

# National Exams December 2017

98-Ind-A1, Operations Research

3 hours duration

## NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. Any non-communicating calculator is permitted. This is an Open Book exam. Note to candidates: You must indicate the type of calculator being used. i.e. write the name and model designation of the calculator, on the first left hand sheet of the exam workbook.
3. There may be more questions than you are able to answer in the allotted time. Although the total value of the questions is 180, any marks achieved will be considered toward the 100 total requirements.

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1. A company has one item in its inventory that requires special storage. The company estimates the storage cost at \$2.00 per unit per year. The demand for the item is 40,000 units per year, and the ordering cost is \$16.00 per order.
    - a. Assuming shortages are not allowed, write an expression for the total yearly cost in terms of the order quantity  $Q$ , and determine the optimal order quantity.
    - b. Assuming shortages are allowed and cost \$4.00 per unit per year. write an expression for the total yearly cost in terms of the order quantity  $Q$  and the maximum shortage level  $s$ , and determine the optimal order quantity.
  
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  2. A company that conducts surveys, is going to bid on a job that has the following requirements
    - a. There must be at least 300 personal interviews
    - b. There must be at least 500 interviews (personal or telephone) conducted at night.
    - c. Of the day interviews, at least 60% of them must be by telephone.
    - d. There must be at least 1000 total interviews ((personal or telephone)

The cost of doing an interview is as follows:

	Personal	Telephone
Day	\$2.00	\$1.00
Night	\$2.10	\$1.20

In preparing the bid, the company wants to know the minimum cost of meeting the requirements.

Define the decision variables, and set up the linear programming model.  
Do not solve.

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3. A company buys tractors at a cost of \$6,500 and sells them for \$10,000. A charge of \$2500 is incurred for each order of tractors, regardless of the size of the order. The company estimates the holding charge for one tractor to be 500 \$/tractor/mth.
- a. If there is a monthly demand for 15 tractors what is the optimal ordering policy, and the resulting monthly inventory (i.e. holding and ordering) cost?

If instead the demand is not constant and the forecast for the next 4 months is 20, 25, 12 and 3, use dynamic programming to determine the optimal ordering policy and the minimal cost over the four-month period.

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4. Joe is selling Christmas trees to pay for his college tuition. He purchases trees for \$10 each and sells them for \$25 each. The number of trees he can sell is normally distributed with a mean of 100 and a standard deviation of 30. How many trees should Joe purchase?

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5. You own a jewellery store with one designated parking spot. All customers arrive by car, and use the designated parking spot. For security reasons the store can only accommodate one customer at a time. The sales person monitors the parking spot, and immediately upon arrival, she meets each customer at the car, serves them and returns them to their car at end of the session. She is suggesting that provision be made for one additional spot at \$100/day where a customer can wait in the car in case there is already another customer in the store. Your records show that on average the sales person serves 10 customers per 7-hr day, and on average spends  $\frac{1}{2}$  hour with each customer. 15% of customer visits result in sales, and the average profit per sale is \$1000. Based on the assessment that 10% of customers who find the parking spot full never return, should you proceed to arrange for a second parking spot?

- 20 6. Consider the LP problem: Maximize  $z=5x_1+x_2+2x_3$  such that  $x_1+x_2+x_3\leq 6$ ,  $6x_1+x_3\leq 8$ ,  $x_2+x_3\leq 2$ ,  $x_i \geq 0$ , along with the final equations:

$$x_1 = 1 + (1/6)x_2 - (1/6)x_5 + (1/6)x_6$$

$$x_3 = 2 - x_2 - x_6$$

$$x_4 = 3 - (1/6)x_2 + (1/6)x_5 + (5/6)x_6$$

$$z = 9 - (1/6)x_2 - (5/6)x_5 - (7/6)x_6$$

- a. Find the range of values for the objective function coefficient of  $x_1$  for which the current basis remains optimal.
- b. Find the range of values for the right hand side coefficient of the constraint  $6x_1+x_3\leq 8$  for which the current basis remains optimal.
- c. What is the shadow price for each of the constraints? What is the meaning of these values?
- d. What is the reduced cost of each of the decision variables  $x_1$ ,  $x_2$ ,  $x_3$ ? What is the meaning of these values?
- 20 7. An automotive manufacturer has four plants to annually manufacture 500,000 cars of each of its three models: X, Y, and Z. Each plant can only manufacture a single model, and each car can only be made at a single plant. If plants 3 and 4 are used, then labour union contracts dictate that plant 1 must also be used. The fixed costs of operating each plant and the variable costs for producing one car of each model are:

<i>Plant</i>	<i>Fixed Cost</i>	<i>Model X Var. Cost</i>	<i>Model Y Var. Cost</i>	<i>Model Z Var. Cost</i>
<b>1</b>	\$7 billion	\$12,000	\$16,000	\$9,000
<b>2</b>	\$6 billion	\$15,000	\$18,000	\$11,000
<b>3</b>	\$4 billion	\$17,000	\$19,000	\$12,000
<b>4</b>	\$2 billion	\$19,000	\$22,000	\$14,000

Formulate an Integer Program to minimize the annual production cost.

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8. The best selling college statistics text, *The Thrill of Statistics*, sells 5 million copies every fall. Some users keep the book, and some sell it back to the bookstore. Suppose that 90% of all students who buy a new book sell it back, 80% of all students who buy a once-used book sell it back, and 60% of all students who buy a twice-used book sell it back. If a book is used four or more times the cover falls off, and it must be discarded. In the steady state, how many new copies of the book will the publisher be able to sell each year?

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9. We are thinking of filming the Don Harnett story. We know that if the film is a flop we will lose 4 M\$, and if the film is a success we will earn 15M\$. We currently believe there is a 10% chance that the film will be a success. Before filming we have the option of paying the noted movie critic Roger Alert 1 M\$ for his view of the film. In the past, Alert has predicted 60% of all actual hits to be hits, and 90% of all actual flops to be flops. Assuming that we want to maximize our expected profit, should we pay Alert his exorbitant fee?