National Exams December 2017

16-Mec-B10, Finite Element Analysis

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. This is an OPEN BOOK EXAM.
   Any non-communicating calculator is permitted.
3. FIVE (5) questions constitute a complete exam paper.
   The first five questions as they appear in the answer book will be marked. The questions are to be solved within the context of the finite element method.
4. Each question is of equal value.
5. Some questions require an answer in essay format. Clarity and organization of the answer are important.

Question 1. [20 marks]

(a) [4 marks] Briefly explain the meaning of geometric isotropy in a sentence or two.

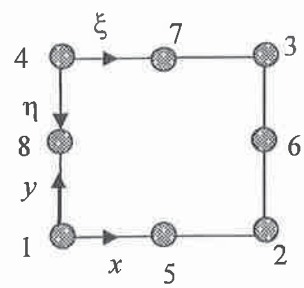
(b) [6 marks] State whether each of the following interpolations of the field variable u has geometric isotropy. Explain.

(i) $u(x,y) = C_1 + C_2x + C_3y$
(ii) $u(x,y) = C_1 + C_2x + C_3y + C_4x^2 + C_5xy + C_6y^2 + C_7x^2y + C_8xy^2$
(iii) $u(x,y) = C_1 + C_2x^2 + C_3y^2 + C_4x^2y^2$

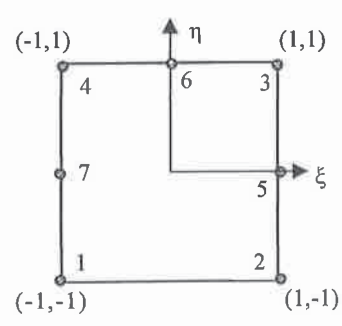
(c) [10 marks] Consider the square element below for which the field variable u is interpolated in the Cartesian *x*,*y* coordinate axes centred at node 1 as

$$u(x,y) = C_1 + C_2x + C_3y + C_4x^2 + C_5y^2 + C_6x^2y + C_7xy^2 + C_8x^2y^2$$

Assume the length of each side of the element is L and use the ξ,η coordinate axes centred at node 4 (see figure) to show that the element does not have geometric isotropy.



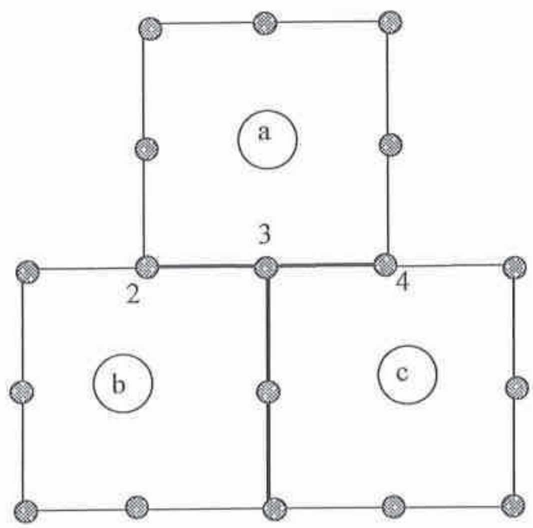
Question 2. [20 marks]



(a) [14 marks] Determine the shape functions (N_i, i = 1 to 7) of the seven-node transition element in natural/local coordinates (ξ,η) such that $-1 \le \xi,\eta \le 1$.

(b) [3 marks] Does the shape function $N_2(\xi,\eta)$ satisfy C^0 continuity along the sides that contain node 2? Explain.

(c) [3 marks] A finite element mesh is generated using eight-noded Serendipity elements, namely a, b, and c, as shown in the figure. What is the major problem with the 2-3-4 interface?



