National Exams December 2017

98-Phys-B5, Control

3 hours duration

NOTES:

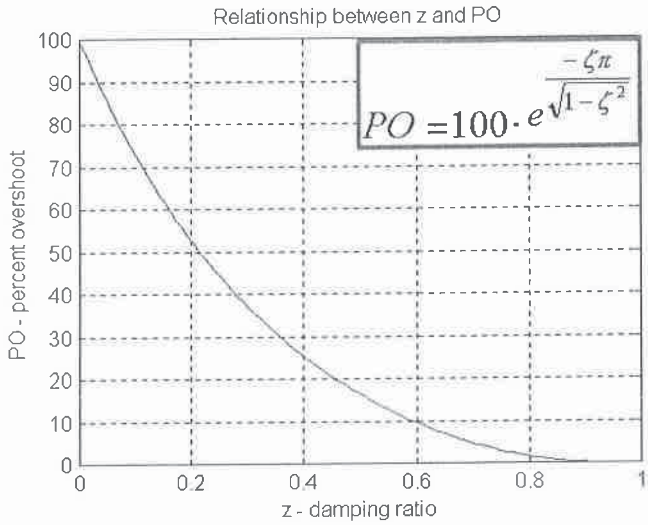
1. This is a CLOSED BOOK EXAM. Only an approved Casio or Sharp calculator is permitted. Candidates are allowed to use a double-sided, **handwritten**, 8.5 by 11" formula and notes sheet. Otherwise, there are no restrictions on the content of the formula sheet. The sheet must be signed and submitted together with the examination paper.
2. If doubt exists as to interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
3. Five (5) questions constitute a complete paper. **YOU ARE REQUIRED TO COMPLETE QUESTION 1 AND QUESTION 2.** Choose three (3) more questions out of the remaining six. Clearly indicate the questions which should be marked - otherwise, only the first five answers provided will be marked. Each question is of equal value. Weighting of topics within each question is indicated. Partial marks will be given. Clearly show steps taken in answering each question.
4. **Use exam booklets to answer the questions - clearly indicate which question is being answered.**

YOUR MARKS		
QUESTIONS 1 AND 2 ARE COMPULSORY:		
Question 1	20	
Question 2	20	
CHOOSE THREE OUT OF THE REMAINING SIX QUESTIONS:		
Question 3	20	
Question 4	20	
Question 5	20	
Question 6	20	
Question 7	20	
Question 8	20	
TOTAL:	100	

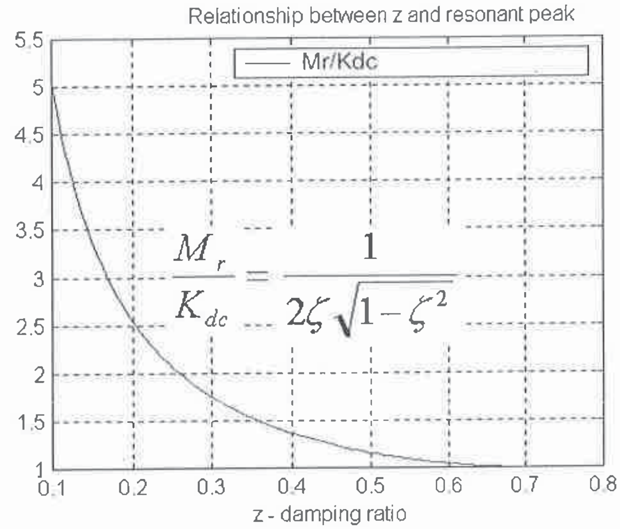
A Short Table of Laplace Transforms

Laplace Transform	Time Function
1	$\sigma(t)$
$\frac{1}{s}$	$1(t)$
$\frac{1}{(s)^2}$	$t \cdot 1(t)$
$\frac{1}{(s)^{k+1}}$	$\frac{t^k}{k!} \cdot 1(t)$
$\frac{1}{s+a}$	$e^{-at} \cdot 1(t)$
$\frac{1}{(s+a)^2}$	$te^{-at} \cdot 1(t)$
$\frac{a}{s(s+a)}$	$(1-e^{-at}) \cdot 1(t)$
$\frac{a}{s^2+a^2}$	$\sin at \cdot 1(t)$
$\frac{s}{s^2+a^2}$	$\cos at \cdot 1(t)$
$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at} \cdot \cos bt \cdot 1(t)$
$\frac{b}{(s+a)^2+b^2}$	$e^{-at} \cdot \sin bt \cdot 1(t)$
$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$\left(1-e^{-at} \cdot \left(\cos bt + \frac{a}{b} \cdot \sin bt\right)\right) \cdot 1(t)$
$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \cdot \sin\left(\omega_n\sqrt{1-\zeta^2}t\right) \cdot 1(t)$
$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$\left(1-\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \cdot \sin\left(\omega_n\sqrt{1-\zeta^2}t+\cos^{-1}\zeta\right)\right) \cdot 1(t)$
$F(s) \cdot e^{-Ts}$	$f(t-T) \cdot 1(t)$
$F(s+a)$	$f(t) \cdot e^{-at} \cdot 1(t)$
$sF(s) - f(0+)$	$\frac{df(t)}{dt}$
$\frac{1}{s}F(s)$	$\int_{0+}^{+\infty} f(t)dt$

