NATIONAL EXAMS May 2013
07-Elec-B2 Advanced Control Systems

3 hours duration

NOTES:

1. If a doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Candidates may use one of two calculators, a Casio FX-991 or a Sharpe EL-540.

3. This is a closed-book examination. Tables of Laplace and z-transforms are attached as page 4 and page 5.

4. Any four questions constitute a complete paper. Only the first four questions as they appear in your answer book will be marked.

5. All questions are of equal value.

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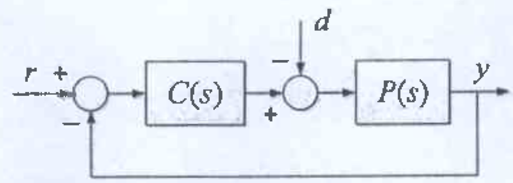
1. Consider the automobile cruise control system shown below with $C(s)=\dfrac{K}{10s+1}$ and $P(s)=\dfrac{4}{(3s+1)^2}$.



(a) With $K=2$ and $r=4$ determine the steady state error between r and y when the grade is level, $d=0$.

(b) Suddenly the grade increases such that $d=1$. Determine the new steady state error

(c) Justify whether or not it is possible to decrease the steady state error to 25% of that computed in part (b) by increasing K.

2. Consider the system,

$$\dot{x}(t)=Ax(t)+Bu(t),\qquad A=\begin{bmatrix}0 & 1-2\alpha & 0\\ 1 & 0 & 0\\ 0 & -2 & 0\end{bmatrix},\ B=\begin{bmatrix}\alpha\\ 0\\ 1\end{bmatrix}$$

$$y(t)=Cx(t)+Du(t),\qquad C=\begin{bmatrix}1 & 0 & 1\end{bmatrix},\ D=0$$

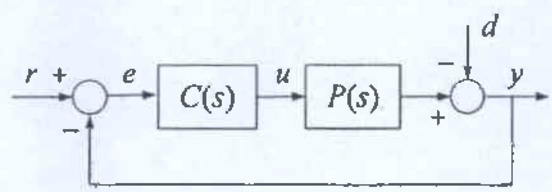
(a) What are the conditions for controllability? Justify your answer.

(b) What are the conditions for observability? Justify your answer.

(c) Determine the system poles and establish stability.

3. Consider the feedback system below with $P(s)=\dfrac{1}{(4s+1)(5s+1)}$.

(a) Determine a *proper* and stable $C(s)$ such that the transfer function that relates e to r is given by, $\dfrac{n(s)}{(s+1)^3}$, $n(s)=b_3s^3+b_2s^2+b_1s+b_0$, where the coefficients, b_i, are to be selected as part of the solution. Recall that $C(s)$ is *proper* if the degree of the numerator is less than or equal to that of the denominator.



(b) Determine the closed loop transfer function that relates r to y.

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4. Several experiments are conducted on an unknown plant:

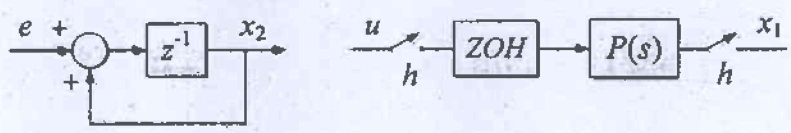
   When a step of magnitude 2 is applied to the input, the steady state output is 10.
   When a sinusoid of frequency 8 rad/sec is applied, the phase lag at the output is 90°.
   When a sinusoid of frequency 5 rad/sec is applied, the phase lag at the output is 15°.

(a) Assume the system, $P(s)$, is second order system and has no finite zeros. Find the parameters of the second order model.

(b) For the model identified in (a) determine the maximum overshoot for a unit step input.

(c) Justify whether the system is stable or not when a controller, $C(s) = 1/s$, is cascaded with $P(s)$ in a negative (unity) feedback loop.

5. Consider the system below. It consists of a discrete component driven by e and a sampled data component with a uniform sample period, h, a zero order hold, and a continuous plant,
$P(s) = \dfrac{1}{s+1}$.



(a) Find a discrete time model that relates $u(k)$ to $x_1(k)$.

