PROFESSIONAL ENGINEERS OF ONTARIO

ANNUAL EXAMINATIONS – May 2016

07-Mec-B10 Finite Element Analysis

3 hours duration

INSTRUCTIONS:

1. **If doubt exists as to the interpretation of any of the questions, the candidate is urged to submit a clear statement of the assumption(s) made with the answer.**

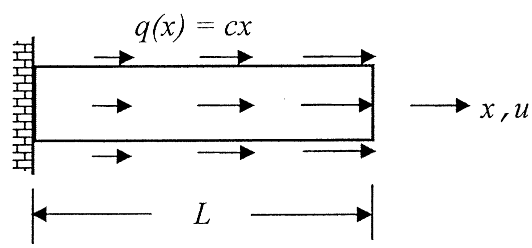
2. **This examination paper is open book; candidates are permitted to make use of any textbooks, references or notes.**

3. **Any non-communicating calculator is permitted. Candidates must indicate the type of calculator(s) that they have used by writing the name and model designation of the calculator(s) on the first inside left hand sheet of the first examination workbook.**

4. **Candidates are required to attempt any five questions. The questions are to be solved within the context of the finite element method.**

5. **The questions are equally weighted. Indicate which five questions are to be marked on the cover of the first examination workbook.**

Question 1. [20 marks] A cantilevered bar is loaded by a linearly varying distributed load $q(x) = cx$ as shown in the figure - note that c is a constant. The cross-sectional area and length of the bar are denoted by A and L, respectively, and it is made of a material with Young's modulus of elasticity E. The system governing equation can be written as



$$EA\frac{d^2u(x)}{dx^2} + cx = 0 \quad 0 < x < L$$

$$subject\ to:\ u(0) = 0 \quad and \quad EA\left.\frac{du(x)}{dx}\right|_{x=L} = 0$$

Use the collocation method to determine an approximate cubic polynomial solution with evaluation points at $x = \frac{1}{4}L$ and $x = \frac{3}{4}L$.

Question 2. [20 marks] A field variable $f(x,y) = x^2y^3$ is defined over a rectangular domain $\Omega = \{\Re^{2+}: 4 \le x \le 6, 2 \le y \le 8\}$. Given the expression

$$g = \int_2^8\int_4^6 x^2y^3dxdy$$

and assume the following bilinear interpolation shape functions are used to discretize the spatial/geometric variables x and y:

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta),\ N_2 = \frac{1}{4}(1+\xi)(1-\eta),\ N_3 = \frac{1}{4}(1+\xi)(1+\eta),\ N_4 = \frac{1}{4}(1-\xi)(1+\eta)$$

where $-1 \le \xi,\eta \le 1$ for the local coordinates ξ,η.

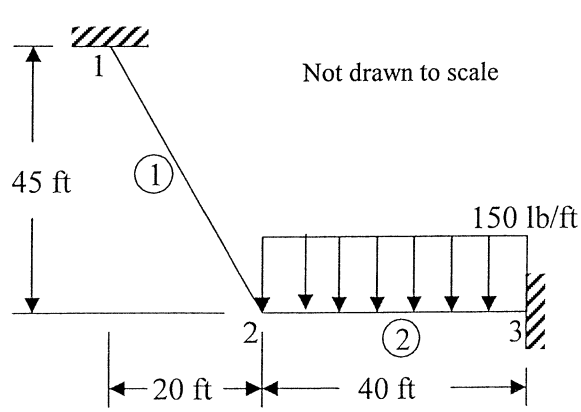
(a) [15 marks] Use the Gauss quadrature numerical integration method to evaluate g.

(b) [5 marks] Explain any similarity or difference between your answer and the exact solution $g = 51680$.

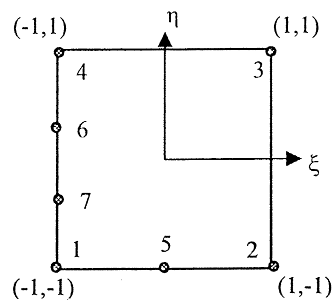
Question 3. [20 marks] A rigid plane frame arrangement is shown in the figure. The frame is fixed at the ends, identified as nodes 1 and 3, and supports a downward acting uniformly distributed load of 150 lb/ft. Take the Young's modulus of elasticity $E = 30 \times 10^6$psi, the cross-sectional area of each element of the frame $A = 10$ in^2, and the moment of inertia $I = 200$ in^4. Determine

(a) [12 marks] the displacements and rotations at node 2; and

(b) [8 marks] the forces in each element and the reactions.



Question 4. [20 marks]



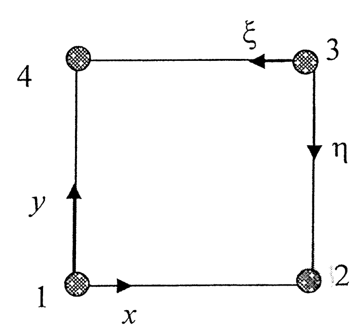
(a) [14 marks] Determine the shape functions (N_i, i = 1 to 7) of the seven-node transition element in natural/local coordinates (ξ, η) such that $-1 \leq \xi, \eta \leq 1$.
(b) [4 marks] Evaluate the shape function N_4 at the sixth node and the centroid of the element.
(c) [2 marks] Assume the field variables of the problem are displacement components denoted by u and v for the ξ and η directions, respectively. If the nodal displacement components are zero except $v_1 = v_2 = v_5 = -0.025$ mm, determine an expression for the field variables, u and v, in the natural/ local coordinates (ξ, η).

