National Exams December 2016

07-Mec-B10, Finite Element Analysis

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. This is an OPEN BOOK EXAM.
   Any non-communicating calculator is permitted.

3. FIVE (5) questions constitute a complete exam paper.
   The first five questions as they appear in the answer book will be marked. The questions are to be solved within the context of the finite element method.

4. Each question is of equal value.

5. Some questions require an answer in essay format. Clarity and organization of the answer are important.

Question 1. [20 marks] A field variable $f(x,y) = x^2(y^3+1)$ is defined over a rectangular domain $\Omega = \{\Re^{2+}: 2 \le x \le 6, 1 \le y \le 7\}$. Given the expression

$$g = \int_1^7 \int_2^6 x^2\,(y^3+1)dxdy$$

and assume the following bilinear interpolation shape functions are used to discretize the spatial/geometric variables *x* and *y*:

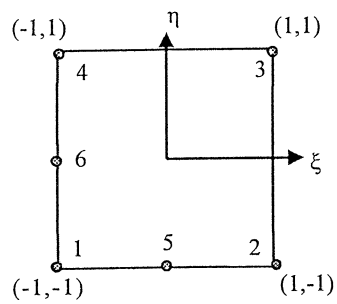
$$N_1 = \frac{1}{4}(1-\xi)(1-\eta),\ N_2 = \frac{1}{4}(1+\xi)(1-\eta),\ N_3 = \frac{1}{4}(1-\xi)(1+\eta),\ N_4 = \frac{1}{4}(1+\xi)(1+\eta)$$

where $-1 \le \xi, \eta \le 1$ for the local coordinates ξ, η.

(a) [15 marks] Use the Gauss quadrature numerical integration method to evaluate *g*.

(b) [5 marks] Explain any similarity or difference between your answer and the exact solution *g* = 42016.

Question 2. [20 marks]

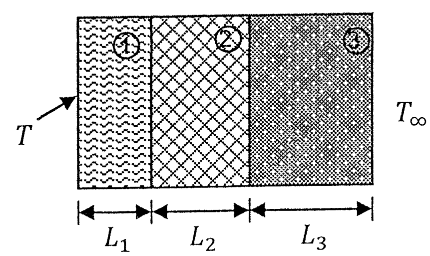


(a) [14 marks] Determine the shape functions (N_i, i = 1 to 6) of the six-node transition element in natural/local coordinates (ξ, η) such that $-1 \le \xi, \eta \le 1$.

(b) [4 marks] Evaluate the shape function N_4 at the sixth node and at the centroid of the element.

(c) [2 marks] Assume the field variables of the problem are displacement components denoted by u and v for the ξ and η directions, respectively. If the nodal displacement components are zero except $v_1 = v_2 = v_5 = -0.025$ mm, determine an expression for the field variables, u and v, in the natural/ local coordinates (ξ, η).

Question 3. [20 marks]



The composite wall shown in the figure is made of three materials denoted by the numbers 1, 2 and 3. The inside wall temperature is $T = 300$ °C and the outside air temperature is $T_\infty = 50$ °C with a convention coefficient of $h = 20$ W/(m^2 °C). The thermal conductivities of the materials are $K_1 = 60\ W/(\text{m °C})$, $K_2 = 30\ W/(\text{m °C})$, and $K_3 = 10\ W/(\text{m °C})$. The thickness of

each material is $L_1 = 2.5$ cm, $L_2 = 4$ cm, and $L_3 = 8$ cm. Employing only 3 elements, one each across a material
(a) [15 marks] determine the interface temperatures,
(b) [5 marks] determine the heat flux through the 8-cm portion.

Question 4. [20 marks]

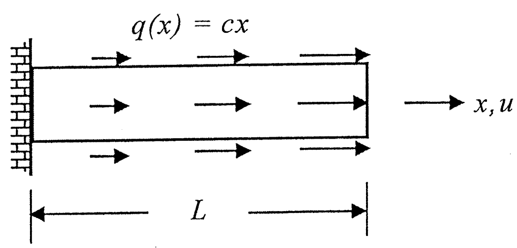
(a) [3 marks] Briefly explain in a sentence or two the difference between basis function and shape function.

(b) [3 marks] Briefly explain in a sentence or two why finite element solutions improve with increasing number of elements.

(c) [4 marks] Briefly explain in a sentence or two why a discretization implemented using Bubnov-Galerkin method (i.e., Galerkin method) is identical to that obtained using Ritz method.

(d) [10 marks] An analyst notices a very slow convergence during a finite element analysis of a Timoshenko beam. On closer examination of the field variables, it was observed that the rotation was interpolated by a linear function and the transverse displacement by a quadratic function. How can this slow convergence problem be resolved?