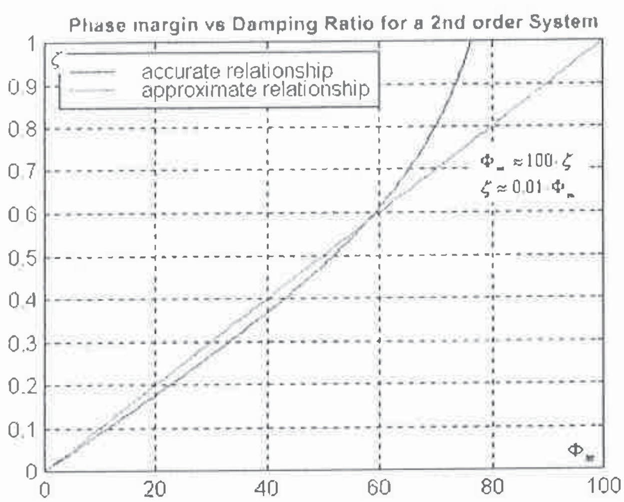
Useful Plots & Formulae



PO vs. Damping Ratio



Resonant Peak vs. Damping Ratio



Second Order Model:

$$G_m(s) = K_{dc}\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

ζ – Damping Ratio (zeta), of the model

ω_n – Frequency of Natural Oscillations of the model

K_{dc} – DC Gain of the model

Definitions for Controllability Matrix, $\mathbf{M_c}$, and Observability Matrix, $\mathbf{M_o}$:

$$\mathbf{M}_c = [\mathbf{B} \quad \mathbf{AB} \quad \mathbf{A}^2\mathbf{B}] \qquad \mathbf{M}_o = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \end{bmatrix}$$

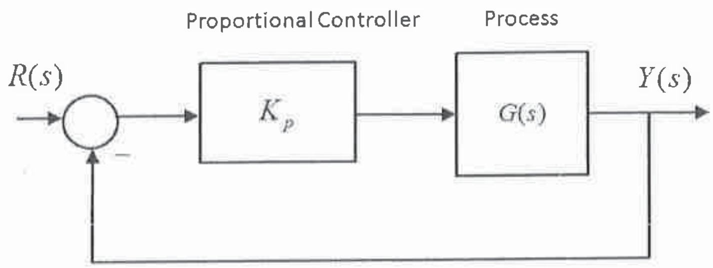
Definition for Transfer Function from State Space:

$$G(s) = \mathbf{C} \cdot (s\mathbf{I} - \mathbf{A})^{-1} \cdot \mathbf{B} + \mathbf{D}$$

Question 1 (Compulsory)

System Stability: Root Locus Construction Rule re. Crossovers with Imaginary Axis, Frequency Response: Gain Margin and the Frequency of Crossover, Routh Array and Routh-Hurwitz Criterion of Stability.

A unit feedback control system is working under a Proportional Controller, as shown below.



The process transfer function is described as follows:

$$G(s) = \frac{100(s+20)(s+10)}{(s+1)^3(s+100)}$$

The Root Locus for this system is shown in Figure Q1.1 and the Open Loop Frequency Response plot is shown in Figure Q1.2. Your task is to investigate the stability of the closed loop system using s-domain analysis by finding: a) the value of the Proportional Controller Gain or Gains at which the closed loop system becomes marginally stable ($K_p = K_{crit}$) and the corresponding frequency or frequencies of marginally stable oscillations (ω_{osc}), and b) the range or ranges of safe operating gains for the Proportional Controller.

1) **(10 points)** Figure Q1.1 shows that there are two crossovers of the Root Locus with the Imaginary axis. They are identified as: $s_1 = j2.25$ rad/sec and $s_2 = j12.5$ rad/sec. Use the Root Locus **Magnitude Criterion** to determine the corresponding values of critical gains (K_{crit}). Finally, use the Root Locus sketch to interpret the corresponding range or ranges of safe operating gains.

Magnitude Criterion: $K = \frac{1}{\lvert G(s^*) \rvert}$ where s^* is a coordinate of a point on the Root Locus

1) **(2 points)** Verify your results from above on Figure Q1.2 by reading the crossover frequencies and the corresponding values of the open loop gains.

2) (**8 points)** Confirm your findings by applying the Routh-Hurwitz Criterion of Stability.

