National Exams May 2016

98-Phys-B5 , Systems & Control

3 hours duration

NOTES:

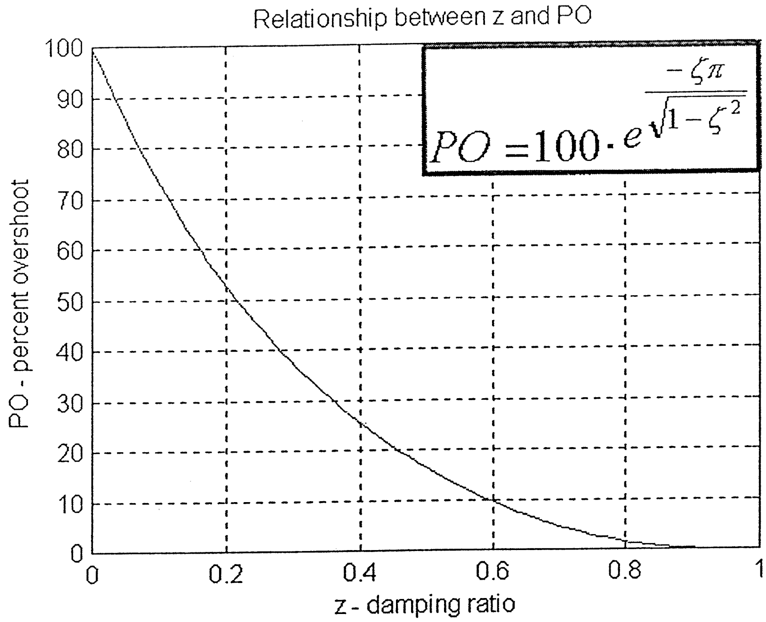
1. This is a CLOSED BOOK EXAM. Only an approved Casio or Sharp calculator is permitted. Candidates are allowed to use a double-sided, **handwritten**, 8.5 by 11" formula and notes sheet. Otherwise, there are no restrictions on the content of the formula sheet. The sheet must be signed and submitted together with the examination paper.
2. If doubt exists as to interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
3. Five (5) questions constitute a complete paper. **YOU ARE REQUIRED TO COMPLETE QUESTION 1 AND QUESTION 2.** Choose three (3) more questions out of the remaining six. Clearly indicate answers to which questions should be marked - otherwise, only the first five answers provided will be marked. Each question is of equal value. Weighting of topics within each question is indicated. Partial marks will be given. Clearly show steps taken in answering each question.
4. **Use exam booklets to answer the questions - clearly indicate which question is being answered.**

YOUR MARKS		
QUESTIONS 1 AND 2 ARE COMPULSORY:		
Question 1	20	
Question 2	20	
CHOOSE THREE OUT OF THE REMAINING SIX QUESTIONS:		
Question 3	20	
Question 4	20	
Question 5	20	
Question 6	20	
Question 7	20	
Question 8	20	
TOTAL:	100	

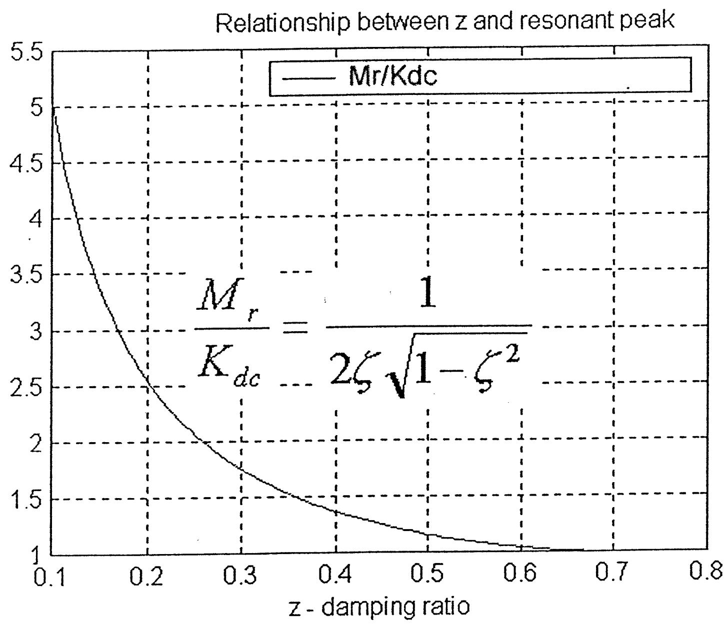
A Short Table of Laplace Transforms

Laplace Transform	Time Function
1	$\sigma(t)$
$\frac{1}{s}$	$1(t)$
$\frac{1}{(s)^2}$	$t \cdot 1(t)$
$\frac{1}{(s)^{k+1}}$	$\frac{t^k}{k!} \cdot 1(t)$
$\frac{1}{s+a}$	$e^{-at} \cdot 1(t)$
$\frac{1}{(s+a)^2}$	$te^{-at} \cdot 1(t)$
$\frac{a}{s(s+a)}$	$(1-e^{-at}) \cdot 1(t)$
$\frac{a}{s^2+a^2}$	$\sin at \cdot 1(t)$
$\frac{s}{s^2+a^2}$	$\cos at \cdot 1(t)$
$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at} \cdot \cos bt \cdot 1(t)$
$\frac{b}{(s+a)^2+b^2}$	$e^{-at} \cdot \sin bt \cdot 1(t)$
$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$\left(1 - e^{-at} \cdot \left(\cos bt + \frac{a}{b} \cdot \sin bt\right)\right) \cdot 1(t)$
$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \cdot \sin\left(\omega_n\sqrt{1-\zeta^2}t\right) \cdot 1(t)$
$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$\left(1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \cdot \sin\left(\omega_n\sqrt{1-\zeta^2}t + \cos^{-1}\zeta\right)\right) \cdot 1(t)$
$F(s) \cdot e^{-Ts}$	$f(t-T) \cdot 1(t)$
$F(s+a)$	$f(t) \cdot e^{-at} \cdot 1(t)$
$sF(s) - f(0+)$	$\frac{df(t)}{dt}$
$\frac{1}{s}F(s)$	$\int_{0+}^{+\infty} f(t)dt$

