NATIONAL EXAMS
May 2019

17-Phys-A6, Solid State Physics

3 hours duration

NOTES:

1. If doubt exits as to the interpretation of any question, the candidate must submit with the answer paper, a clear statement of any assumption made.

2. Candidates may use one of two calculators, the Casio or Sharp approved models.

3. This is a CLOSED BOOK EXAM.
   Useful constants and equations have been annexed to the exam paper.

4. **Any FIVE (5) of the SEVEN (7)** questions constitute a complete exam paper.
   The first five questions as they appear in the answer book will be marked.

5. When answering questions, candidates must clearly indicate units for all parameters used or computed.

MARKING SCHEME

Questions	*Marks*				
1	(a) 3	(b) 3	(c) 5	(d) 5	(e) 4
2	(a) 3	(b) 3	(c) 8	(d) 6	
3	(a) 12	(b) 8			
4	(a) 5	(b) 5	(c) 4	(d) 6	
5	(a) 4	(b) 4	(c) 4	(d) 4	(e) 4
6	(a) 3	(b) 3	(c) 6	(d) i. 4	(d) ii. 4
7	(a) 4	(b) 5	(c) 4	(d) 7	

1. A face centered cubic (*fcc*) Bravais lattice is shown in Figure P1 where basis vectors $\mathbf{a}_1$, $\mathbf{a}_2$ and $\mathbf{a}_3$ define a parallelepiped.

3 pts (a) Briefly explain the difference between a *unit cell* and a *primitive cell*.

3 pts (b) Express the primitive translation vectors $\mathbf{a}_1$, $\mathbf{a}_2$ and $\mathbf{a}_3$ in terms of the Cartesian unit vectors **x**, **y**, **z** and the cube edge length a.

5 pts (c) Given that when closely packed, each atom occupies a volume $V = \frac{\pi\sqrt{2}}{24}a^3$, use the volume of the parallelepiped to determine the *packing fraction* of the *fcc* lattice.

5 pts (d) Find the primitive translation vectors $\mathbf{b}_1$, $\mathbf{b}_2$ and $\mathbf{b}_3$ for the corresponding *reciprocal lattice*.

4 pts (e) What are the Miller indices of the plane containing the grey area shown in Figure P1?

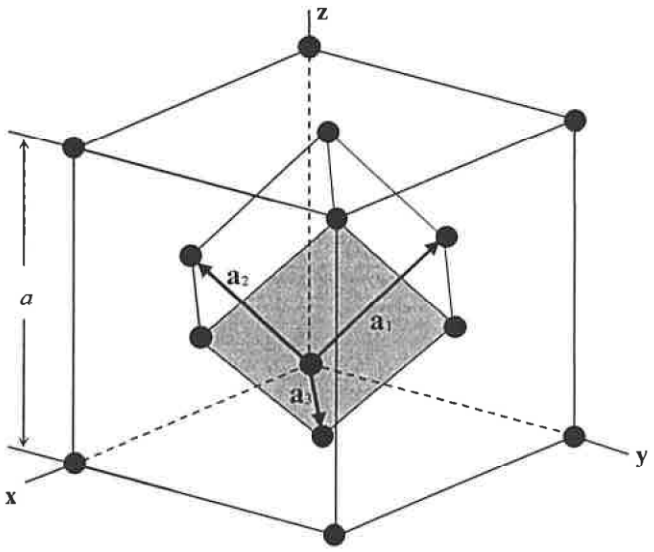

Figure P1

2. The solid curve in Figure P2a represents the total energy per molecule $U(r)$ as a function of ionic separation r for a typical ionic crystal such as KCl, NaCl, or ZnS. In these crystals, this energy is basically the sum of two types of energy contributions; dashed curves $A(r)$ and $B(r)$ represent typical variations of these two components.

3 pts (a) Briefly explain the origin and impact of the energy contribution of type $A(r)$.

3 pts (b) Briefly explain the origin and impact of the energy contribution of type $B(r)$.

8 pts (c) For the data shown on the graph of Figure P2a, determine the value for the equilibrium ionic separation r_o.

6 pts (d) Determine the Madelung constant α for the line of equally spaced ions shown in Figure P2b.