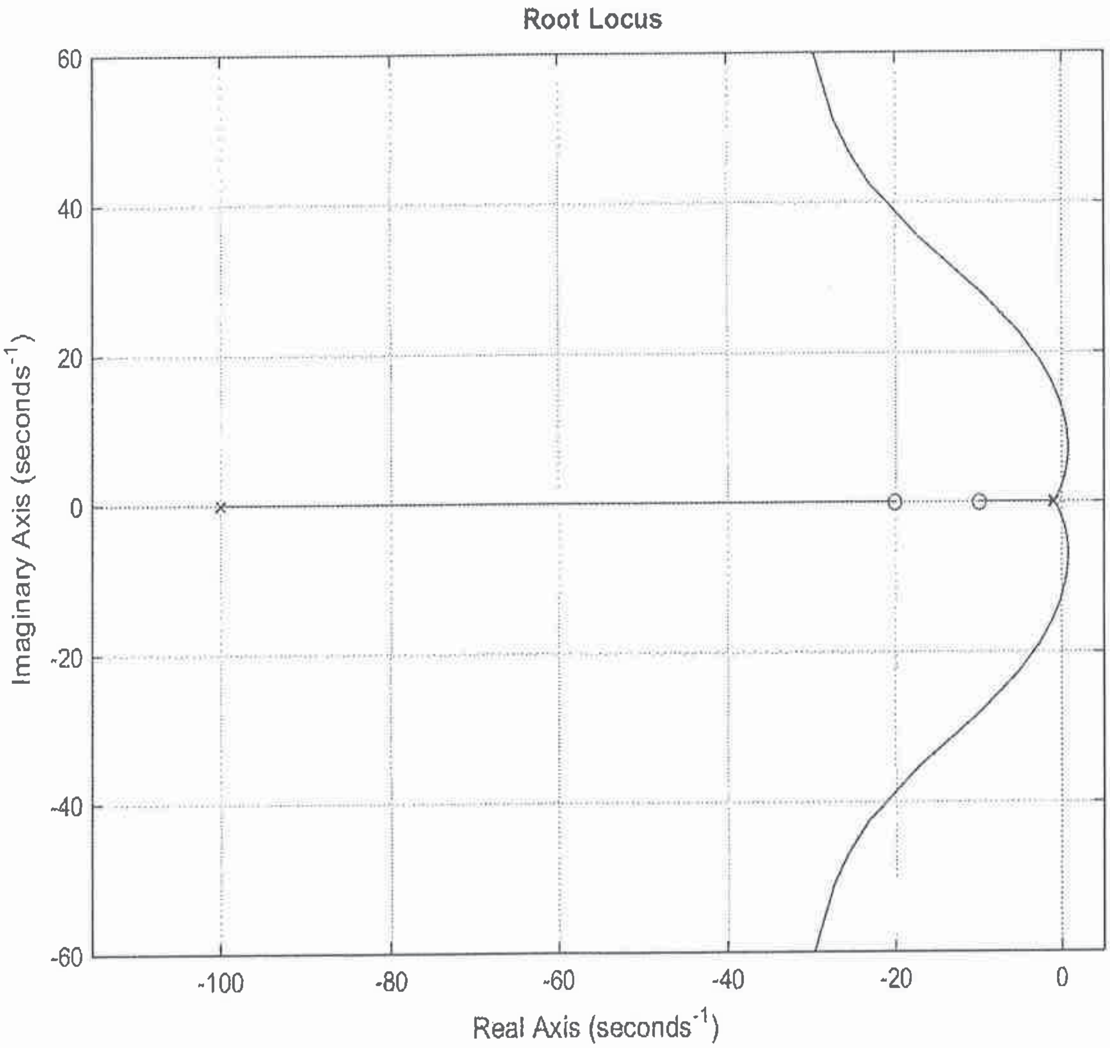


Figure Q1.1 – Root Locus Plot for Stability Question

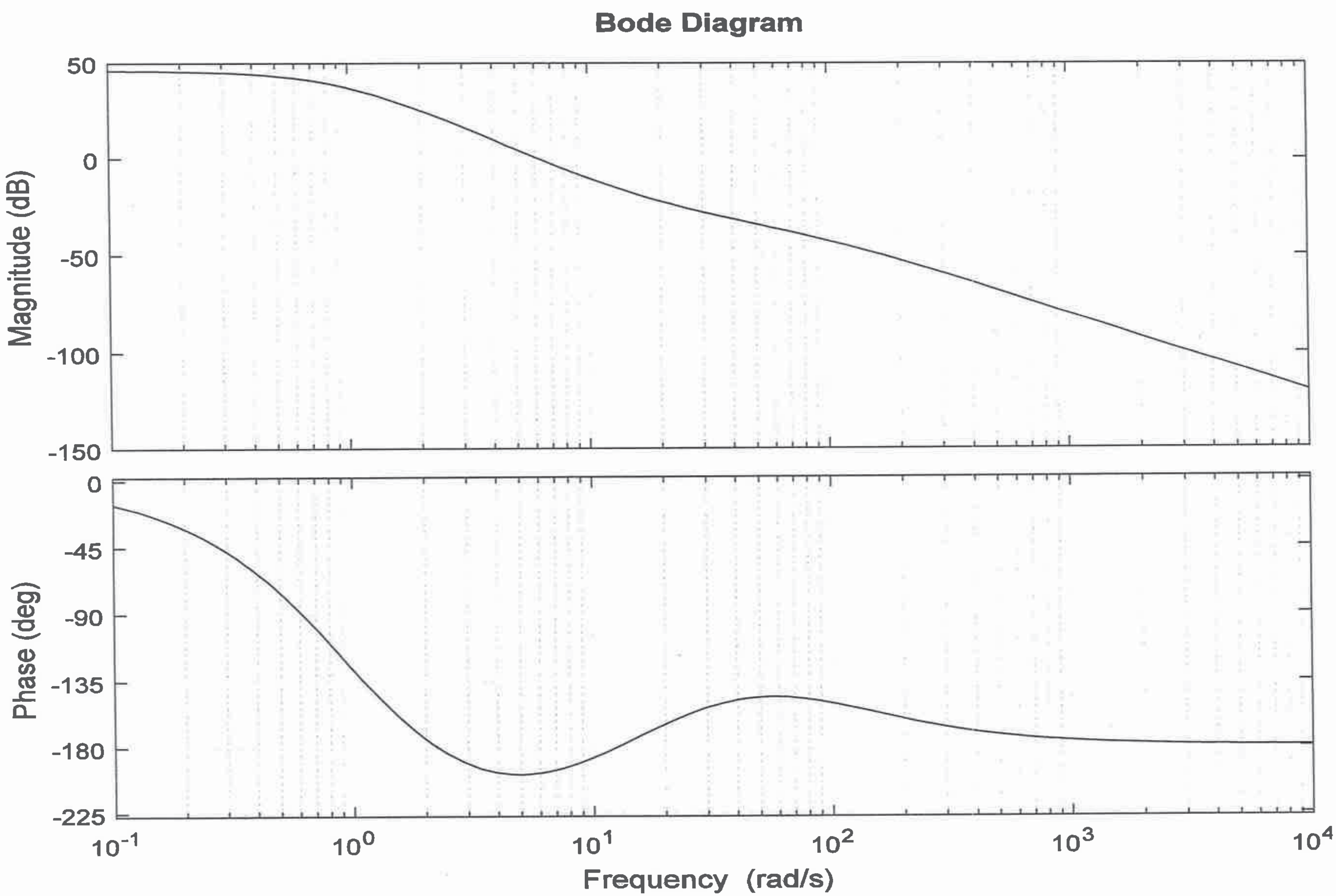
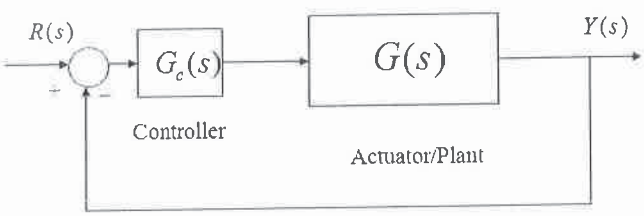


Figure Q1.2 - Open Loop Frequency Response Plot for Stability Question

Question 2 (Compulsory)

Controller Design in Frequency Domain – Lag Controller, Second Order Dominant Poles Model, Step Response Specifications.