Useful Plots & Second Order Model



PO vs. Damping Ratio



Resonant Peak vs. Damping Ratio

Second Order Model:

ζ – Damping Ratio (zeta), of the model

ω_n – Frequency of Natural Oscillations of the model

$$G_m(s) = K_{dc} \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

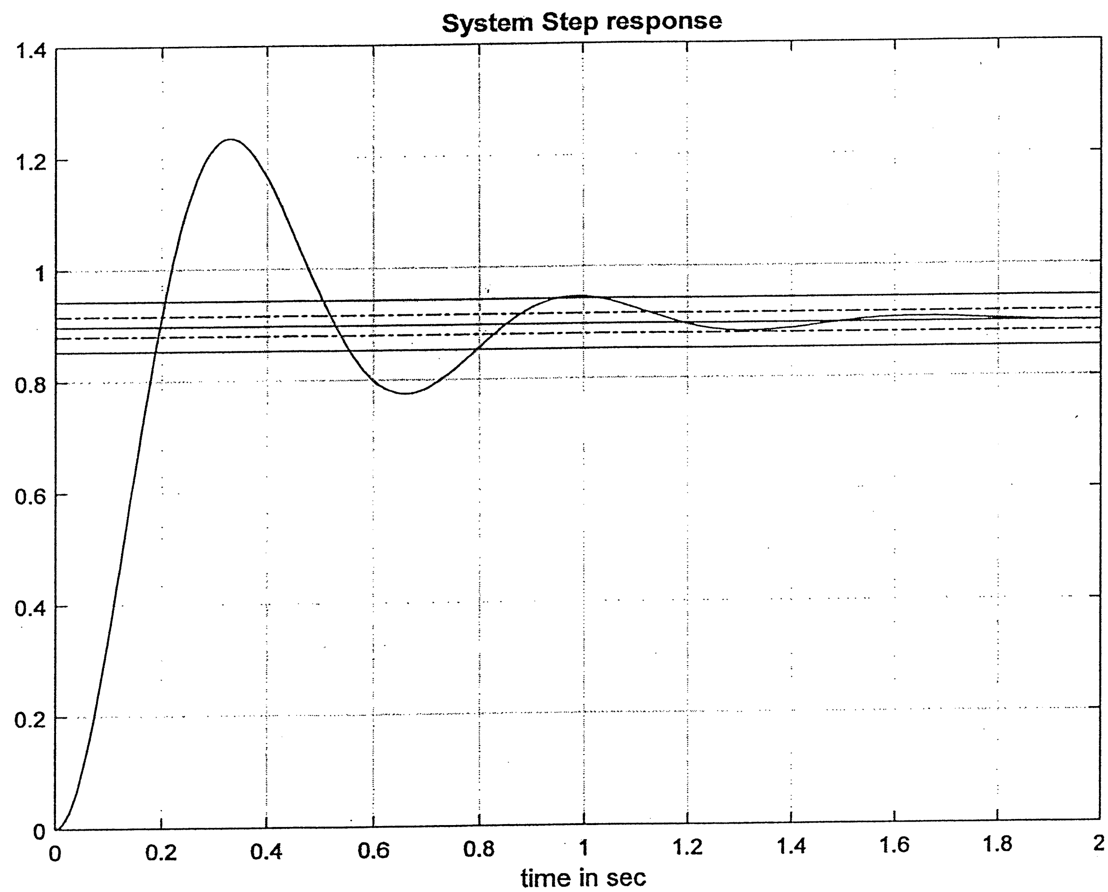
K_{dc} – DC Gain of the model

Question 1 (Compulsory)

Basic Definitions and Concepts of Control.

1) **(2 marks)** Consider a step response to **a unit reference input** of a control system as shown. Estimate the following specs: Percent Overshoot, Steady State Error, Settling Time.

$PO =$	$e_{ss(step)\%} =$	$T_{settle(5\%)} =$



2) **(2 marks)** Consider the same step response to a unit reference input as shown in item 1 above. What are your estimates of the closed loop damping ratio, ζ, the frequency of natural oscillations, ω_n, and the DC gain, K_{dc}?

$\zeta =$	$\omega_n =$	$K_{dc} =$

3) **(2 marks)** Consider a Closed Loop (CL) control system described by the following transfer function, $G(s)$:

$$G(s) = \frac{22.5}{s^2 + s + 25}$$

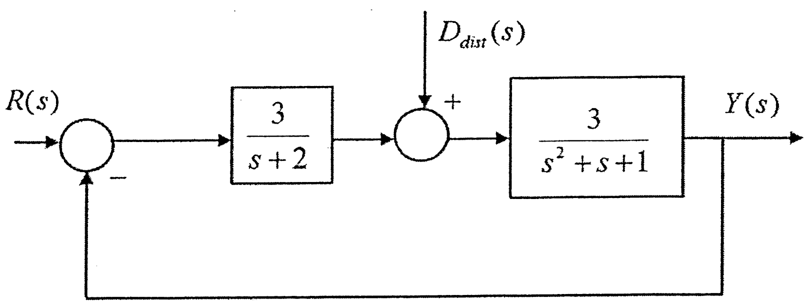
What are your estimates of: damping ratio ζ, frequency of natural oscillations ω_n, DC gain, K_{dc}?