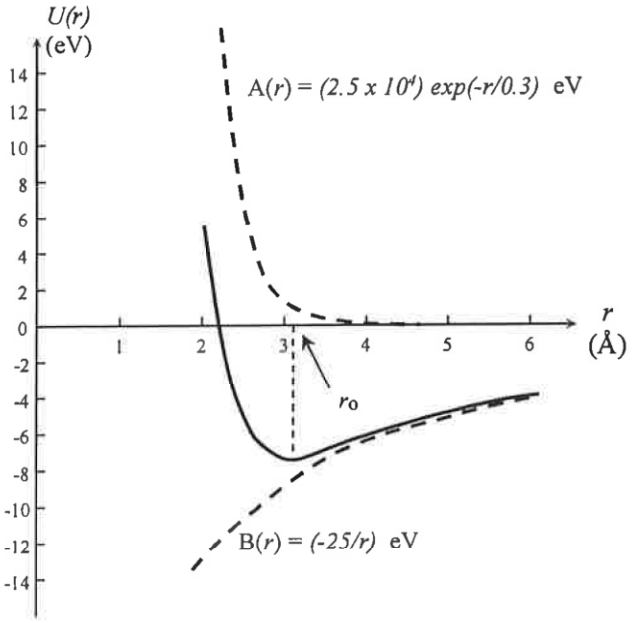

Figure P2a

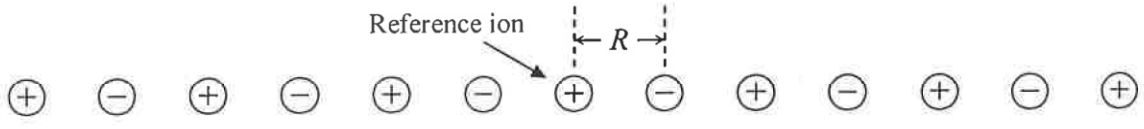

Figure P2b

3. To study a crystal of diatomic molecules such as H_2, consider the normal vibration modes of the linear chain shown in Figure P3a. Here we suppose that the force constants between nearest-neighbor atoms of mass M are alternately $C_1 = C$ and $C_2 = 10C$. The nearest-neighbor atoms are separated by *a/2*.

Equations of motion are given by

$$M\frac{d^2u_s}{dt^2} = C_1[v_s - u_s] - C_2[v_{s-1} - u_s] \quad \text{and} \quad M\frac{d^2v_s}{dt^2} = C_1[u_s - v_s] - C_2[u_{s+1} - v_s]$$

with solutions in the form of traveling waves u_s and v_s having different amplitudes u and v on alternate planes such as

$$u_s = ue^{i(sKa-\omega t)} \quad \text{and} \quad v_s = ve^{i(sKa-\omega t)}$$

This leads to the following two homogeneous linear equations:

$$[(C_1 + C_2) - \omega^2 M]u - [C_1 + C_2e^{-iKa}]v = 0 \qquad (1)$$

$$-[C_1 + C_2e^{+iKa}]u + [(C_1 + C_2) - \omega^2 M]v = 0 \qquad (2)$$

12 pts (a) The dispersion relation ω(K) in the *first Brillouin zone* is shown in Figure P3b. Find expressions for optical phonon frequencies ω_1 and ω_2, and for acoustical phonon frequency ω_3 in terms of parameters C and M.
Hint: a solution to the set of equations (1) and (2) exists only if the determinant of the coefficients of the unknowns *u* and *v* is zero.

8 pts (b) Briefly explain the origin of the terms "optical phonon" and "acoustical phonon".

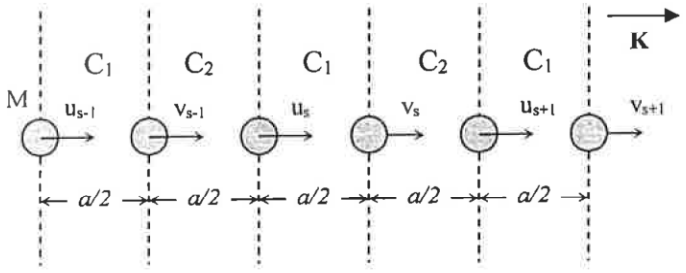

Figure P3a

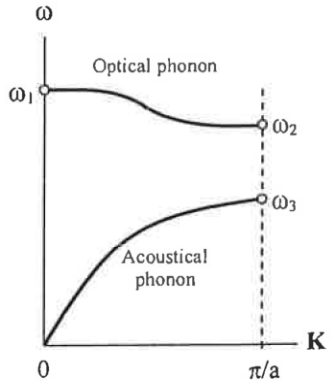

Figure P3b

4. Particles which behavior follows the Fermi-Dirac distribution are called *fermions*. Electrons in good conducting metals behave closely to *fermions*. The 3-dimensional Fermi surface of free electrons is shown in Figure P4a. The energy-wavevector relationship for a free electron is shown in Figure P4b and the energy-wavevector relationship for a *nearly free* electron in a monatomic linear lattice with ions separated by a lattice constant a appears in Figure P4c.

