

# Professional Engineers of Ontario

Annual Examinations – **December 2014**

07-Elec-B3

Digital Communication Systems

3 Hours Duration

## Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumptions made.
2. This is a closed book exam. A PEO-approved non-programmable calculator is permitted.
3. There are **5 questions** on this exam. **Any 4 questions constitute a complete paper.** Only the first 4 questions as they appear in your answer book will be marked.
4. Marks allocated to each question are noted in the left margin. A complete paper is worth 100 marks.

**(25 marks) Question 1.** This question concerns link budgeting.

- (10 marks) a. Consider a wireless system with transmitter power of 500 mW, antenna gains of 9 dB, receiver losses of 6 dB, receiver noise figure of -168 dBm/Hz, a bandwidth of 10 MHz, and a fading margin requirement of 6 dB. Aside from free-space losses, no other gains or losses are present other than path loss. If the receiver requires a signal-to-noise ratio of at least 2 dB, what is the maximum allowed path loss (in dB)?
- (10 marks) b. Using a free-space path loss of  $30 \log_{10}(4 \pi df/c)$ , where  $d$  represents the distance from transmitter to receiver,  $f$  represents the carrier frequency, and  $c$  represents the speed of light ( $c = 3.0 \times 10^8$  m/s), and assuming a carrier frequency of 1.5 GHz, is the signal-to-noise criterion satisfied when  $d = 200$  m? Show all work.
- (5 marks) c. If a radio transmits at a power level of 20 dBm, what is the radio's power output in W?

**(25 marks) Question 2.** This question concerns modulation schemes.

- (7 marks) a. Briefly explain how a signal is transmitted using the rectangular 16QAM constellation.
- (7 marks) b. Suppose the available system bandwidth is 1 MHz. Using 16QAM symbols, and pulses that satisfy the Nyquist criterion, what is the maximum data rate (in bits per second) at which information can be transmitted over this link?
- (7 marks) c. The information-theoretic capacity of a system (in bits per second) is given by  $C = B \log_2(1 + S)$ , where  $B$  is the system bandwidth (in Hz), and  $S$  is the signal-to-noise ratio. For what value of  $S$  can the data rate from part b be achieved? (Assume a value of 4 Mbit/s if you did not complete part b.)
- (4 marks) d. Suppose a vendor approaches you and tells you about a new technology which, according to the vendor's claim, allows data transmission at a rate faster than the capacity  $C$ . Are the vendor's claims reasonable? Why or why not?

**(25 marks) Question 3.** This question concerns source coding.

- (10 marks) a. You are given a source with six letters: A, B, C, D, E, F. The probabilities of these letters are:  $\Pr(A) = 0.11$ ;  $\Pr(B) = 0.05$ ;  $\Pr(C) = 0.28$ ;  $\Pr(D) = 0.17$ ;  $\Pr(E) = 0.25$ ;  $\Pr(F) = 0.14$ . Find a Huffman code for this source.
- (5 marks) b. What is the entropy of the source in part a?
- (5 marks) c. What is a "prefix code"? Is a Huffman code a prefix code? Explain.
- (5 marks) d. Give one advantage and one disadvantage of Huffman codes.

(25 marks) **Question 4.** This question concerns signal detection.

(5 marks) a. Consider signals  $s_0(t)$  and  $s_1(t)$ , which are used to modulate the binary symbols “0” and “1”, respectively, where

$$s_0(t) = \begin{cases} 1, & 0 \leq t \leq T; \\ 0 & \text{elsewhere} \end{cases}$$

and  $s_1(t) = -s_0(t)$ . Sketch the two signals, and sketch the impulse response of the matched filter  $m(t)$ , assuming the filter is matched to  $s_0(t)$ , and assuming the filter output is sampled at time  $T$ .

(5 marks) b. Sketch the convolution of  $s_0(t)$  and  $m(t)$  as a function of  $t$ .

(5 marks) c. At the sampling instant (time  $T$ ), the matched filter output is corrupted by additive Gaussian noise with zero mean and variance  $\sigma^2$ . Give the optimal decision rule assuming that 0 and 1 are equiprobable.

(10 marks) d. Given that

$$\frac{1}{2} \operatorname{erfc} \left( \frac{t - \mu}{\sqrt{2\sigma^2}} \right) = \int_t^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left( -\frac{(x - \mu)^2}{2\sigma^2} \right) dx$$

and given your decision rule from part c, express the probability of error given that a 1 was sent in terms of erfc.

(25 marks) **Question 5.** This question concerns error-control coding.

(5 marks) a. Briefly discuss the difference between a block code and a convolutional code.

(5 marks) b. What is meant by the “minimum Hamming distance” of a code? How is the minimum Hamming distance related to the number of errors a code can correct?

(5 marks) c. Consider a binary Hamming code with the following parity check matrix:

Give the corresponding generator matrix.

(10 marks) d. Give an example illustrating how to correct a single bit error using the Hamming code from part c.