$\zeta =$ $\omega_n =$ $K_{dc} =$

4) **(2 marks)** Consider now a step response to **a unit reference input** of the control system above. Estimate the following specs: Percent Overshoot, Steady State Error, Settling Time.

$PO =$ $e_{ss(step)\%} =$ $T_{settle(5\%)} =$

5) **(2 marks)** Consider a block diagram of a Type 0 system, shown below. If the steady state value of the input reference signal is $r_{ss} = 2$ V/V, and the steady state value of disturbance signal is $d_{dist(ss)} = 1$ V/V, what is the steady state value of the system output, y_{ss} ?



$y_{ss} =$

6) **(2 marks)** Consider a time function describing a step response of a certain control system:

$$y(t) = [-0.139 + 0.0833t + 0.889\, t \cdot e^{-3t} + 0.333 \cdot e^{-3t} + 0.75t \cdot e^{-2t} - 0.75e^{-2t}] \cdot 1(t)$$

Match it to an appropriate expression describing the system transfer function G(s) below. HINT – no calculations are required!

☐ $G(s) = \frac{3}{s \cdot (s+2)(s+3)}$ ☐ $G(s) = \frac{3}{(s+2)^2(s+3)^2}$ ☐ $G(s) = \frac{3}{(s+2)(s+3)}$ ☐ $G(s) = \frac{3}{s(s+2)^2(s+3)^2}$

7) **(1 mark)** To have an "ideal" tracking of a reference signal by a closed loop control system under Proportional Control:

- ❑ The open loop gain should be as close to zero as possible
- ❑ The open loop gain should be as close to one as possible
- ❑ The open loop gain should be as close to infinity as possible
- ❑ The open loop gain should be as high as possible without destabilizing the system

8) **(3 marks)** Consider an equivalent unit feedback system, where the Closed Loop transfer function is described as:

$$G_{closed}(s) = \frac{5s^2 + 10s + 3}{s^4 + 6s^3 + 12s^2 + 10s + 3}$$

System Type is N =

What are: System Type, Error Constants and Steady State Errors?

$K_{pos} =$	$K_v =$	$K_a =$
$e_{ss(step)} =$	$e_{ss(ramp)} =$	$e_{ss(parab)} =$

9) **(2 marks)** To have an "ideal" disturbance rejection by a Closed Loop control system under Proportional Control:

- ❑ The Open Loop gain should be as close to zero as possible
- ❑ The Open Loop gain should be as close to one as possible
- ❑ The Open Loop gain should be as close to infinity as possible
- ❑ The Open Loop gain should be as high as possible without destabilizing the system

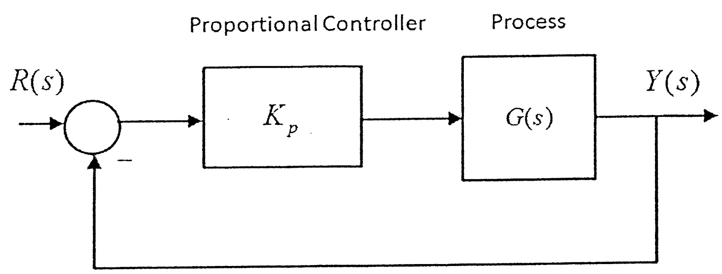
10) **(2 marks)** To have an "ideal" disturbance rejection by a Closed Loop control system under Proportional Control:

- ❑ The Closed Loop gain should be as close to zero as possible
- ❑ The Closed Loop gain should be as close to one as possible
- ❑ The Closed Loop gain should be as close to infinity as possible
- ❑ The Closed Loop gain should be as high as possible without destabilizing the system

Question 2 (Compulsory)

System Stability in the s-Domain and in the Frequency Domain: Bode Plots, Root Locus Plots and Routh-Hurwitz Criterion of Stability.

A unit feedback control system is to be stabilized using a Proportional Controller, as shown in Figure Q2.1.



The process transfer function is described as follows:

$$G(s) = \frac{(s+50)^2}{(s+4)^3}$$

Figure Q2.1

Your task is to investigate the stability of the closed loop system using s-domain analysis by finding: a) the value of the Proportional Controller Gain or Gains at which the closed loop system becomes marginally stable ($K_p = K_{crit}$) and the corresponding frequency or frequencies of marginally stable oscillations (ω_{osc}), and b) the range or ranges of safe operating gains for the Proportional Controller.

1) **(6 marks)** Root Locus sketch for the system in question is provided in Figure Q2.2. Use it to read off the frequency or frequencies (ω_{osc}) corresponding to the crossover(s) of the Root Locus with the Imaginary axis. Next, use the Root Locus Magnitude Criterion (shown below) to determine the corresponding values of critical gains (K_{crit}). Finally, use the Root Locus sketch to interpret the corresponding range or ranges of safe operating gains.

   Magnitude Criterion: $K = \dfrac{1}{|G(s^*)|}$ where s^* is a coordinate of the crossover with the Im axis

2) **(4 marks)** The frequency response (Bode plot) for the process G(s) is shown in Figure Q2.3. Use it to verify the above calculations of the critical value (or values) of the gain, K_{crit}, and the corresponding frequency (or frequencies) of marginally stable oscillations, ω_{osc}.

3) **(10 marks)** Finally, apply the Routh-Hurwitz Criterion of Stability to verify the above calculations of the critical value (or values) of the gain, K_{crit}, and the corresponding frequency (or frequencies) of marginally stable oscillations, ω_{osc}.

