National Examinations December 2019

16-Elec-A3, Signals and Communications

3 hours duration

Notes:

1) If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumption made.

2) This is a CLOSED BOOK exam. A Casio or Sharp approved calculator is permitted.

3) Answer all 5 questions.

4) All 5 questions are of equal value.

1. Consider the signal $x(t) = 4\cos(2\pi f_0 t)$, where $f_0 = 120$ Hz. This signal is input to a full wave rectifier which gives the output $y(t)$ as $y(t) = |x(t)|$,

a) Determine the fundamental frequency of $y(t)$.

b) Write $y(t)$ as a Fourier series (i.e. the real Fourier series).

c) If the signal $y(t)$ is input to an ideal band-pass filter with center frequency 500 Hz and bandwidth 400 Hz, determine the output signal $z(t)$.

d) Determine the average power of the signal $y(t)$ and the signal $z(t)$.

2) In a modulation scheme the message signal is given by $m(t) = \cos(2\pi f_m t) + \frac{1}{2}\sin(4\pi f_m t)$ where f_m is an audio frequency and the carrier is equal to $A\cos(2\pi f_c t)$, where $f_c = 30 f_m$.

a) Plot $m(t)$ in the time interval $[0, 2/f_m]$.

b) Plot the spectrum of the message signal exactly (i.e. the signal in the frequency domain) showing both the real and imaginary components. What is the bandwidth?

c) Plot the spectrum of the modulated signal assuming DSB modulation, again showing precisely the real and imaginary components. What is the bandwidth of the modulated signal?

d) Plot the spectrum of the signal assuming lower-sideband SSB. What is the bandwidth of the modulated signal?

e) Give a block diagram of a system to recover the message signal exactly from the DSB signal. The system should work for arbitrary message signals with the same message signal bandwidth.

f) Give a block diagram of a system which takes the DSB signal in b) and outputs a DSB signal with carrier frequency equal to $40 f_m$.

3) A PCM system with uniform quantization is used to transmit a speech signal. The bandwidth of the signal is equal to 8 KHz. Assume that the signal has a dynamic range that varies between -3 and 3 volts (i.e. peak to peak value equal to 6V). Upon reconstruction of the signal the quantization noise is not to exceed 0.5% of the peak of the signal and the number of quantization levels must be a power of 2.

a) Determine the smallest number of quantization levels that meets the above constraints.

b) Determine the quantization noise power.

c) Model the signal as a sinusoid and determine the SNR for the reconstructed signal.

d) Assume that we change the above quantization scheme by doubling the number of quantization levels in c). What is the resulting change in SNR in dB?

e) Determine the number of bits required to represent each sample assuming the number of quantization levels in a).

f) Determine the bit rate of the resulting PCM signal.

4. A discrete linear time invariant system has an impulse response given by $h(n) = u(n) - u(n-4) + \frac{1}{2}\delta(n-4)$, where $u(\cdot)$ is the step function and $\delta(\cdot)$ is the discrete delta function.

a) Give a block diagram of the system for arbitrary inputs.

b) Give an equation relating the output $y(n)$ to the input $x(n)$ in the time domain.

c) Give the transfer function for the system.

d) Assuming that the system models a sampled data system, give the frequency response for the system assuming a sampling frequency of 50 KHz. Your result should be a mathematical expression that can be evaluated and plotted.

e) If the input to the system is the signal $x(n) = 1 - \frac{|n-6|}{6}$ for $0 \leq n \leq 12$ and zero elsewhere, determine the output of the system $y(n)$.

f) Is the system bounded input bounded output stable? Justify your answer.

5. An FM modulator has a frequency deviation constant of 5 KHz/V. Assume that a message signal is input to the modulator as follows: $m(t) = 2u(t) - 4u(t-T) + 2u(t-2T)$, where $u(\cdot)$ is the step function.

a) Determine the peak frequency deviation for the FM signal.

b) Give a mathematical expression for the FM signal that can be evaluated and plotted if the carrier frequency is 100 KHz. Assume that the average power of the FM signal is 1.

c) Give an approximate value for the bandwidth of the FM signal for each of the following three cases:

(i) $T = 1$ s

(ii) $T = 1$ ms

(iii) $T = 1\ \mu$s

d) Suppose an FM signal has the form $x(t) = A\cos(100000t + \phi(t))$ where $\phi(t)$ is some baseband signal with bandwidth less than 2 KHz. If this FM signal is input to an FM demodulator give an expression for the demodulated signal.