(b) Let $e(k) = r(k) - x_1(k)$. Taking $x_1(k)$ and $x_2(k)$ as state variables, $u(k)$ as input, and $x_1(k)$ as output, find a state space model for the system (assume $r = 0$).

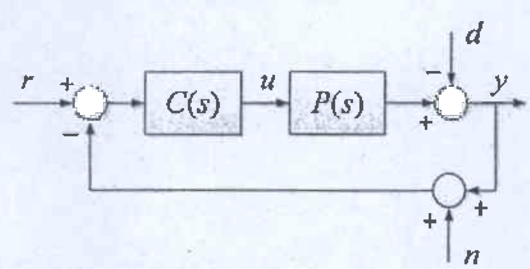
(c) Determine the statefeedback gain, K, for a statefeedback controller, $u(k) = -Kx(k)$, such that the closed loop poles are all at zero.

6. Consider the (continuous time) feedback system below with, $C(s) = \dfrac{K}{s}$, $P(s) = \dfrac{e^{-s/8}}{s+1}$.

(a) Determine the range of K that results in closed loop stability.

(b) Determine the phase margin when $K = 1$ and sketch the associated Nyquist plot.

(c) The system is stable and operating with a sensor bias, $n(t) = 0.3$, disturbance, $d(t) = 0$, and set-point, $r(t) = 1$. Determine the tracking error, $e(t) = r(t) - y(t)$, as a function of K.



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Inverse Laplace Transforms	
$F(s)$	$f(t)$
$\dfrac{A}{s+\alpha}$	$Ae^{-\alpha t}$
$\dfrac{C+jD}{s+\alpha+j\beta}+\dfrac{C-jD}{s+\alpha-j\beta}$	$2e^{-\alpha t}\left(C\cos\beta t+D\sin\beta t\right)$
$\dfrac{A}{(s+\alpha)^{n+1}}$	$\dfrac{At^{n}e^{-\alpha t}}{n!}$
$\dfrac{C+jD}{\left(s+\alpha+j\beta\right)^{n+1}}+\dfrac{C-jD}{\left(s+\alpha-j\beta\right)^{n+1}}$	$\dfrac{2t^{n}e^{-\alpha t}}{n!}\left(C\cos\beta t+D\sin\beta t\right)$

Inverse z-Transforms	
$F(z)$	$f(nT)$
$\dfrac{Kz}{z-a}$	Ka^{n}
$\dfrac{\left(C+jD\right)z}{z-re^{j\varphi}}+\dfrac{\left(C-jD\right)z}{z-re^{-j\varphi}}$	$2r^{n}\left(C\cos n\varphi-D\sin n\varphi\right)$
$\dfrac{Kz}{\left(z-a\right)^{r}}$, $r=2,3\ldots$	$\dfrac{Kn\left(n-1\right)\ldots\left(n-r+2\right)}{\left(r-1\right)!a^{r-1}}a^{n}$

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Table of Laplace and z-Transforms
(*h* denotes the sample period)

$f(t)$	$F(s)$	$F(z)$
unit impulse	1	1
unit step	$\frac{1}{s}$	$\frac{z}{z-1}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$	$\frac{z}{z-e^{\alpha h}}$
t	$\frac{1}{s^2}$	$\frac{hz}{(z-1)^2}$
$\cos \beta t$	$\frac{s}{s^2+\beta^2}$	$\frac{z(z-\cos \beta h)}{z^2-2z\cos \beta h+1}$
$\sin \beta t$	$\frac{\beta}{s^2+\beta^2}$	$\frac{z\sin \beta h}{z^2-2z\cos \beta h+1}$
$e^{-\alpha t} \cos \beta t$	$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$	$\frac{z(z-e^{-\alpha h}\cos \beta h)}{z^2-2ze^{-\alpha h}\cos \beta h+e^{-2\alpha h}}$
$e^{-\alpha t} \sin \beta t$	$\frac{\beta}{(s+\alpha)^2+\beta^2}$	$\frac{ze^{-\alpha h}\sin \beta h}{z^2-2ze^{-\alpha h}\cos \beta h+e^{-2\alpha h}}$
$t\,f(t)$	$-\frac{dF(s)}{ds}$	$-zh\frac{dF(z)}{dz}$
$e^{-\alpha t} f(t)$	$F(s+\alpha)$	$F(ze^{\alpha h})$