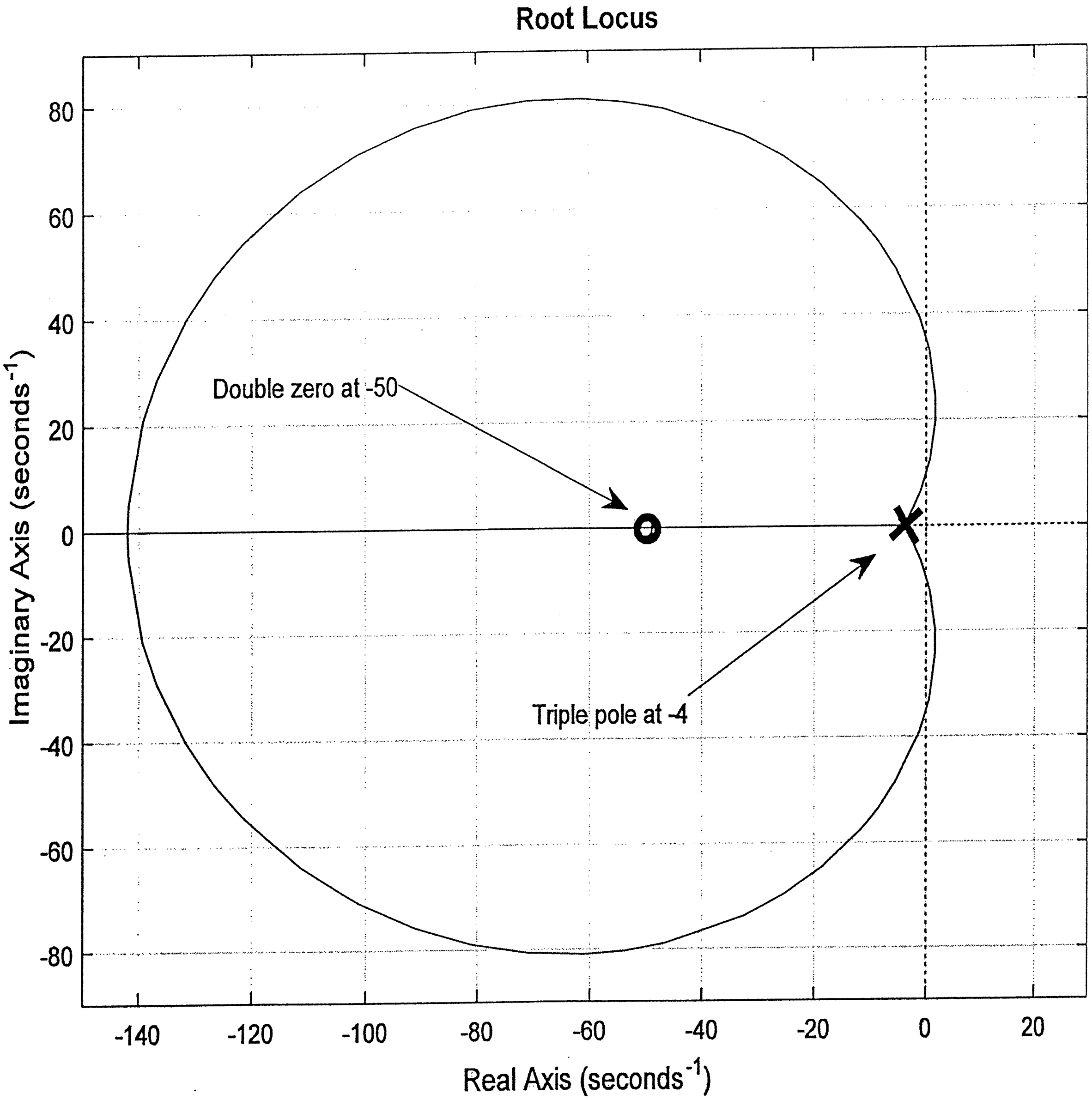


Figure Q2.2 – Root Locus Plot for the System in Question 2

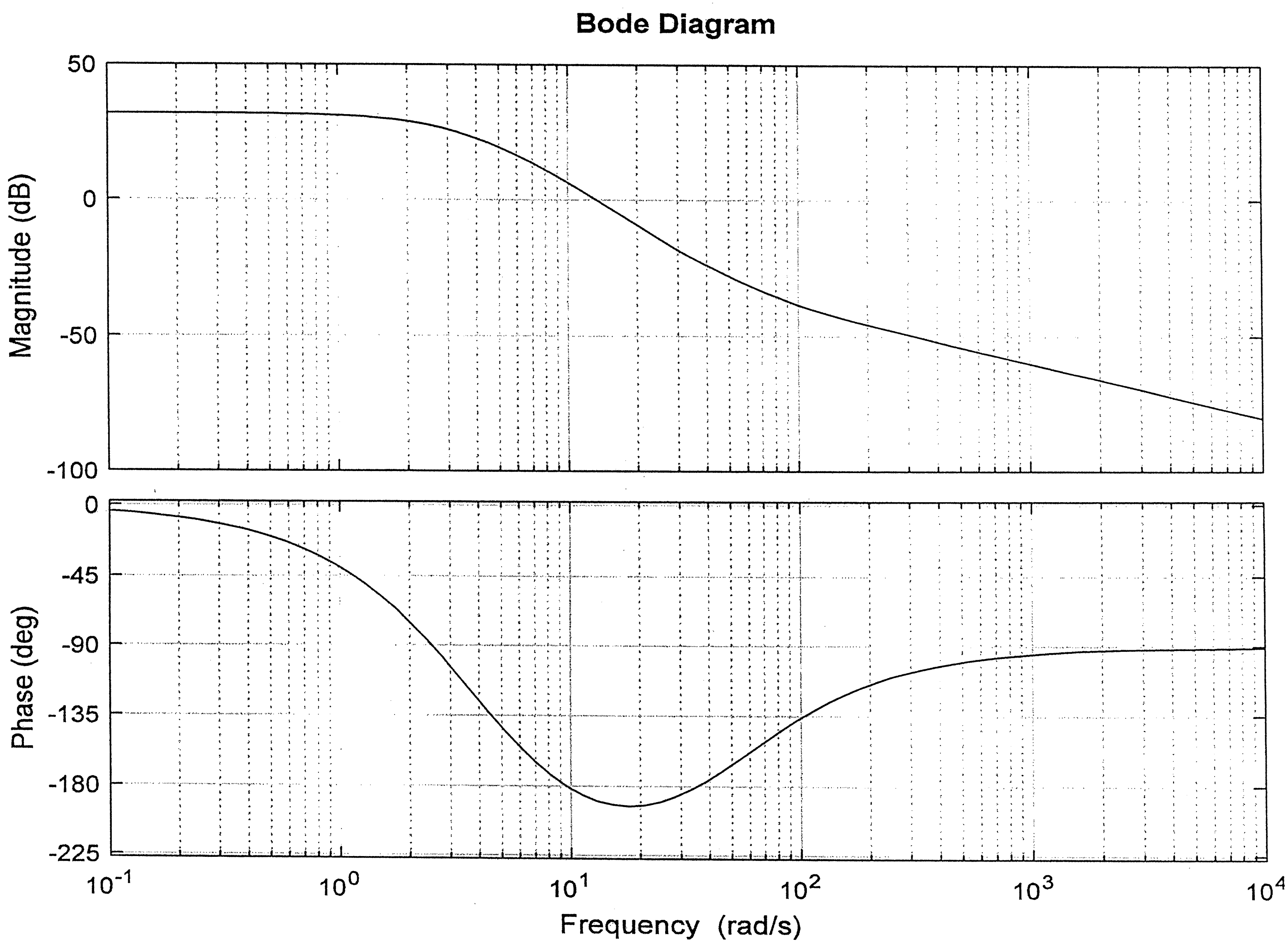


Figure Q2.3 – Frequency Response of G(s) in Question 2

Question 3

State Space vs. Transfer Function Representations, Controllability and Observability, Steady State Errors to Step Inputs.

Consider a certain closed loop control system shown below:

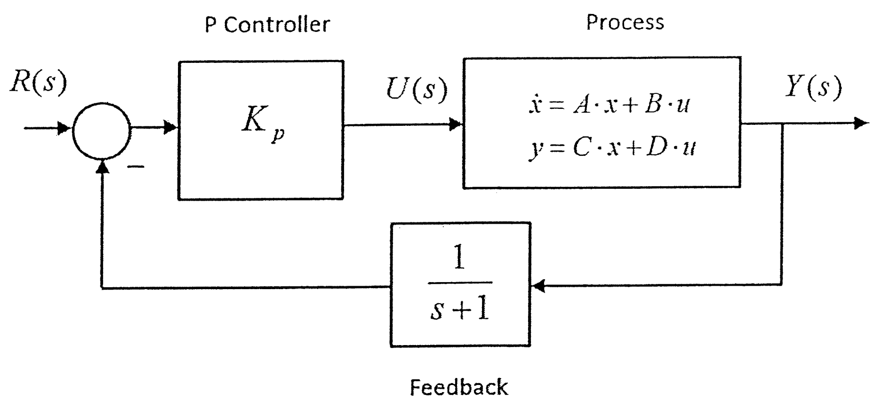


Figure Q3.1 - Closed Loop System in Question 3

1) **(5 marks)** If the state space description of the Process is as follows, find the Open Loop transfer function of the system shown.

$$\dot{x_1} = x_2$$
$$\dot{x_2} = -x_1 - x_2 + u$$
$$y = x_1 + x_2 + u$$

2) **(5 marks)** Determine if the Open Loop (OL) system is controllable and observable.

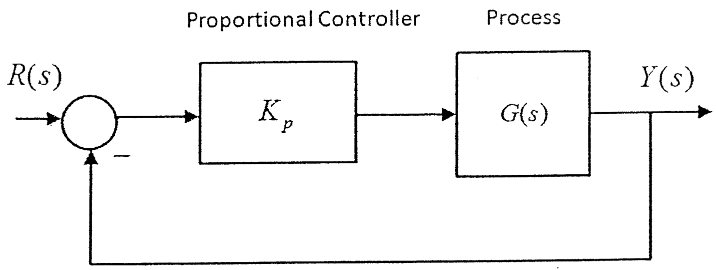
3) **(10 marks)** Find the range of Proportional Controller gains such that the Closed Loop (CL) system is

   a) Stable;
   b) The CL system step response has a Steady State Error, $e_{ss(step)\%}$, of no more than 5%.

Question 4

Root Locus Analysis and Gain Selection, Stability, Second Order Model, System Damping.

A unit feedback control system is to be stabilized using a Proportional Controller, as shown in Figure Q4.1.



The process transfer function is described as follows:

$$G(s) = \frac{10}{s(s^2 + 10s + 36)}$$

Figure Q4.1

1) **(5 marks)** A Root Locus plot for this system is shown in Figure Q4.2. Calculate asymptotic angles, the location of the centroid and angles of departure from the complex open loop poles. Next, clearly superimpose these on the plot in Figure Q4.2.