Consider a unit feedback closed loop control system, as shown below.



The system is to operate under Lag Control. The Lag Controller transfer function is as follows:

$$G_c(s) = K_c \cdot \frac{\tau\alpha s + 1}{\tau s + 1} = \frac{a_1 s + a_0}{b_1 s + 1}$$

Where τ is the so-called Lag Time Constant and $\alpha < 1$.

The process transfer function G(s) is as follows:

$$G(s) = \frac{30(s+2)}{(s+0.1)^2(s+20)^2}$$

The uncompensated Open Loop Frequency Response plot is shown in Figure Q2.1. The design requirements are:

- The Steady State Error for the unit step input for the compensated closed loop system is to be **one half** of the Steady State Error for the uncompensated system;
- Percent Overshoot of the compensated closed loop system is to be no more than 10%.

1) **(3 marks)** Calculate the Position Constant for the uncompensated system (K_{pos_u}), then the Position Constant for the compensated system (K_{pos_c}), such that it would meet the design requirements.
2) **(3 marks)** Read off the Phase Margin of the uncompensated system (Φ_{m_u}) and then decide what value of the Phase Margin for the compensated system (Φ_{m_c}) would meet the design requirements.
3) **(8 marks)** What is the resulting crossover frequency of the compensated system, ω_{cp_c}? Sketch an approximate shape of the compensated Open Loop Frequency Response plots in Figure Q2.1 – clearly indicate the DC gain, the crossover frequency and the Phase Margin. Calculate the appropriate Lag Controller parameters and the Controller transfer function.
4) **(6 marks)** Next, estimate the compensated closed loop step response specs: $e_{ss(step\%)}$, $T_{rise(0-100\%)}$, $T_{settle(\pm 2\%)}$ and PO.

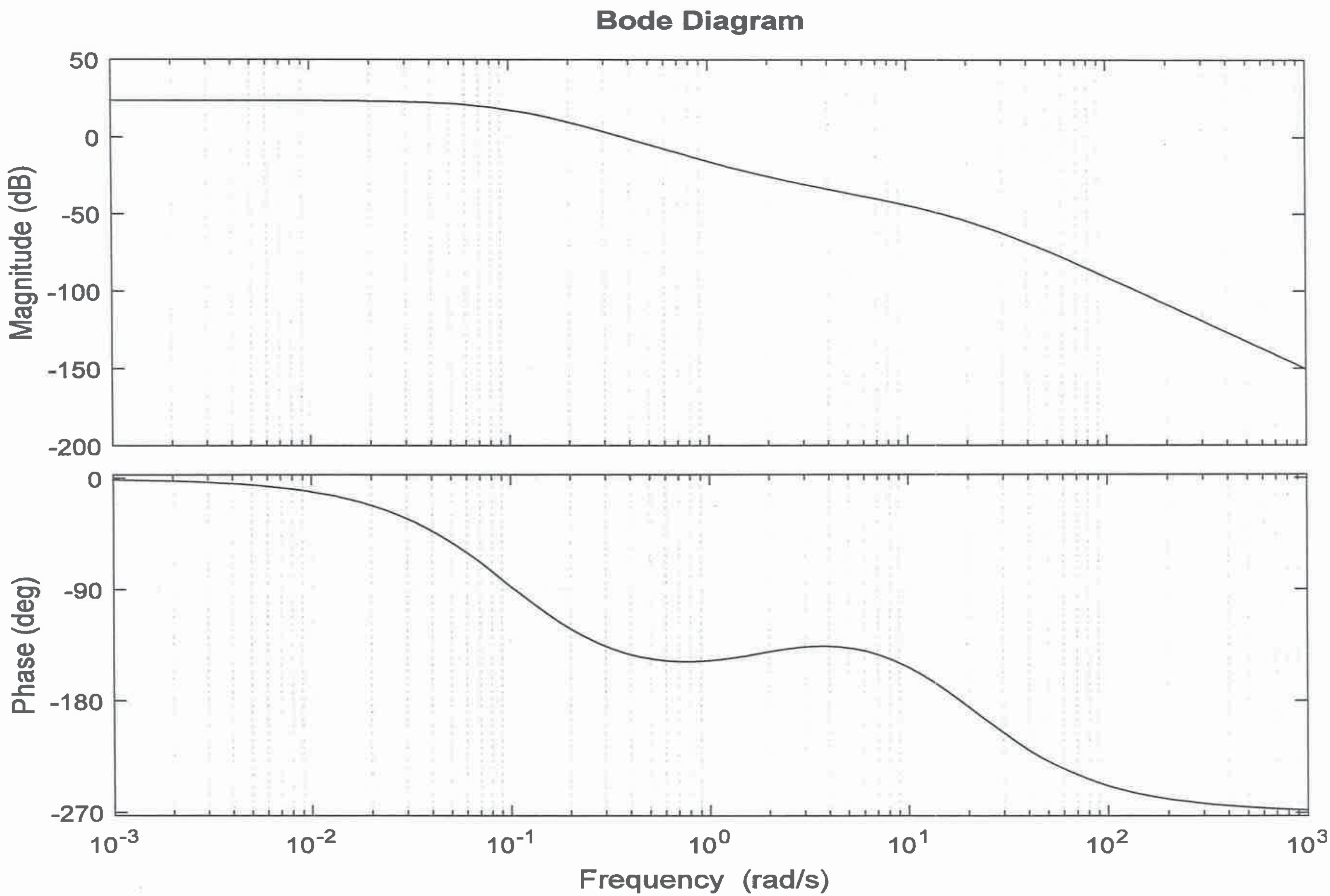
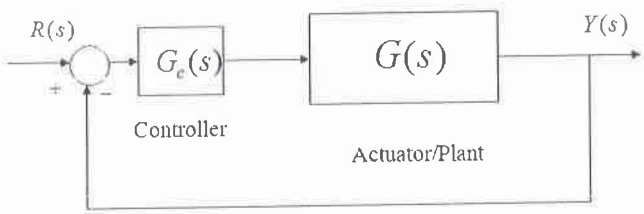