Question 5. [20 marks] A cantilevered bar is loaded by a linearly varying distributed load $q(x) = cx$ as shown in the figure - note that c is a constant. The cross-sectional area and length of the bar are denoted by A and L, respectively, and it is made of a material with Young's modulus of elasticity E. The system governing equation can be written as



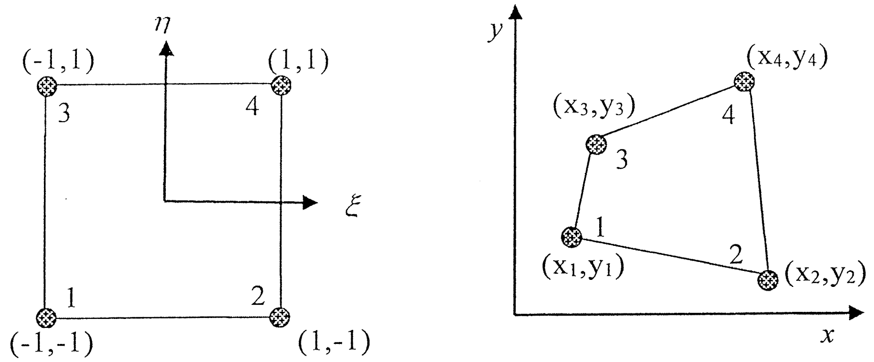
$$EA\frac{d^2u(x)}{dx^2} + cx = 0 \quad 0 < x < L$$

$$\text{subject to}: \quad u(0) = 0 \quad \text{and} \quad EA\left.\frac{du(x)}{dx}\right|_{x=L} = 0$$

Use the least square method to determine an approximate cubic polynomial solution with evaluation points at $x = \frac{1}{4}L$ and $x = \frac{3}{4}L$.

Question 6. [20 marks]

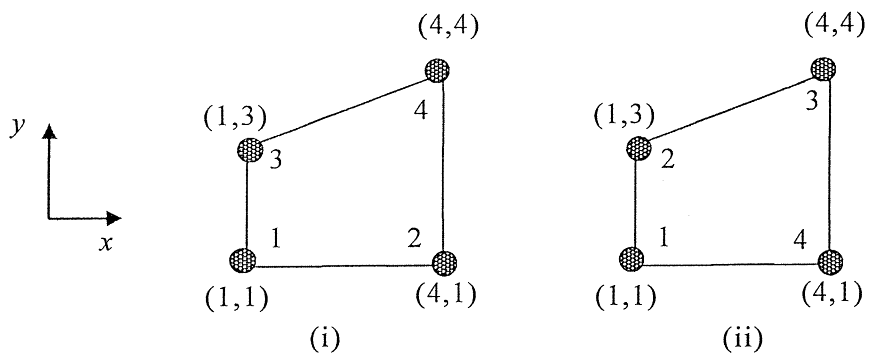
(a) [2 marks] What is an isoparametric element?
(b) [10 marks] The four-node isoparametric quadrilateral element shown below is used to map a region in the parent domain into that in the global domain.



The shape functions of the element are given as

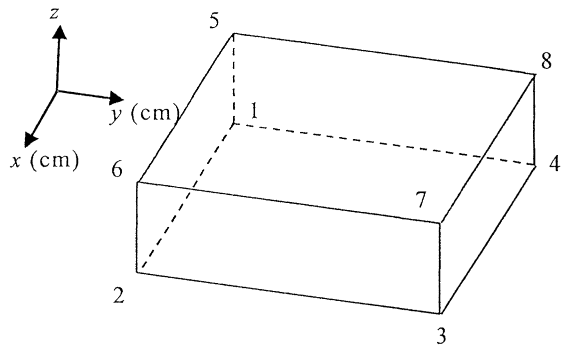
$$N_1 = \frac{1}{4}(1-\xi)(1-\eta),\ N_2 = \frac{1}{4}(1+\xi)(1-\eta),\ N_3 = \frac{1}{4}(1-\xi)(1+\eta),\ N_4 = \frac{1}{4}(1+\xi)(1+\eta)$$

Determine the Jacobian matrix of the element.
(c) [4 marks] Use the Jacobian matrix obtained in part (b) to evaluate the Jacobian of the following elements.



(d) [4 marks] What can be inferred or concluded from the Jacobian expressions obtained in part (c) in light of the mapping of the coordinates?

Question 7. [20 marks]



1: (2,3,2)
2: (7,3,2)
3: (7,9,2)
4: (2,9,2)
5: (2,3,4)
6: (7,3,4)
7: (7,9,4)
8: (2,9,4)

Not to scale

(a) [4 marks] Determine the shape functions (N_i, i = 1 to 8) of an eight-node hexahedron element in natural/local coordinates (ξ, η, ζ) such that $-1 \leq \xi, \eta, \zeta \leq 1$. The node numbering is identical to that shown in the above representative global element.
(b) [3 marks] Evaluate the shape function N_8 at the second node, the eighth node, and the centroid of the element.
(c) [3 marks] Assume the field variables of the problem are displacement components denoted by $u, v,$ and w in the ξ, η, and ζ directions, respectively. If the nodal displacement components are zero except $u_2 = u_3 = u_6 = u_7 = 0.025$ mm, express the field variables in the natural/ local coordinates (ξ, η, ζ).
(d) [7 marks] Determine the Jacobian matrix and evaluate the Jacobian of the above element.
(e) [3 marks] Determine the normal strain $\varepsilon_x = \frac{\partial u}{\partial x}$ and shear strain $\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$ at the centre of the element.