2) **(5 marks)** Locate and label the coordinate of the crossover with the Imaginary axis (i.e. the marginal stability condition) on the Root Locus plot in Figure Q4.2 and read off the corresponding value of the frequency of marginal stability oscillations, ω_{osc}. Next, calculate the corresponding value of the critical gain, K_{crit}, at which the system becomes marginally stable.

3) **(7 marks)** It is required that the unit step response of the Closed Loop system exhibits Percent Overshoot of approximately 5%. Determine the corresponding Proportional Gain Value, K_{op}, and calculate an estimate of the Rise Time, $T_{rise(0-100\%)}$.

4) **(3 marks)** Finally, consider the two step responses shown in Figure Q4.3. One corresponds to the actual system response at the Proportional Gain value found in item 3) above, the other corresponds to the second order ("dominant poles") model. Identify the traces (i.e. write A or B), and provide a <u>very brief</u> explanation to support your choices.

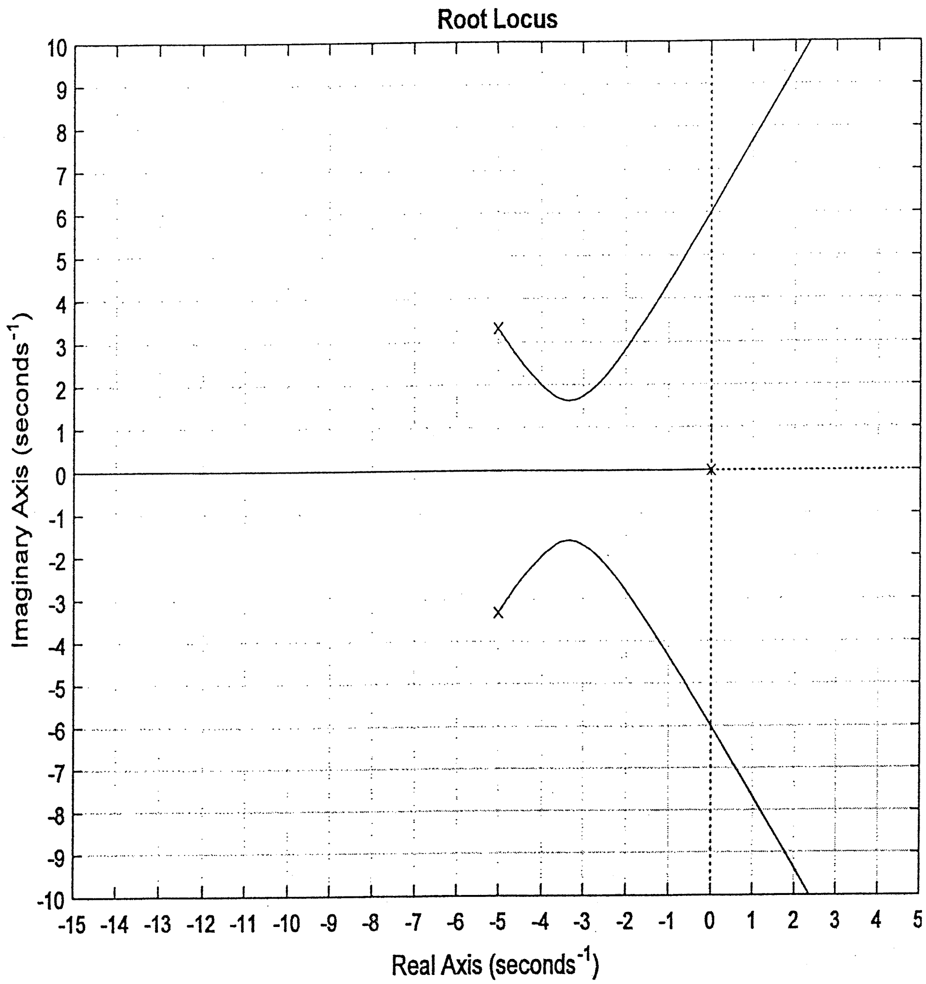


Figure Q4.2 – Root Locus of the System in Question 4

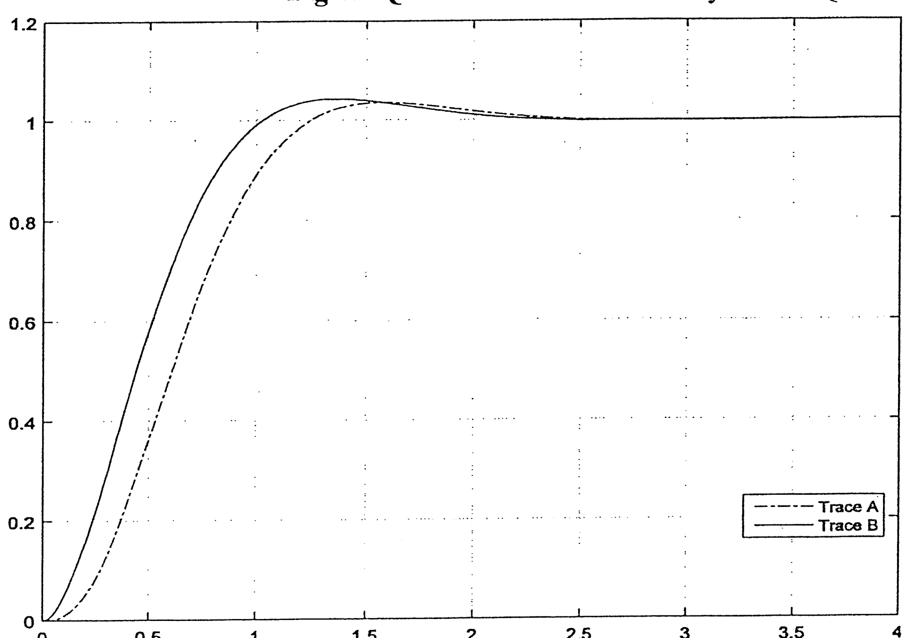


Figure Q4.3 – Comparison of Step Responses in Question 4

The actual System response is shown as

TRACE__________

The Model response is shown as

TRACE__________

Question 5

PID Controller Design by Pole Placement, Response Specifications, Second Order Model.

