only the first four** questions as they appear in the answer book will be marked.

SECTION A: Fluid Mechanics

A1. Air at 70°F and 14.7 psi flows over a flat plate with a free stream velocity of 50 feet/sec. The kinematic viscosity of air at these conditions is 1.8×10^{-4} ft²/sec.

a) **[5 points]** Determine the boundary layer thickness at a point 5 feet from the leading edge.

b) **[10 points]** Redo part (a) taking into account that part of the boundary layer is laminar and the rest is turbulent. The transition from laminar to turbulent occurs at a Reynolds number of 3.2×10^5.

c) **[10 points]** Determine the total drag force for part (a).

A2. Stenosis (narrowing of a blood vessel or valve) arises during artherosclerosis and could block an artery, depriving the tissue downstream of oxygen. Consider a symmetric stenosis as shown in the figure below:
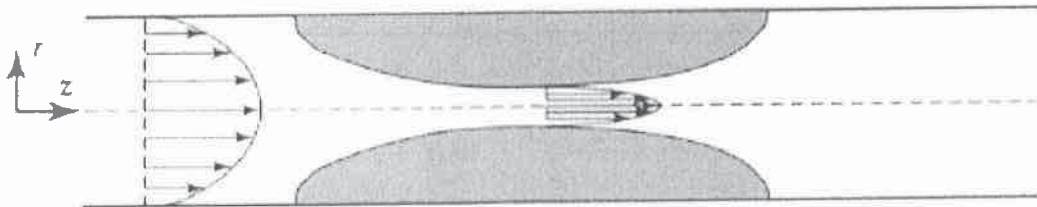
Assume that the velocity profile within the stenosis of radius $R_i(z)$ has the same shape as the profile outside the stenosis and it is represented as

$$V_z(r) = V_{max}\ \{1 - (r^2/R^2)\}$$

Outside the stenosis, the radius equals R_0 and the maximum velocity (V_{max}) is constant. Within the stenosis, the radius of the fluid channel $R(z)$ is represented as

$$R(z) = R_0\ \{1 - 0.5[1 - 4(z/L)^2]^{1/2}\}$$

The origin of the z-axis is the midpoint of the stenosis.

a) **[12 points]** Develop an expression for V_{max} in a stenosis in terms of the volumetric flow rate Q, cylindrical tube of radius R_0, and distance along the stenosis z/L.

b) **[13 points]** Compute the ratio of shear stress acting on the surface of the stenosis ($r = R_i$) at $z = 0$ relative to the value of shear stress outside the stenosis.

SECTION B: Heat Transfer

B1. A 15-mm spherical Ni/Al_2O_3 catalyst is used to catalyze the endothermic, steam reforming of methane to give hydrogen and water. The heat of reaction is 40 kcal/mole and the methane reaction rate is 0.05 mol/m^{2}s. At a point in the packed-bed reactor, the gas has a superficial mass velocity of 3500 kg/m^{2}s and is at 800°C. What is the steady-state temperature of the catalyst surface?

DATA FOR THE REACTION MIXTURE:

Heat capacity = 3.2 cal/g.K

Viscosity = 0.03 cP

Thermal conductivity = 0.33 J/m.s.K

B2. A fluid is flowing upward over a thin, flat vertical plate under steady-state laminar conditions. The temperature of the plate is higher than the external fluid temperature, and mixed convection (forced and free) exists. The Prandtl number for the fluid is 0.2423. Develop the criteria in terms of relevant dimensionless numbers that can show when free, forced, or mixed convection will prevail over the situation.

SECTION C: Mass Transfer

C1. For the flow of a fluid over a flat membrane of length 10 cm at an average velocity of 5 cm/s, determine the length-average mass transfer coefficient.

DATA FOR THE SYSTEM:
Kinematic viscosity $= 0.01$ cm^2/s
Diffusivity $= 5 \times 10^{-6}$ cm^2/s

C2. Oxygen is consumed in body tissues, or by cells maintained in vitro, at a rate that is often nearly independent of the O_2 concentration. As a model for a tissue region or aggregate of cells, consider stead-state O_2 diffusion in a sphere of radius r_0 with zeroth-order consumption of O_2. Assume that the O_2 concentration at the outer surface ($r = r_0$) is maintained at C_0.

a) **[10 points]** Determine the O_2 concentration profile $C(r)$ as a function of radius.

b) **[15 points]** If r_0 and the rate of O_2 consumption are sufficiently large, no O_2 will reach a central core (defined by $r < r_c$). For the central core the assumption of zeroth-order kinetics is no longer valid because no O_2 is available to react. Determine the criteria when an oxygen-free central core will exist and find an expression for r_c.