Figure Q2.1 - Open Loop Frequency Response Plot for Lag Design

Question 3

Controller Design in Frequency Domain – Lead Controller, Second Order Dominant Poles Model, Step Response Specifications.

Consider a unit feedback closed loop control system, as shown below.



The system is to operate under Lead Control. The Lead Controller transfer function is as follows:

$$G_c(s) = K_c \cdot \frac{\tau s + 1}{\alpha \tau s + 1} = \frac{a_1 s + a_0}{b_1 s + 1}$$

Where τ is the so-called Lead Time Constant and $\alpha < 1$.

The process transfer function G(s) is as follows:

$$G(s) = \frac{2500}{(s+1)(s+5)(s+50)}$$

The uncompensated Open Loop Frequency Response plot is shown in Figure Q3.1. The design requirements are:

- The Steady State Error for the unit step input for the compensated closed loop system is to be no more than 4%;
- Percent Overshoot of the compensated closed loop system is to be no more than 20%;
- The Settling Time, $T_{settle(\pm 2\%)}$, is to be no more than 0.5 seconds.

1) **(3 marks)** Calculate the Position Constant for the uncompensated system (K_{pos_u}), then the Position Constant for the compensated system (K_{pos_c}) that would meet the design requirements.
2) **(5 marks)** Read off the Phase Margin (Φ_{m_u}) and the crossover frequency $\left(\omega_{cp_u}\right)$ of the uncompensated system. Next, decide what their values should be ($\Phi_{m_c}, \omega_{cp\_c}$) so that the compensated system meets the design requirements.
3) **(12 marks)** Sketch an approximate shape of the compensated Open Loop Frequency Response plots in Figure Q3.1 – clearly indicate the DC gain, the crossover frequency and the Phase Margin. Calculate the appropriate Lead Controller parameters and the Controller transfer function. Next, estimate the compensated closed loop step response specs: $e_{ss(step\%)}$, $T_{rise(0-100\%)}$, $T_{settle(\pm 2\%)}$ and PO.