Consider a closed loop positioning control system working under Proportional + Integral + Rate Feedback Control, as shown in Figure Q5.1:

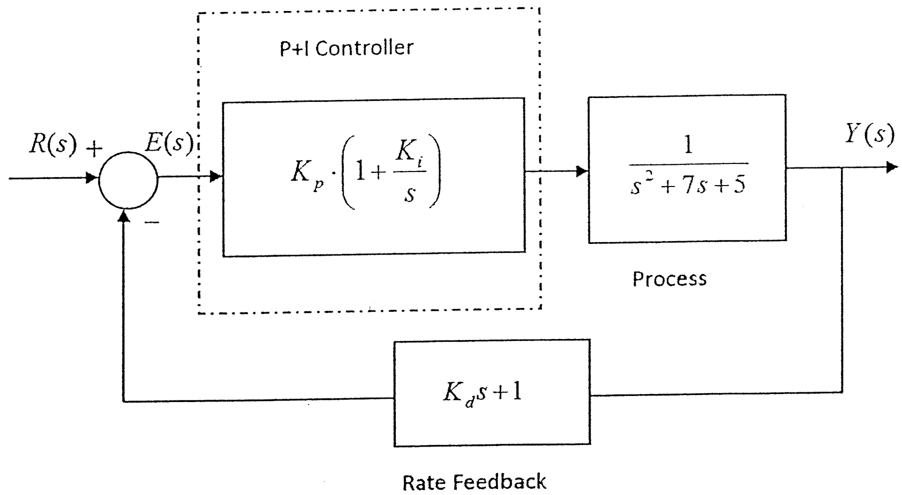


Figure Q5.1

1) **(5 marks)** Derive the Closed Loop system transfer function in terms of Controller Gains K_p and K_d and K_i, and write the system Characteristic Equation, $Q(s) = 0$.

2) **(5 marks)** The compensated Closed Loop step response of this system is to have the following specifications:

   a. Percent Overshoot, $PO = 10\%$

   b. Settling Time, $T_{settle(\pm 2\%)} = 1$ sec

Determine the Closed Loop system damping ratio, ζ, and the frequency of natural oscillations, ω_n, to meet the transient response requirements.

3) **(10 marks)** Choose the pole locations for the Closed Loop system so that system two complex conjugate ("dominant") poles correspond to the desired second order model (above) and the third real pole equals to the value of Integral Gain K_i so that **a pole-zero cancellation** in the Closed Loop transfer function occurs. Compute the required Controller Gains K_p, K_d, and K_i.

Question 6

Lead/Lag Controller Design by Pole Placement, Response Specifications, Second Order Model.

Consider a unit feedback control system where the process is a first order transfer function G(s), and the Controller transfer function is described below as $G_c(s)$:

$$G(s) = \frac{1}{(s + 0.5)}$$

$$G_c(s) = \frac{a_1 s + a_0}{b_1 s + 1}$$

1) **(5 marks)** Derive the closed loop transfer function, $G_{cl}(s) = \frac{Y(s)}{R(s)}$, in terms of the controller parameters, a_1, a_0 and b_1, and write the system Characteristic Equation, $Q(s) = 0$.

2) **(5 marks)** The compensated Closed Loop step response of this system is to have the following specifications:
   a. Percent Overshoot, $PO = 10\%$
   b. Settling Time, $T_{settle(\pm 2\%)} = 2$ sec
   c. Steady State Error, $e_{ss(\%)} = 5\%$.

Determine an appropriate model, $G_m(s)$, that would meet these specifications.

3) **(7 marks)** Next, find controller parameters (a_1, a_0 and b_1) and identify the nature of the controller i.e. is it a Lead or Lag Controller.

4) **(3 marks)** Briefly describe how the actual closed loop system response would compare with the model response.

Question 7

Lead Controller Design in Frequency Domain, Response Specifications, Second Order Model.

Consider uncompensated open loop frequency response plots shown in Figure Q7.1. The closed loop system is to be compensated by a Lead Controller $G_c(s)$:

$$G_c(s) = \frac{a_1 s + a_0}{b_1 s + 1}$$

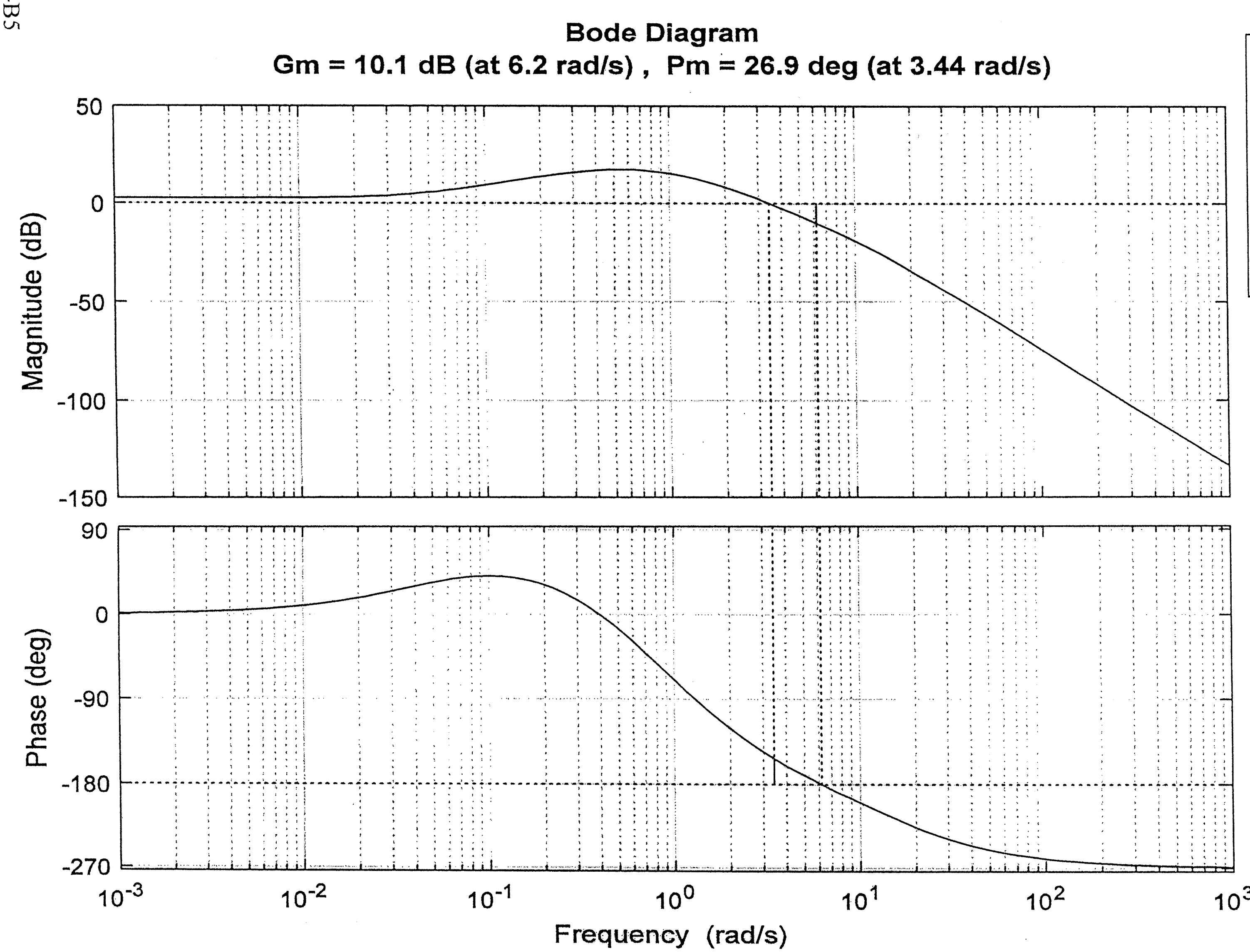
Design requirements are:

- The Steady State Error for the unit step input for the compensated closed loop system is to be no more than 25%
- Percent Overshoot of the compensated closed loop system is to be no more than 20%
- The Settling Time, $T_{settle(\pm 2\%)}$, is to be no less than 1 second.

1. **(6 marks)** Use the information provided in Figure Q7.1 to determine the Position Constant of the uncompensated system, (K_{pos_u}) and to estimate the uncompensated system step error, in %, $e_{ss(step)\%\_u}$. Next, calculate the Position Constant of the compensated system (K_{pos_c}) that would meet the design requirement and calculate the required value of the Controller DC gain parameter, a_0.

2. **(4 marks)** Decide on the Phase Margin of the compensated system, Φ_{m_c}, and on the corresponding Crossover Frequency, ω_{cp_c}, that would meet the design requirements above.

3. **(10 marks)** Calculate the remaining Lead Controller parameters (a_1, b_1) and the Controller transfer function $G_c(s)$. Show the general shape of the compensated frequency response by overlaying it on top of the uncompensated plot in Figure Q7.1.



$\Phi_{m\_u} = 27^0$

$\omega_{cpu} = 3.44$ rad/s

$K_{dc\_u} = 2.5$ dB

Figure Q7.1 – Open Loop Uncompensated System Frequency Response in Question 7

Question 8

PART A (10 marks) - *System Stability in the s-domain and in the frequency domain: Nyquist Criterion*

Consider a unit feedback loop system under Proportional Control (gain K). The transfer function of its open loop is described as follows:

$$G_{open}(s) = K \cdot \frac{(1+s)}{s(s-5)}$$

1) Sketch a polar plot of the normalized open loop transfer function $\frac{G_{open}(j\omega)}{K}$; calculate all relevant coordinates, including the crossovers with the Imaginary and Real axis, and clearly indicate the direction of increasing frequency on the resulting polar plot.
2) Apply the Nyquist criterion of stability to establish the range of values of the Proportional Controller gain, K_p, that will result in a stable closed loop system response. NOTE: clearly show the chosen clockwise (CW) Γ path in the s-plane, and the resulting Nyquist Contour.

PART B (10 marks) – *State Space System Representations, Analytical Solution for Dynamic System Response.*

A certain system is described by a state space model:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -3 & -5 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix} \cdot u$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0] \cdot u$$

Find the complete solution, $y(t), t \geq 0$, if $u(t) = 1, t \geq 0$ (unit step input), and $x_1(0) = 0, x_2(0) = 0$.