APPENDIX

Summary of the Conservation Equations

Table A.1 The Continuity Equation

$$\frac{\partial \rho}{\partial t} + \left(\nabla \cdot \rho \vec{u} \right) = 0 \tag{1.1}$$

Rectangular coordinates (x, y, z)

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}\left(\rho u_x \right) + \frac{\partial}{\partial y}\left(\rho u_y \right) + \frac{\partial}{\partial z}\left(\rho u_z \right) = 0 \tag{1.1a}$$

Cylindrical coordinates (r, θ, z)

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}\left(\rho r u_r \right) + \frac{1}{r}\frac{\partial}{\partial \theta}\left(\rho u_\theta \right) + \frac{\partial}{\partial z}\left(\rho u_z \right) = 0 \tag{1.1b}$$

Spherical coordinates (r, θ, ϕ)

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho r^2 u_r \right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(\rho u_\theta \sin\theta \right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\left(\rho u_\phi \right) = 0 \tag{1.1c}$$

Table A.2 The Navier-Stokes equations for Newtonian fluids of constant ρ and μ

$$\frac{\partial \vec{u}}{\partial t} + \left(\vec{u} \cdot \nabla \right)\vec{u} = -\frac{1}{\rho}\nabla P + \vec{g} + \nu \left(\nabla^2 \vec{u} \right) \tag{A2}$$

Rectangular coordinates (x, y, z)

x-component
$$\frac{\partial u_x}{\partial t} + u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} + u_z \frac{\partial u_x}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + g_x + \nu \left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_x}{\partial z^2} \right) \tag{A2a}$$

y-component
$$\frac{\partial u_y}{\partial t} + u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y} + u_z \frac{\partial u_y}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + g_y + \nu \left(\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_y}{\partial z^2} \right) \tag{A2b}$$

z-component
$$\frac{\partial u_z}{\partial t} + u_x \frac{\partial u_z}{\partial x} + u_y \frac{\partial u_z}{\partial y} + u_z \frac{\partial u_z}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + g_z + \nu \left(\frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial y^2} + \frac{\partial^2 u_z}{\partial z^2} \right) \tag{A2c}$$

Cylindrical coordinates (r, θ, z)

r-component

$$\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_r}{\partial \theta} + u_z \frac{\partial u_r}{\partial z} - \frac{u_\theta^2}{r}$$

$$= -\frac{1}{\rho} \frac{\partial P}{\partial r} + g_r + \nu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial (r u_r)}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial u_\theta}{\partial \theta} + \frac{\partial^2 u_r}{\partial z^2} \right]$$

(A2d)

θ-component

$$\frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\theta}{\partial \theta} + u_z \frac{\partial u_\theta}{\partial z} + \frac{u_r u_\theta}{r}$$

$$= -\frac{1}{\rho r} \frac{\partial P}{\partial \theta} + g_\theta + \nu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial (r u_\theta)}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial u_r}{\partial \theta} + \frac{\partial^2 u_\theta}{\partial z^2} \right]$$

(A2e)

z-component

$$\frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_z}{\partial \theta} + u_z \frac{\partial u_z}{\partial z}$$

$$= -\frac{1}{\rho} \frac{\partial P}{\partial z} + g_z + \nu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2} \right]$$

(A2f)

Spherical coordinates (r, θ, ϕ)

r-component

$$\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_r}{\partial \theta} + \left(\frac{u_\phi}{r \sin \theta} \right) \frac{\partial u_r}{\partial \phi} - \frac{u_\theta^2}{r} - \frac{u_\phi^2}{r} = -\frac{1}{\rho} \frac{\partial P}{\partial r} + g_r$$

$$+ \nu \left[\frac{1}{r^2} \frac{\partial^2}{\partial r^2} \left(r^2 u_r \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial u_r}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 u_r}{\partial \phi^2} \right]$$

(A2g)

θ-component

$$\frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\theta}{\partial \theta} + \left(\frac{u_\phi}{r \sin \theta} \right) \frac{\partial u_\theta}{\partial \phi} + \frac{u_r u_\theta}{r} - \frac{u_\phi^2}{r} \cot \theta = -\frac{1}{\rho r} \frac{\partial P}{\partial \theta} + g_\theta$$

$$+ \nu \left[\begin{array}{c} \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial u_\theta}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(u_\theta \sin \theta \right) \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 u_\theta}{\partial \phi^2} \\ + \frac{2}{r^2} \frac{\partial u_r}{\partial \theta} - \frac{2 \cot \theta}{r^2 \sin \theta} \frac{\partial u_\phi}{\partial \phi} \end{array} \right]$$

(A2h)

ϕ-component

$$\frac{\partial u_\phi}{\partial t} + u_r \frac{\partial u_\phi}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\phi}{\partial \theta} + \frac{u_\phi}{r \sin \theta} \frac{\partial u_\phi}{\partial \phi} + \frac{u_r u_\phi}{r} + \frac{u_\theta u_\phi}{r} \cot \theta = -\frac{1}{\rho r \sin \theta} \frac{\partial P}{\partial \phi}$$

$$+ g_\phi + \nu \left[\begin{array}{c} \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial u_\phi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(u_\phi \sin \theta \right) \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 u_\phi}{\partial \phi^2} \\ + \frac{2}{r^2 \sin \theta} \frac{\partial u_r}{\partial \phi} + \frac{2 \cot \theta}{r^2 \sin \theta} \frac{\partial u_\theta}{\partial \phi} \end{array} \right]$$