5 pts (a) Supposing that electrons in copper (Cu) can be considered free electrons, find the value (in **eV**) of their *Fermi energy* ε_F if the concentration of electrons in Cu is $8.45\times10^{22}/\text{cm}^3$.

5 pts (b) What is the speed of a copper electron which has a *wavevector* $k = 2\times10^8\ \text{cm}^{-1}$?

4 pts (c) At room temperature (300 °K), what is the probability that an electron would occupy an energy level that is 0.12 eV above the *chemical potential* of the free electron gas?

6 pts (d) Briefly explain the origin of the energy band gap displayed in Figure P4c for the *nearly free electron* model.

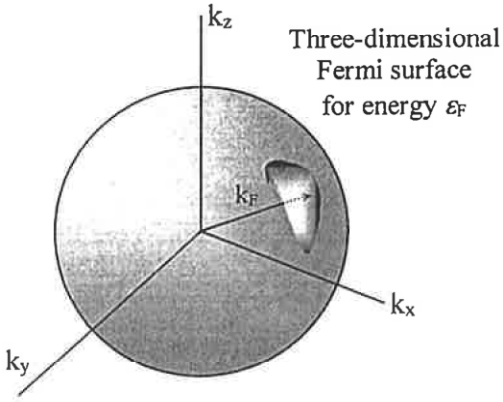

Figure P4a

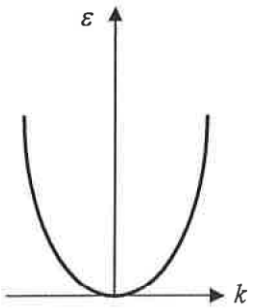

Figure P4b

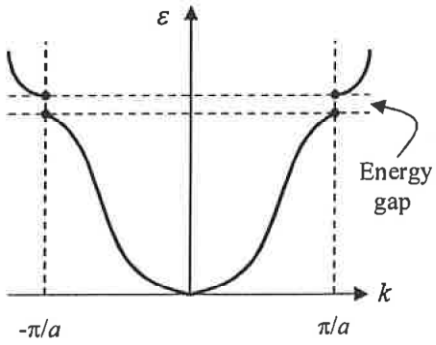

Figure P4c

5. Energy band characteristics of *intrinsic* and *extrinsic* Silicon (Si) appear in Figures P5a, P5b and P5c. Measurements made on *intrinsic* Si also indicate that electrons have an *effective mass* of $1.1m$ and holes have an effective mass of $0.56m$ where m is the mass of an electron at rest.

4 pts (a) For *intrinsic* Si, calculate how far above the valence band E_c the Fermi level μ_i is situated when the temperature is T = 300 °K.

4 pts (b) Calculate the electron concentration in *intrinsic* Si when the temperature is T = 300 °K.

4 pts (c) Briefly explain why the fermi level μ_n of the *n*-type Si is closer to the conduction band E_c and the impact it has on the properties of Si.

4 pts (d) Calculate the hole concentration p in the *n*-type Si when the temperature is T = 300 °K.

4 pts (e) Draw and clearly annotate the energy band diagram of a *p-n junction* formed by adjoining the *p*-type and *n*-type Si materials with energy band characteristics shown in Figure P5b and P5c respectively.

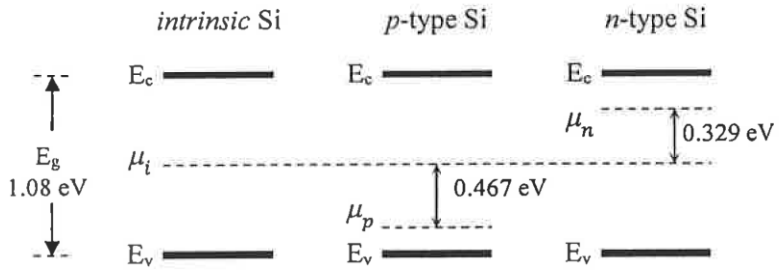

Figure P5a **Figure P5b** **Figure P5c**

6. The following questions refer to magnetism present or induced in crystal lattices.

3 pts (a) Briefly explain what is meant by *electron spin.*

3 pts (b) Experimental data on inert gases shows the following values of *molar* susceptibilities in units of 10^{-6} cm^3/mole:

He: -1.9 **Ne:** -7.2 Ar: -19.4 **Kr:** -28.0 **Xe:** -43.0

State if these inert gases are *paramagnetic* or *diamagnetic* and briefly explain the source of this magnetic property.