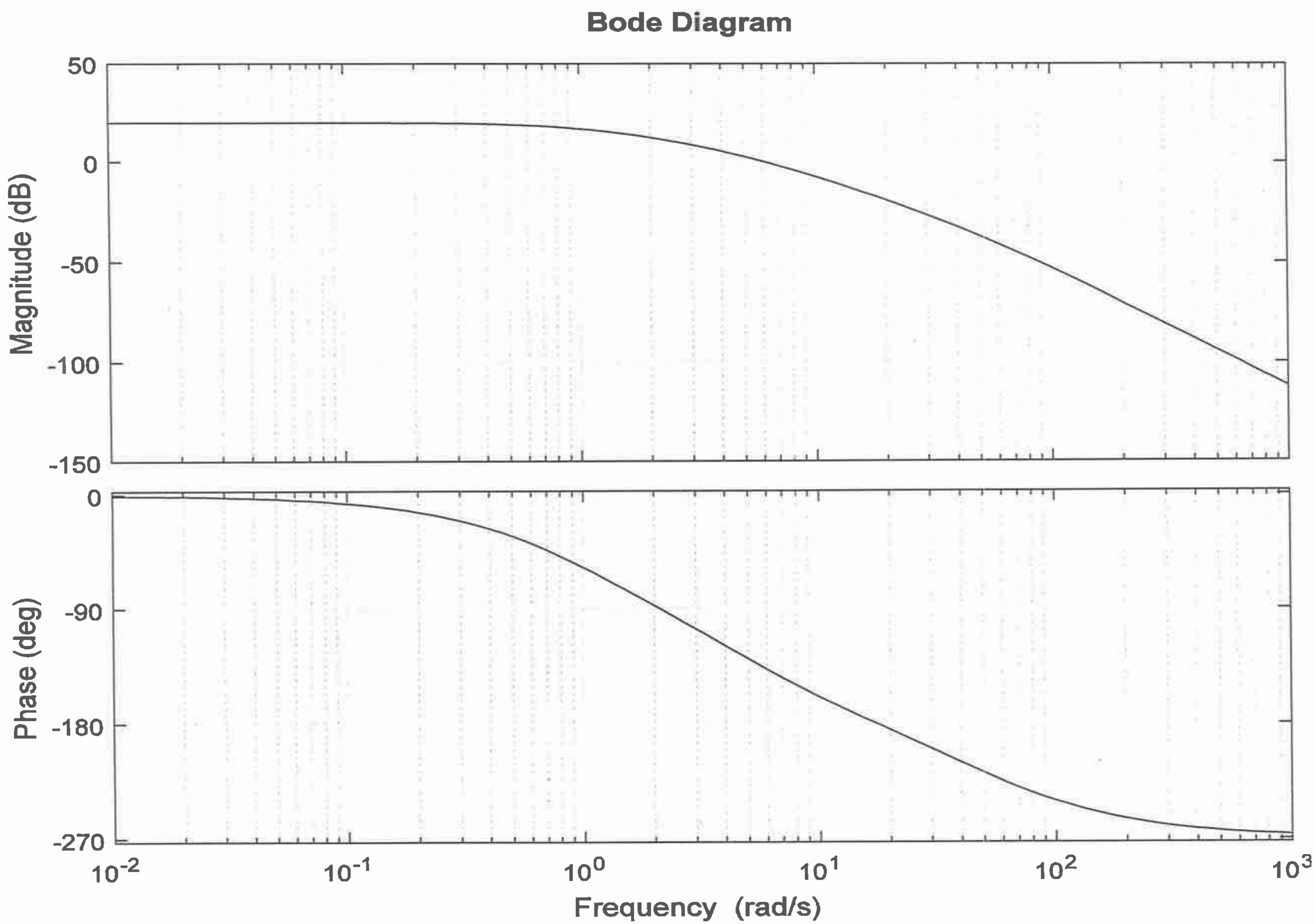


Figure Q3.1 - Open Loop Frequency Response Plots for Lead Design

Question 4

System Stability in the Frequency Domain: Polar Plots, Nyquist Criterion of Stability.

Consider a unit feedback loop system under Proportional Control (gain K). The transfer function of its open loop is described as follows:

$$G_{open}(s) = K \cdot \frac{(s+1)}{s(s-1)}$$

1) **(10 marks)** Sketch a polar plot of the normalized open loop transfer function, $\frac{G_{open}(j\omega)}{K}$; note that you do not have the frequency response plots available to read off the coordinates of the crossovers with the Imaginary and Real axis, and thus you have to calculate these crossovers using the Fourier Transfer Function $G_{open}(j\omega)$. Clearly indicate the direction of increasing frequency on the resulting polar plot.
2) **(10 marks)** Next, apply the Nyquist criterion of stability to establish the range of values of the Proportional Controller gain, K_p, that will result in a stable closed loop system response. NOTE: clearly show the chosen clockwise (CW) Γ path in the s-plane, and the resulting Nyquist Contour.

Question 5

State Space Model from Transfer Functions, Pole Placement by State Feedback Method, Steady State Errors to Step and Ramp Inputs.

Consider a linear system described by the following transfer function:

$$\frac{Y(s)}{U(s)} = \frac{2s^2 + 6s + 20}{s^3 + 10s^2 + 2s + 30}$$

1) **(10 marks)** Derive a set of state equations in the Controller Canonical Form.
2) **(10 marks)** State-variable feedback is to be applied according to the following equation:

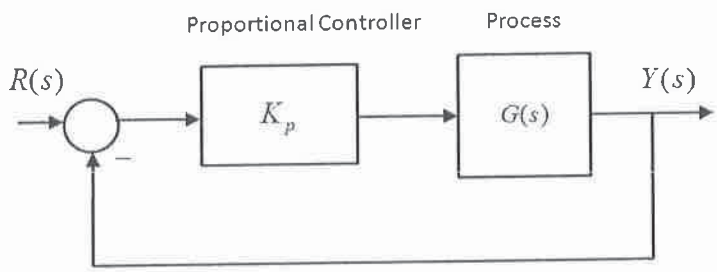
$$u = K \cdot \left(r - \mathbf{k}^T \cdot \mathbf{x}\right)$$

Determine the values of the gain constant K and the state feedback vector $\mathbf{k}$ so that the closed loop system will have poles at: -20 and $-2 \pm j2$, and the steady-state error to a step input will be zero.

Question 6

Second Order Models from Dominant Poles, Open and Closed Loop Bode Plots, System Damping, Performance Specifications.

A unit feedback control system is working under a Proportional Controller, as shown below. The gain value is set at $K_p = 3$.



The process transfer function is described as follows:

$$G(s) = \frac{10(s+2)}{(s+0.1)^2(s+20)^2}$$

1) **(5 marks)** The closed loop transfer function of the system has four poles: $p_1 = -0.1308 + j0.3794$, $p_2 = -0.1308 - j0.3794$, $p_3 = -18.8$, and $p_4 = -21.14$. Determine the second order dominant poles model of the closed loop transfer function, given the locations of the closed loop poles, and derive its transfer function, $G_{m1}(s)$.

2) **(5 marks)** Note that the uncompensated Open Loop transfer function for this system is the same as in Question 2 (Lag Design). Thus, determine the second order dominant poles model, $G_{m2}(s)$, from the information provided in the Open Loop Bode plot in Figure Q2.1.

3) **(5 marks)** Determine the second order dominant poles model, $G_{m3}(s)$, from the information provided in the Closed Loop Bode plot in Figure Q6.1.

4) **(5 marks)** How do the three models compare? Use the one you consider the most accurate to estimate the following closed loop step response specifications: $e_{ss(step\%)}$, $T_{rise(0-100\%)}$, $T_{settle(\pm 2\%)}$ and PO.