(A2i)

Table A.3 The Energy Equation for Incompressible Media

$$\rho c_P \left[\frac{\partial T}{\partial t} + (\vec{u} \cdot \nabla)(T) \right] = \left[\nabla \cdot k \nabla T \right] + \dot{T}_G \qquad \text{(A3)}$$
Rectangular coordinates (x, y, z) $$\rho c_P \left[\frac{\partial T}{\partial t} + u_x \frac{\partial T}{\partial x} + u_y \frac{\partial T}{\partial y} + u_z \frac{\partial T}{\partial z} \right] = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z} \right) + \dot{T}_G \qquad \text{(A3a)}$$
Cylindrical coordinates (r, θ, z) $$\rho c_P \left[\frac{\partial T}{\partial t} + u_r \frac{\partial T}{\partial r} + \frac{u_\theta}{r} \frac{\partial T}{\partial \theta} + u_z \frac{\partial T}{\partial z} \right] = \frac{1}{r}\frac{\partial}{\partial r}\left(rk \frac{\partial T}{\partial r} \right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k \frac{\partial T}{\partial \theta} \right) + \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z} \right) + \dot{T}_G \qquad \text{(A3b)}$$
Spherical coordinates (r, θ, ϕ) $$\rho c_P \left[\frac{\partial T}{\partial t} + u_r \frac{\partial T}{\partial r} + \frac{u_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{u_\phi}{r \sin\theta} \frac{\partial T}{\partial \phi} \right] =$$ $$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 k \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(k \sin\theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial}{\partial \phi}\left(k \frac{\partial T}{\partial \phi} \right) + \dot{T}_G \qquad \text{(A3c)}$$

Table A4: The continuity equation for species A in terms of the molar flux

$$\frac{\partial C_A}{\partial t} = -\left(\nabla \cdot \vec{N}_A \right) + \dot{R}_{A,G} \qquad \text{(4.)}$$
Rectangular coordinates (x, y, z) $$\frac{\partial C_A}{\partial t} = -\left(\frac{\partial [N_A]_x}{\partial x} + \frac{\partial [N_A]_y}{\partial y} + \frac{\partial [N_A]_z}{\partial z} \right) + \dot{R}_{A,G} \qquad \text{(4a)}$$
Cylindrical coordinates (r, θ, z) $$\frac{\partial C_A}{\partial t} = -\left\{ \frac{1}{r}\frac{\partial}{\partial r}[rN_A]_r + \frac{1}{r}\frac{\partial}{\partial \theta}[N_A]_\theta + \frac{\partial}{\partial z}[N_A]_z \right\} + \dot{R}_{A,G} \qquad \text{(4b)}$$
Spherical coordinates (r, θ, ϕ) $$\frac{\partial C_A}{\partial t} = -\left\{ \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 [N_A]_r \right) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}\left([N_A]_\theta \sin\theta \right) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \phi}[N_A]_\phi \right\} + \dot{R}_{A,G} \qquad \text{(4c)}$$

Table A.5: The continuity equation for species A

$$\frac{\partial C_A}{\partial t} + (\vec{u} \cdot \nabla)C_A = D_A \nabla^2 C_A + \dot{R}_{A,G} \tag{5}$$

Rectangular coordinates (x, y, z)

$$\frac{\partial C_A}{\partial t} + u_x \frac{\partial C_A}{\partial x} + u_y \frac{\partial C_A}{\partial y} + u_z \frac{\partial C_A}{\partial z} = \frac{\partial}{\partial x}\left(D\frac{\partial C_A}{\partial x}\right) + \frac{\partial}{\partial y}\left(D\frac{\partial C_A}{\partial y}\right) + \frac{\partial}{\partial z}\left(D\frac{\partial C_A}{\partial z}\right) + \dot{R}_{A,G} \tag{5a}$$

Cylindrical coordinates (r, θ, z)

$$\frac{\partial C_A}{\partial t} + u_r \frac{\partial C_A}{\partial r} + \frac{u_\theta}{r}\frac{\partial C_A}{\partial \theta} + u_z \frac{\partial C_A}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(rD\frac{\partial C_A}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(D\frac{\partial C_A}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(D\frac{\partial C_A}{\partial z}\right) + \dot{R}_{A,G} \tag{5b}$$

Spherical coordinates (r, θ, ϕ)

$$\frac{\partial C_A}{\partial t} + u_r \frac{\partial C_A}{\partial r} + \frac{u_\theta}{r}\frac{\partial C_A}{\partial \theta} + \frac{u_\phi}{r\sin\theta}\frac{\partial C_A}{\partial \phi} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 D\frac{\partial C_A}{\partial r}\right)$$

$$+ \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(D\sin\theta\frac{\partial C_A}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial}{\partial \phi}\left(D\frac{\partial C_A}{\partial \phi}\right) + \dot{R}_{A,G} \tag{5c}$$