National Exams – May 2015

07–Mec-B6 Advanced Fluid Mechanics

3 hours duration

Instructions:

1. If doubt exists as to the interpretation of any question the candidate is urged to submit with the answer paper a clear statement of the assumptions made.
2. Candidates may use any approved Sharp/Casio calculator.
   The exam is OPEN BOOK.
3. Any FIVE (5) out of the 6 questions constitute a complete exam paper for a total of 100 MARKS.
   The first five questions as they appear in the answer book will be marked.
4. Each question is of equal value (20 marks) and question items are marked as indicated.
5. Clarity and organization of the answer are important.

(20) **Question 1**

Consider the ideal flow given by the velocity potential function

$$\phi = \frac{-A}{2\pi} \ln r$$

where A is a positive constant.

(5) (a) Determine the stream function ψ.

(5) (b) Sketch the equipotential lines and and the stream lines of this flow.

(5) (c) Calculate the radial velocity V_r and identify the flow pattern.

(5) (d) Give the physical meaning of the constant A.

(20) **Question 2**

A large pressurized air reservoir (a) contains air at temperature T_a=100°C and at a constant pressure of P_a=300 kPa. The air passes through a convergent-divergent nozzle from reservoir (a) to another large reservoir (b), as shown in Figure 1. The throat area of the nozzle is A_T=9 cm^3 and the exit area is A_E=31.5 cm^3.
A mercury manometer reads h = 15 cm between the throat and reservoir (b).
Assume frictional losses are negligible and that the pressurized air density is negligible compared with mercury.
Air properties: γ=1.4, R=287 J/(kg K), C_P=1004.5 J/(kg K)
Mercury properties: ρ=13,550 kg/m^3

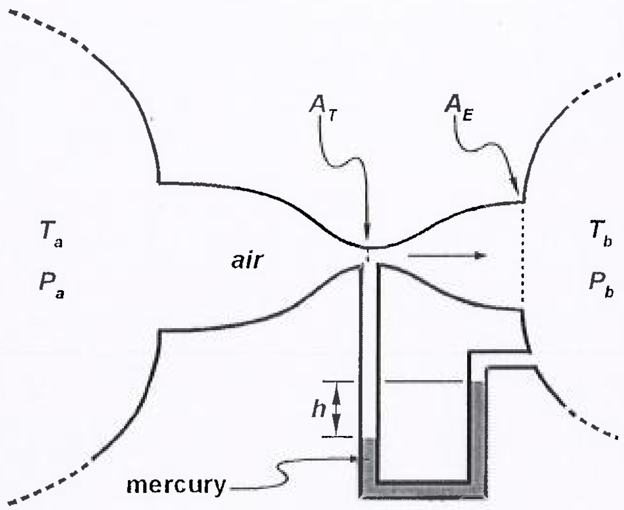

Figure 1: Convergent-divergent nozzle between two large air reservoirs.

(5) (a) Estimate the downstream reservoir pressure.

(5) (b) Is there a normal shock wave in the flow?

(5) (c) If there is a normal shock wave, does it stand in the exit plane or farther upstream?

(5) (d) What would be the mercury manometer reading if the nozzle were operating exactly at supersonic design conditions, i.e. with an *ideal expansion*?

(20) **Question 3**

Air flows through an insulated constant diameter horizontal duct as shown in Figure 2. The inlet conditions are P_1=700 kPa, T_1=50°C and V_1=150 m/s. The diameter of the pipe is D=15 cm and the flow is "choked". Determine the net force of the fluid on the pipe.