6 pts (c) Evaluate the *molar* susceptibility of an inert gas having the following properties:

Atomic mass: 4.002 Density: 0.214 g/cm^3
Number of electrons: 2 Average atomic radius: 1.7 x10^{-9} cm

(d) The variation of the susceptibility of a crystal as a function of temperature is shown in Figure P6.

4 pts i. State if the crystal is *paramagnetic* or *diamagnetic* and briefly explain why the susceptibility decreases with temperature.

4 pts ii. Evaluate the *Curie constant* of the crystal.

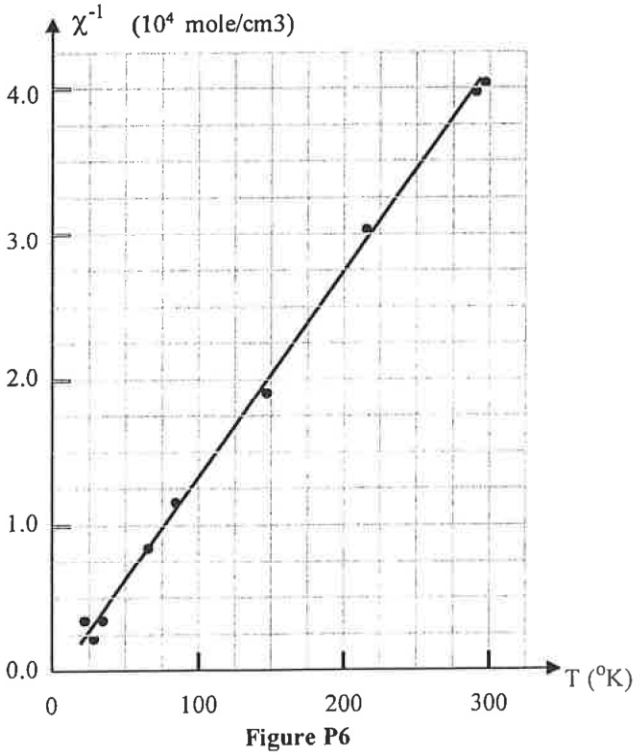

Figure P6

7. The following questions refer to the presence of defects or impurities in crystal lattices. The graph of Figure P7 shows the diffusivity of arsenic (As) into a silicon (Si) host crystal.

4 pts (a) Briefly explain what the expression *color center* means for alkali halide crystals (e.g. NaCl, KCl, CsCl) and state two methods that can be used to color these types of crystals.

5 pts (b) Experimental measurements conducted at T = 1000 °K show that 0.0001 % of lattice sites in a copper (Cu) crystal become Schottky defects. Evaluate the energy (in **eV**) it takes to create a Schottky defect in this Cu crystal at T = 1000 °K.

4 pts (c) Briefly explain the main purpose of diffusing As atoms into an Si host crystal.

7 pts (d) From the graph shown in Figure P7, specify (without any calculation) the value of the *diffusion constant* and then extract the *activation energy* of the diffusing As atoms.

