National Exams May 2013

07-Mec-B6, Advanced Fluid Mechanics

3 hours duration

Notes:

1. If doubt exists as to the interpretation of any question the candidate is urged to submit with the answer paper a clear statement of the assumptions made.
2. Candidates may use any non-communicating calculator. The exam is OPEN BOOK.
3. Answer and 2 of 3 questions in each of Part A and B.
4. Weighting: Part A: 40%, Part B: 60%. Part A: Each question is weighted (20). Part B: Each question is weighted (30).

Part A: Answer any 2 of the following 3 questions. Questions have equal weight (20).

Question A1: A spillway consists of a rectangular channel 100m wide. The flow rate of water is to be 3548 m^3/s at an average speed of 10m/s. You are to investigate the resulting hydraulic jump in the laboratory by using a scaled model. The model you have is a rectangular channel 1m wide and your pump can only generate an average speed of 1m/s of water.

a) What must be the Froude number of the flow upstream of the hydraulic jump?
b) What flow rate must you maintain in the laboratory set-up?
c) What is the flow level after the hydraulic jump in your laboratory and in the spillway?
d) What is the Froude number downstream of the jump in the laboratory and the spillway?

Question A2: A super-tanker is 360m long and has a width of 70m and a draft (depth of the hull below the surface) of 25m. Assuming that the drag is mainly due to the skin friction on the bottom and side faces and that the boundary layer develops naturally (i.e. is laminar at the beginning and has transition to turbulence at about Re_x=300,000), determine the force and power required to overcome the skin friction. The ship is travelling in sea water ($\rho = 1020$ kg/m^3;$\nu = 1.37 \times 10^{-6}$ m^2/s) at a speed of 24 km/hr. You may assume, for the turbulent section, that the velocity profile is given by the 1/7-law.

Question A3: A pitot-static tube is introduced in a supersonic flow. Determine the speed of the flowing (free) stream if the tube reads a stagnation pressure of 130 kPa, a static pressure of 110 kPa and a static temperature of 519K (in the proximity of the tube), for: (a) the gas is air (R = 287 J/kg-K, γ= 1.4); (b) the gas is Helium (R = 2077 J/kg-K, γ = 5/3).

Part B: Answer any 2 of the following 3 questions. Questions have equal weight (30).

<u>Question B1:</u> A pressurized, insulated tank is initially filled with air (γ = 1.4; R = 287 J/kg-K) at 10 MPz (absolute) and 30°C. The volume of the tank is 0.5 m^3. The tank has a valve, which when open is shaped like a convergent-divergent nozzle. The nozzle throat area is 1 mm^2 and the exit area is 2 mm^2.

The valve is opened and air exhausts to atmosphere (101.3 kPa, absolute). When the internal pressure of the tank reaches 2 MPa (absolute), the valve is closed again.

a) Determine the temperature of the air inside the tank when the final pressure of 2 MPa is reached. What is the exit speed and temperature of the air at this instant (just before the valve is closed)?
b) Determine the time required for the internal pressure to reach 2 MPa.

In answering these questions, assume that all frictional losses are negligible.

<u>Question B2:</u> A very large water pool is foreseen with a recirculating system consisting of a pipe which provides water at a rate of m = 1 m^3/s (per unit length). The pipe is located at a distance a = 3m from the pool floor. A water drain is located on the pool floor. The drain can be modelled as a sink of strength M. It drains all the water provided by the source m.

a) Determine a suitable function to model this flow. Verify that the floor is correctly modelled.
b) What must be the strength M of the sink used to represent the drain?
c) The system is monitored by measuring the pressure difference along the floor of the pool at the location b = 1m and c = 2m. For the given operating conditions, what will be this pressure difference?

For water, use ρ = 1000 kg/m^3. Neglect gravitational effects.