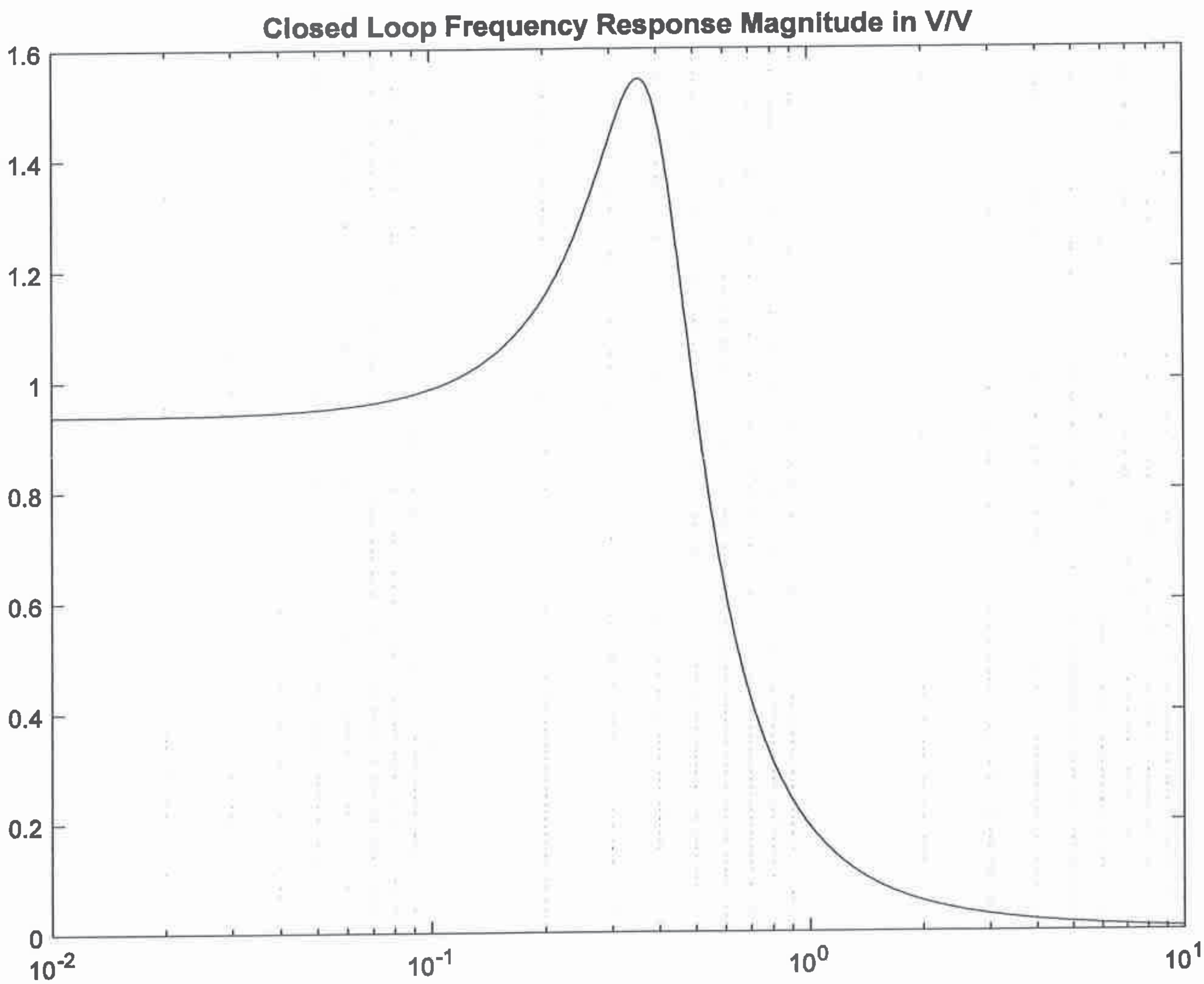
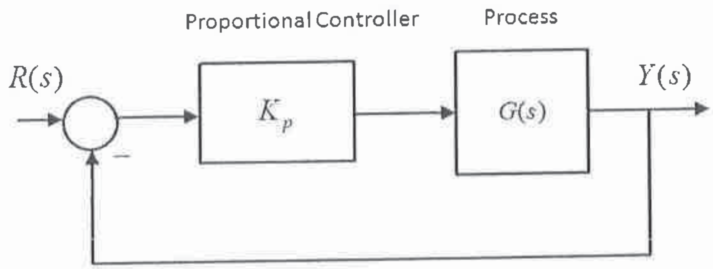


Figure Q6.1 – Closed Loop Magnitude Plot

Question 7

Root Locus Analysis and Gain Selection, Stability, Second Order Model, Step Response Specifications.

A unit feedback control system is working under a Proportional Controller, as shown below.



The process transfer function is described as follows:

$$G(s) = \frac{100}{(s+1)(s+2)(s+5)}$$

1) **(10 marks)** In the space provided in Figure Q7.1, sketch a detailed Root Locus for the system, including crossovers with the imaginary axis, if any, break-away/break-in coordinates, if any, asymptotes, if any, a centroid, etc. if you are using estimates, explain why.

2) **(5 marks)** Determine the value of the operational gain K_{op} that would result in the closed loop system equivalent damping ratio of $\zeta = 0.5$. What is the system Gain Margin at this value of the gain?

3) **(5 marks)** Find the closed loop transfer function at the operational gain as calculated in item 2. Assume that the system closed loop behaviour can be approximated by a second order model. Determine the remaining model parameters K_{dc} and ω_n, and write the model transfer function $G_m(s)$. What will the expected values be of the following specs of the closed loop step response: Percent Overshoot (PO), Settling Time ($T_{settle(\pm 2\%)}$), Rise Time ($T_{rise(0-100\%)}$), and the Steady State Error ($e_{ss(step\%)}$)?