Question 5. [20 marks]

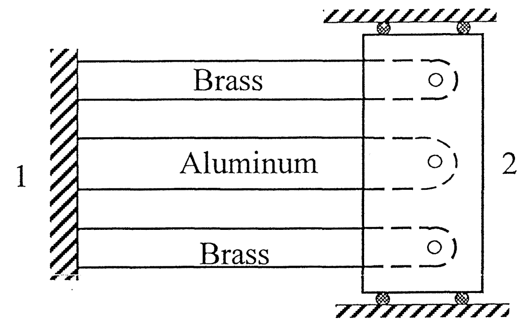
(a) [4 marks] Briefly explain the meaning of geometric isotropy in a sentence or two.
(b) [6 marks] Identify whether each of the following polynomial representation of a field variable variation in an element possess geometric isotropy. Explain.
(i) $u(x) = C_1 + C_2x + C_3x^2 + C_4x^4$
(ii) $u(x, y) = C_1 + C_2x + C_3y + C_4x^2 + C_5xy + C_6y^2 + C_7x^3 + C_8x^2y + C_9xy^2 + C_{10}y^3$
(iii) $u(x, y) = C_1 + C_2x + C_3y + C_4x^2 + C_5xy + C_6y^2 + C_7x^2y + C_8xy^2$
(c) [10 marks] Consider the square element below for which the field variable u is interpolated in the Cartesian x,y coordinate axes attached at node 1 as

$$u(x, y) = C_1 + C_2x + C_3y + C_4xy$$

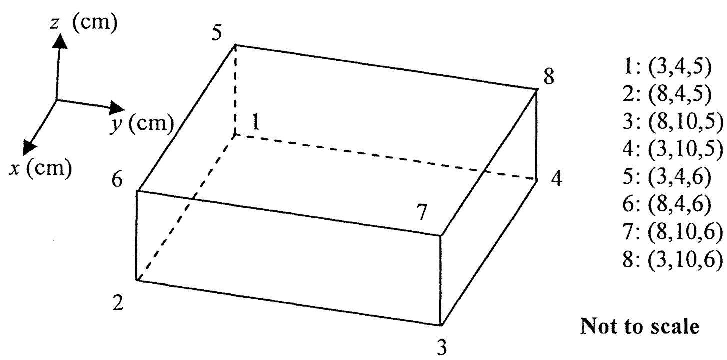
Assuming the length of each side of the element is L, use the ξ, η coordinate axes centred at node 3 to show that the element has geometric isotropy.



Question 6. [20 marks] The shown bar assemblage comprises two outer brass bars and an inner aluminum bar. The three-bar assemblage is subjected to a temperature drop of 20°C. Assume the lengths of each bar $L = 2.5$ m. For the aluminum bar, take $E_{alum} = 70$ GPa, $\alpha_{alum} = 23 \times 10^{-6}$ (mm/mm)/°C, and cross-sectional area $A_{alum} = 10 \times 10^{-4}$ m^2. For the brass bars, take $E_{brass} = 100$ GPa, $\alpha_{brass} = 20 \times 10^{-6}$ (mm/mm)/°C, and cross-sectional area $A_{brass} = 5 \times 10^{-4}$ m^2. Determine
(a) [14 marks] the displacement of node 2; and
(b) [6 marks] the stress in each of the three bars.



Question 7. [20 marks]



Not to scale

(a) [4 marks] Determine the shape functions (N_i, $i = 1$ to 8) of an eight-node hexahedron element in natural/local coordinates (ξ, η, ζ) such that $-1 \leq \xi, \eta, \zeta \leq 1$. The node numbering is identical to that shown in the above representative global element.
(b) [3 marks] Evaluate the shape function N_8 at the second node, the eighth node, and the centroid of the element.
(c) [3 marks] Assume the field variables of the problem are displacement components denoted by $u, v,$ and w in the ξ, η, and ζ directions, respectively. If the nodal displacement components are zero except $v_1 = v_2 = v_5 = v_6 = -0.025$ mm, compute the field variables in the natural/ local coordinates (ξ, η, ζ).
(d) [7 marks] Determine the Jacobian matrix and evaluate the Jacobian of the above element.
(e) [3 marks] Determine the normal strain $\varepsilon_x = \frac{\partial u}{\partial x}$ and shear strain $\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$ at the centre of the element.

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Marking Scheme of Examination Paper

Marking Scheme

1. 20 marks total [1 parts: 20 marks]
2. 20 marks total [2 parts: 15 marks and 5 marks, respectively]
3. 20 marks total [2 parts: 12 marks, and 8 marks, respectively]
4. 20 marks total [3 parts: 14 marks, 4 marks and 2 marks, respectively]
5. 20 marks total [3 parts: 4 marks, 6 marks, and 10 marks, respectively]
6. 20 marks total [2 parts: 14 marks and 6 marks, respectively]
7. 20 marks total [5 parts: 4 marks, 3 marks, 3 marks, 7 marks and 3 marks, respectively]