National Exams May 2019
04-BS-1, Mathematics
3 hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to include a clear statement of any assumptions made along with their answer.
2. Any APPROVED Casio or Sharp CALCULATOR is permitted. This is a CLOSED BOOK exam. However, candidates are permitted to bring ONE AID SHEET written on both sides.
3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
4. All questions are of equal value.

Marking Scheme:

1. (a) 7 marks, (b) 7 marks, (c) 6 marks
2. 20 marks
3. 20 marks
4. 20 marks
5. 20 marks
6. 20 marks
7. 20 marks
8. 20 marks

1. Find the general solutions of the following differential equations:

   (a) $x^2y' + 2xy = 2\sin(3x)$,

   (b) $y' + 2xy^2 = 0$,

   (c) $3y'' + 5y' - 2y = 0$.

   Note that in each case, $'$ denotes differentiation with respect to x.

2. Solve the initial value problem

$$y'' + 4y = 6\cos(2t), \qquad y(0) = 1,\ y'(0) = 0.$$

   Note that $'$ denotes differentiation with respect to x.

3. Solve the initial value problem

$$\begin{aligned} x' &= 4x + 2y, & x(0) &= 0, \\ y' &= 3x - y, & y(0) &= 7. \end{aligned}$$

4. Find the line tangent to the intersection of the surfaces

$$3x^2 + 2y^2 - 2z = 1$$

   and

$$x^2 + y^2 + z^2 - 4y - 2z + 2 = 0$$

   at the point $(1, 1, 2)$.

5. Find the volume of the solid region inside the ellipsoid

$$x^2 + y^2 + 4z^2 = 5$$

   and above the cone

$$z = \sqrt{x^2 + y^2}.$$

6. Let S be the boundary of the region defined by $x^2 + 4y^2 \leq 1$, $x \geq 0$, $y \geq 0$ and $0 \leq z \leq 4$, and let F be the vector function $F(x, y, z) = (y^3, x^3, z^3)$. Evaluate the flux of F across the surface S.

7. Evaluate the line integral $\oint_C \mathbf{v} \cdot d\mathbf{r}$ where C is the curve formed by the intersection of the cylinder $x^2 + y^2 = 4$ and the plane $z + 2x - y = 3$, travelled counterclockwise as viewed from the positive z-axis, and $\mathbf{v}$ is the vector function $\mathbf{v} = x\mathbf{i} + (x - y)\mathbf{j} + yz\mathbf{k}$.

8. Compute the response of the damped mass-spring system modelled by

$$y'' + 3y' + 2y = r(t), \qquad y(0) = 0, \qquad y'(0) = 0,$$

   where r is the square wave

$$r(t) = \begin{cases} 1, & 1 \leq t < 2, \\ 0, & \text{otherwise}, \end{cases}$$

   and $'$ denotes differentiation with respect to time.