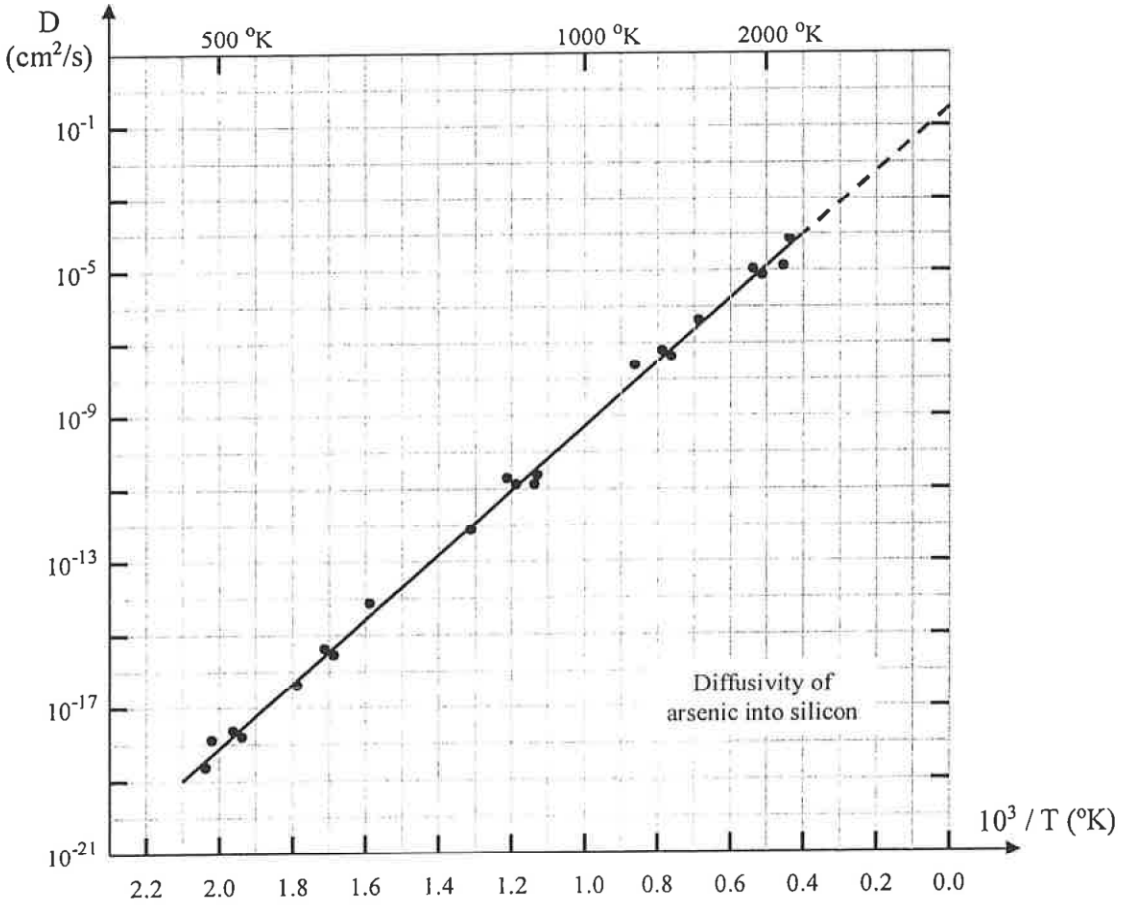

Figure P7

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USEFUL EQUATIONS

(1) $\boldsymbol{T} = u_1\boldsymbol{a}_1 + u_2\boldsymbol{a}_2 + u_3\boldsymbol{a}_3$

(2) $\boldsymbol{G} = v_1\boldsymbol{b}_1 + v_2\boldsymbol{b}_2 + v_3\boldsymbol{b}_3$

(3) $\boldsymbol{p} = \boldsymbol{r} \times \boldsymbol{t} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} (\boldsymbol{x}\ \boldsymbol{y}\ \boldsymbol{z}) = \begin{pmatrix} r_2t_3 - r_3t_2 \\ r_3t_1 - r_1t_3 \\ r_1t_2 - r_2t_1 \end{pmatrix} (\boldsymbol{x}\ \boldsymbol{y}\ \boldsymbol{z})$ where $\boldsymbol{r} = r_1\boldsymbol{x} + r_2\boldsymbol{y} + r_3\boldsymbol{z}$, $\boldsymbol{t} = t_1\boldsymbol{x} + t_2\boldsymbol{y} + t_3\boldsymbol{z}$

(4) $V_{min} = |\boldsymbol{a}_1 \cdot (\boldsymbol{a}_2 \times \boldsymbol{a}_3)|$

(5) $\boldsymbol{b}_1 = 2\pi \frac{\boldsymbol{a}_2 \times \boldsymbol{a}_3}{\boldsymbol{a}_1 \cdot (\boldsymbol{a}_2 \times \boldsymbol{a}_3)}$ $\boldsymbol{b}_2 = 2\pi \frac{\boldsymbol{a}_3 \times \boldsymbol{a}_1}{\boldsymbol{a}_1 \cdot (\boldsymbol{a}_2 \times \boldsymbol{a}_3)}$ $\boldsymbol{b}_3 = 2\pi \frac{\boldsymbol{a}_1 \times \boldsymbol{a}_2}{\boldsymbol{a}_1 \cdot (\boldsymbol{a}_2 \times \boldsymbol{a}_3)}$