Question 3. [20 marks]

A field variable $f(x, y) = x^2(x + y)$ is defined over a rectangular domain $\Omega = \{\Re^{2+}: 1 \le x \le 7, 1 \le y \le 5\}$. For the expression

$$g = \int_1^5 \int_1^7 x^2\,(x + y)dxdy$$

where the spatial/geometric variables x and y are discretized using the following bilinear interpolation shape functions:

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta),\ N_2 = \frac{1}{4}(1+\xi)(1-\eta),\ N_3 = \frac{1}{4}(1+\xi)(1+\eta), N_4 = \frac{1}{4}(1-\xi)(1+\eta)$$

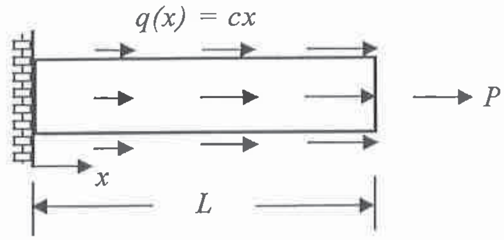
where $-1 \le \xi, \eta \le 1$ for the local coordinates ξ, η,

(a) [15 marks] Use the Gauss quadrature numerical integration method to evaluate g. (*The Jacobian must be provided and the entire evaluation must cast in vector-matrix format as commonly employed in finite element analysis.*)

(b) [5 marks] Explain any similarity or difference between your answer in part (a) and the exact solution g = 3768.

Question 4. [20 marks]

A cantilevered bar is loaded by a constant axial load P and a linearly varying distributed load $q(x) = cx$ as shown in the figure - *note that* c *is a constant*. The cross-sectional area and length of the bar are denoted by A and L, respectively. Assuming the bar is made of a material with Young's modulus of elasticity E, the system governing equation can be written as



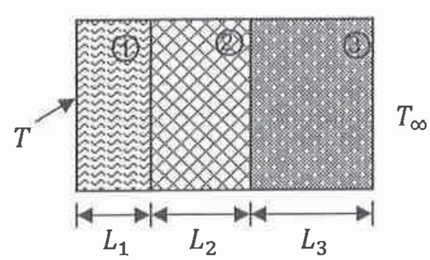
$$EA\frac{d^2u(x)}{dx^2} + cx = 0 \quad 0 < x < L$$

$$\textit{subject to}: \quad u(0) = 0 \quad \textit{and} \quad EA\left.\frac{du(x)}{dx}\right|_{x=L} = P$$

Use the collocation method to determine an approximate cubic polynomial solution with evaluation points at $x = \frac{1}{4}L$ and $x = \frac{3}{4}L$.

Question 5. [20 marks]

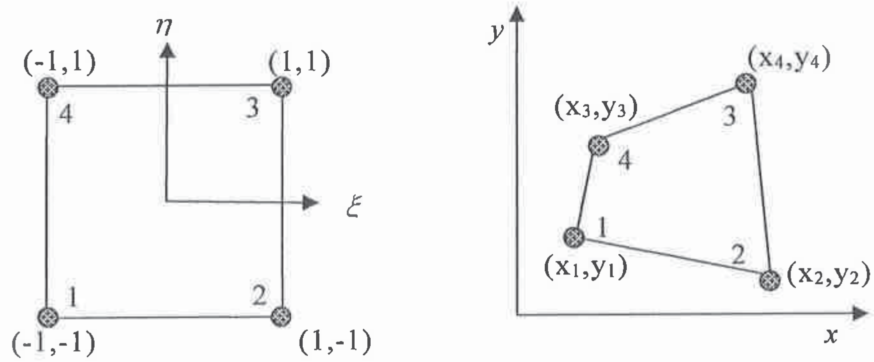
The composite wall shown in the figure is made of three materials denoted by the numbers 1, 2 and 3. The inside wall temperature is $T = 280$ °C and the outside air temperature is $T_\infty = 60$ °C with a convention coefficient of $h = 15\ \mathrm{W/(m^2\ °C)}$. The thermal conductivities of the materials are $K_1 = 60\ W/(\mathrm{m\ °C})$, $K_2 = 30\ W/(\mathrm{m\ °C})$, and $K_3 = 15\ W/(\mathrm{m\ °C})$. The thickness of each material is $L_1 = 3$ cm, $L_2 = 6$ cm, and $L_3 = 9$ cm, and the cross-sectional area A = 1 cm². Employing only 3 elements, one each across a material,



(a) [15 marks] determine the interface temperatures,
(b) [5 marks] determine the heat flux through the 6 cm portion.