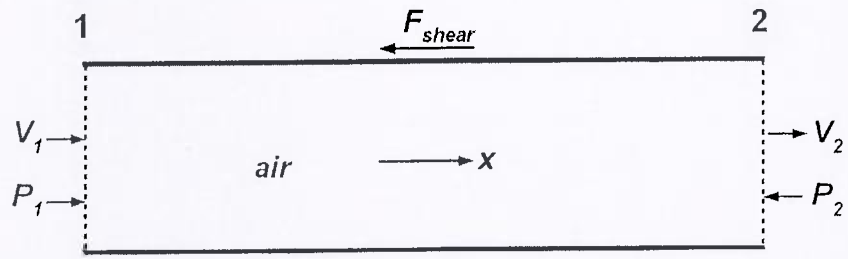

Figure 2: Pipe section with choked flow at the exit.

(20) **Question 4**

The lubrication of bearings is a problem in fluid mechanics. The oil or lubricant between the bearing and the slider is a viscous fluid and most bearings operate in the laminar range with a very small Reynolds number. The spacing between the bearing and the slider (the flow gap) is much smaller than the length of the slider so that the flow becomes fully developed throughout most of the gap. Because the Reynolds number is so small, the inertia of the fluid is negligible compared with the pressure and viscous forces.

Consider a step bearing moving at velocity U, as shown in Figure 3. The spacings h_1 and h_2 are much less than L_1 and L_2 and the width of the bearing in the z direction is assumed to be very large, so that leakage in the z direction can be neglected. To facilitate the analysis, we attach coordinate systems to the slider with the origins at the beginning of each bearing section and we invert the direction of y, such that the relative velocity is $u = 0$ when $y = 0$, and $u = -U$ when $y = h$ in the respective sections.

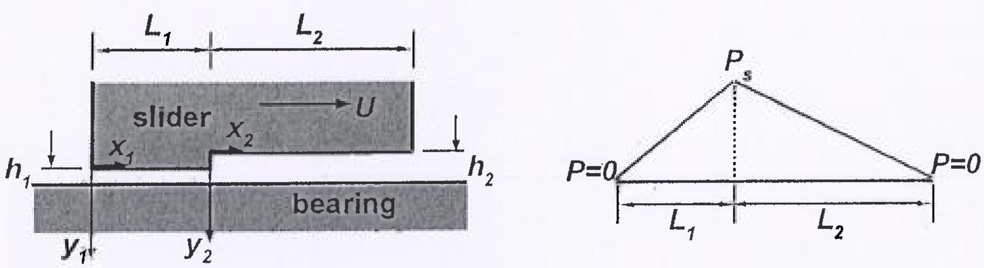

Figure 3: The step bearing and its pressure distribution.

(5) (a) Assuming incompressible flow of a Newtonian fluid, simplify the equations of motion (the Navier-Stokes equations) in x and in y directions.

(5) (b) Find an expression for the total flow rate and for the pressure change between any two points in a single section of the bearing.

(5) (c) Find an expression for the relative pressure distribution in the step bearing, assuming a zero gauge pressure at the inlet and outlet, as shown in Figure 3.

(5) (d) Calculate the total load carrying capacity per unit length of z for a step bearing with the following conditions:
U=0.5 m/s ; μ_{oil}=3.85 (N s)/m^2; ρ_{oil}=900 kg/m^3
h_1=1 mm ; L_1=30 mm
h_2=2 mm ; L_2=100 mm

(20) **Question 5**

The lift force F on a missile is a function of its length L, velocity V, diameter D, angle of attack α, density ρ, viscosity μ, and speed of sound of the air c.

Find the dimensionless π groups and rewrite the function in terms of known π groups, if there are any.

(20) **Question 6**

A thin, flat, two-sided plate of length L and height δ is attached orthogonally to a wall and oriented parallel to an approaching boundary layer flow, i.e. the plate is parallel to the x, y-plane. Assume that the boundary layer flow is fully turbulent and that the velocity profile follows a one-seventh power law, i.e. $u(y) = U_\infty (y/\delta)^{1/7}$.

(15) (a) Derive a formula for the drag coefficient of this plate.

(5) (b) How does this drag compare against the drag on the same plate immersed on a uniform stream with speed U_∞?