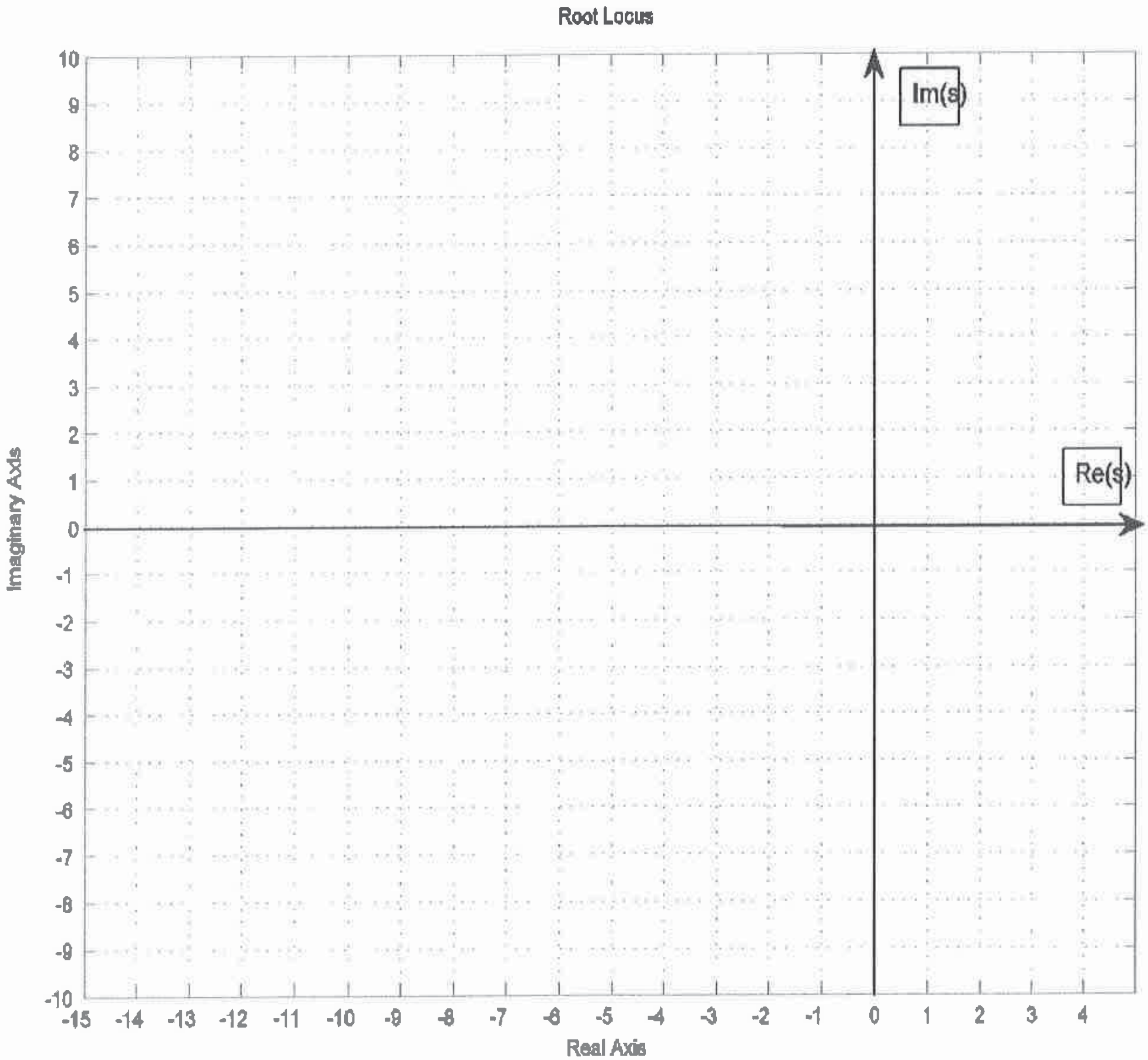


Figure Q7.1 – Place Your Root Locus Graph Here

Question 8

PID Controller Design by Pole Placement, Response Specifications, Second Order Model.

Consider a closed loop positioning control system working under Proportional + Integral + Rate Feedback Control, as shown in Figure Q8.1:

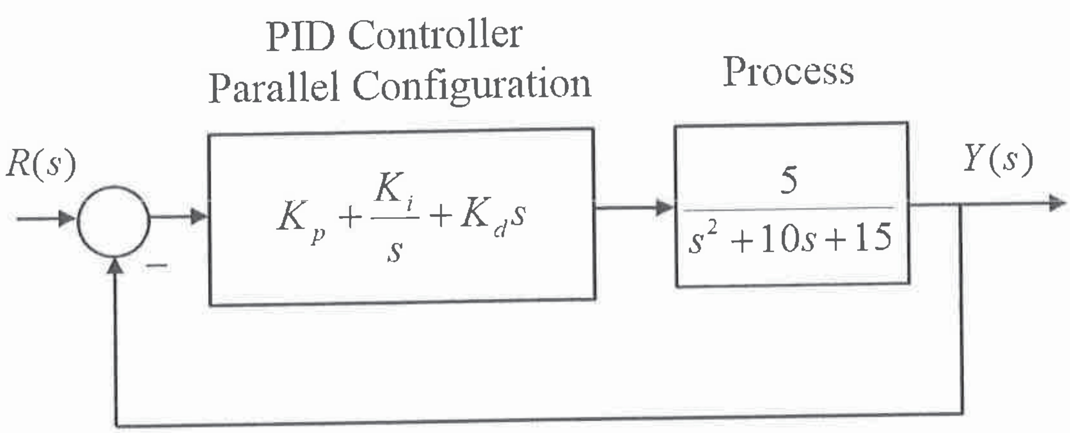


Figure Q8.1

1) **(5 marks)** The compensated Closed Loop step response of this system is to have the following specifications: $PO = 10\%$ and $T_{settle(\pm 2\%)} = 1$ sec. Determine the Closed Loop system damping ratio, ζ and the frequency of natural oscillations, ω_n, to meet the transient response requirements.

2) **(15 marks)** Choose the pole locations for the Closed Loop system so that system two complex conjugate ("dominant") poles correspond to the desired second order model, and that **a pole-zero cancellation** in the Closed Loop Transfer Function occurs. Compute the required Controller Gains K_p, K_d, and K_i.

Note that you are expected to solve a quadratic equation to find the gains in item 2), which means you will have two sets of solutions. Choose ONLY ONE set for your final answer – clearly identify it, and briefly justify your choice. Also note that you should re-write the PID Controller transfer function so that you can identify the two real zeros of the controller.