Question 6. [20 marks]

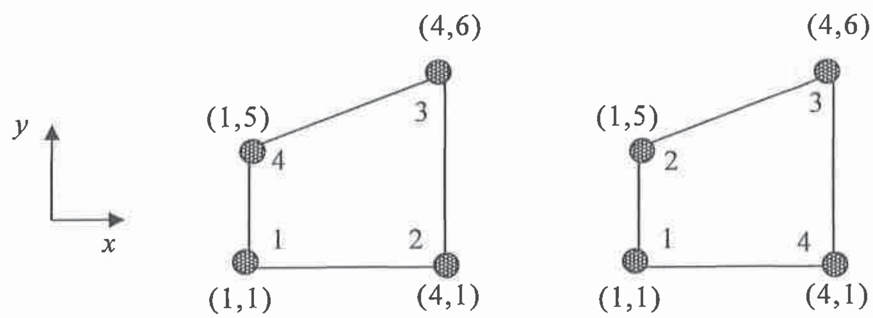
(a) [2 marks] What is an isoparametric element?
(b) [10 marks] The four-node isoparametric quadrilateral element shown below is used to map a region in the parent domain into that in the global domain.



The shape functions of the element are given as

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta),\ N_2 = \frac{1}{4}(1+\xi)(1-\eta),\ N_3 = \frac{1}{4}(1+\xi)(1+\eta), N_4 = \frac{1}{4}(1-\xi)(1+\eta)$$

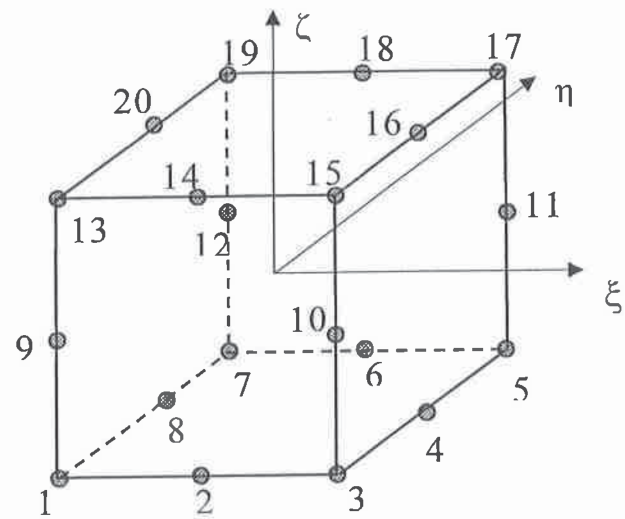
Determine the Jacobian matrix of the element.
(c) [4 marks] Use the Jacobian matrix obtained in part (b) to evaluate the Jacobian of the following elements.



(i) (ii)

(d) [4 marks] What can be inferred or concluded from the Jacobian expressions obtained in part (c) in light of the mapping of the coordinates?

Question 7. [20 marks]



(a) [14 marks] Determine the shape functions (N_i, i = 1 to 20) of the above 20-noded quadratic Serendipity prism in natural/local coordinates (ξ, η, ζ) such that $-1 \leq \xi, \eta, \zeta \leq 1$.
(b) [4 marks] Verify that the shape functions determined in part (a) satisfy the two standard conditions required for acceptable set of shape functions.
(c) [2 marks] Why will a finite element method practitioner choose the 20-noded quadratic Serendipity prism over a quadratic Lagrange prism?