(6) $2d \sin\theta = n\lambda$ $\Delta\boldsymbol{k} = \boldsymbol{G}$ $2\boldsymbol{k} \cdot \boldsymbol{G} = \boldsymbol{G}^2$

(7) $U(R) = 4\epsilon\left[\left(\frac{\sigma}{R}\right)^{12} - \left(\frac{\sigma}{R}\right)^{6}\right]$ $F(R) = -\, dU(R)/dR$

(8) $\frac{\alpha}{R} = \sum_j \frac{\pm}{r_j}$

(9) $M\frac{d^2u_s}{dt^2} = C(u_{s+1} - u_s) - C(u_{s-1} - u_s)$

(10) $f(\varepsilon) = \frac{1}{exp\left[\frac{\varepsilon-\mu}{k_BT}\right]+1}$

(11) $\epsilon_F = \frac{\hbar^2}{2m}k_F^2 = \frac{\hbar^2}{2m}\left(\frac{3\pi^2N}{V}\right)^{\frac{2}{3}}$

(12) $np = 4\left(\frac{k_BT}{2\pi\hbar^2}\right)^3 (m_em_h)^{3/2} \exp\left(\frac{-E_g}{k_BT}\right)$

(13) $n_i = p_i = 2\left(\frac{k_BT}{2\pi\hbar^2}\right)^{3/2} (m_em_h)^{3/4} \exp\left(\frac{-E_g}{2k_BT}\right)$

(14) $\mu_i = \frac{E_g}{2} + \frac{3}{4}\, k_BT\, \ln(m_h/m_e)$

(15) $n = n_i \exp\left(\frac{\mu_n - \mu_i}{k_BT}\right)$ n-type

(16) $p = p_i \exp\left(\frac{\mu_i - \mu_p}{k_BT}\right)$ p-type

(17) $\chi = -\frac{\mu_o NZe^2}{6m}\langle r^2\rangle$ (SI) $\chi = -\frac{NZe^2}{6mc^2}\langle r^2\rangle$ (CGS)

(18) $\frac{n}{N-n} = exp\left(\frac{-E_V}{k_BT}\right)$

(19) $D = D_o exp\left(\frac{-E}{k_BT}\right)$

USEFUL PARAMETERS

Quantity	Symbol	Value	CGS	SI
Light velocity	c	2.998	10^{10} cm/s	10^{8} m/s
Proton's charge	e	1.602		10^{-19} C
		4.803	10^{-10} esu	
Planck's constant	h	6.626	10^{-27} erg · s	10^{-34} J · s
	$\hbar = h/2\pi$	1.055	10^{-27} erg · s	10^{-34} J · s
Avogadro's number	N	6.022 x 10^{23} /mole		
Atomic mass unit	amu	1.66	10^{-24} g	10^{-27} kg
Electron's mass	m	9.11	10^{-28} g	10^{-31} kg
Proton's mass	M_p	1.67	10^{-24} g	10^{-27} kg
Bohr radius: $\hbar^2/me^2$	r_o	5.292	10^{-9} cm	10^{-11} m
Bohr magneton: $e\hbar/2mc$	μ_B	9.274	10^{-21} erg/G	10^{-24} J/T
Energy (electron volt)	eV	1.602	10^{-12} erg	10^{-19} J
Boltzmann's constant	k_B	1.38	10^{-16} erg/K	10^{-23} J/K
		0.862	10^{-4} eV/K	
	eV/k_B	1.16 x 10^{4} K		
Permittivity (free space)	ϵ_o		1	$10^{7}/4\pi c^{2}$ F/m
Permeability (free space)	μ_o		1	4π x 10^{-7} N/A^2

TRIGONOMETRIC IDENTITIES

(1) $sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$

(2) $cos\ \theta = \frac{1}{2}[\exp(i\theta) + \exp(-i\theta)]$

(3) $sin\ \theta = \frac{1}{2i}[\exp(i\theta) - \exp(-i\theta)]$

(4) $cos\ \theta \cong 1 - \frac{1}{2}\theta^2$ for very small θ

(5) $sin\ \theta \cong \theta$ for very small θ