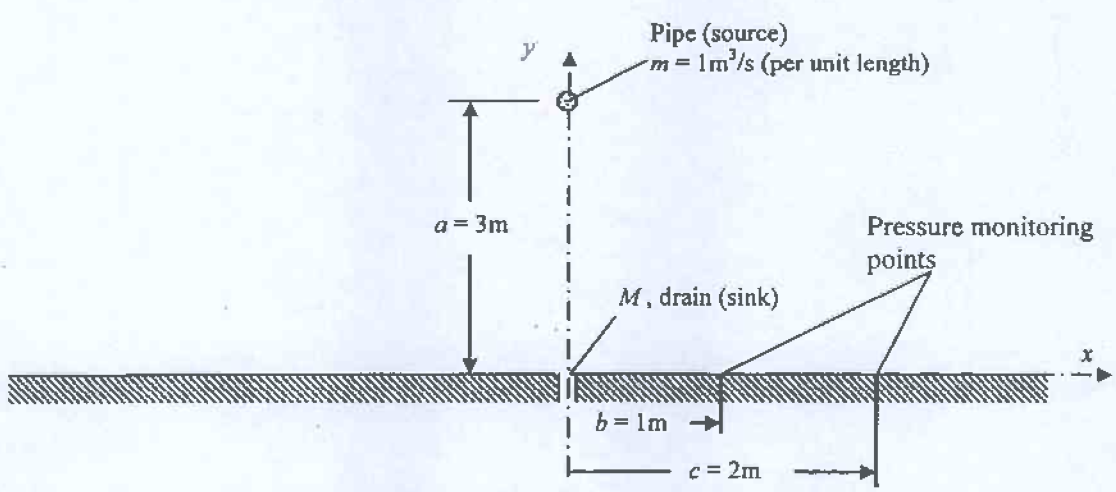


Figure B2: Schematic pool recirculating system.

<u>Question B3:</u> Two immiscible (i.e. do not mix) liquids flow in a very long, horizontal, two-dimensional channel (i.e. negligible spanwise gradients). The channel has a constant height $2h$ = 0.02m. The bottom fluid has a density of 1200 kg/m^3 and a dynamic viscosity of 0.01 Pa-s. The top fluid has a density of 1000 kg/m^3 and a dynamic viscosity of 0.001 Pa-s. The flow is driven by a constant pressure gradient of dP/dx = - 1 Pa/m. The liquids wet the solid walls, which are non-porous. The flow is steady.

a) State the boundary conditions.
b) Determine the velocity profile.
c) What is the shear stress at the walls and the liquid interface?
d) What is the flow speed at the interface?
e) Sketch the velocity profile. Determine where the maximum velocity occurs.

Hint: It is easier if you set the coordinate axis origin at the centre of the channel as shown.

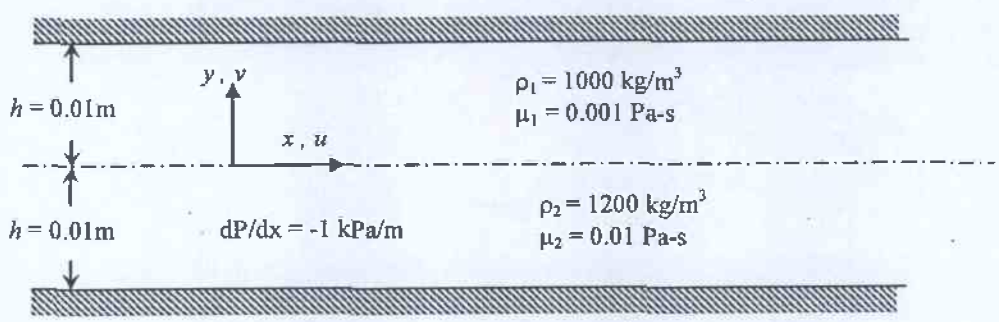


Figure B3: Sketch of the flow in a two-dimensional channel driven by dP/dx = -1kPa/m.

Aid Sheets

Compressible Flow:

Adiabatic flow: $$\frac{T_o}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

Isentropic flow: $$\frac{P_o}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}} \quad ; \quad \frac{\rho_o}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{1}{\gamma - 1}}$$

$$\dot{m} = \rho U A = \sqrt{\frac{\gamma}{R}} \cdot \frac{P_o}{\sqrt{T_o}} \cdot M \cdot \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{\gamma + 1}{2(\gamma - 1)}} A$$

Shock Relations: $$\frac{P_2}{P_1} = \frac{1 + \gamma M_1^{\,2}}{1 + \gamma M_2^{\,2}} = \frac{2\gamma M_1^{\,2}}{\gamma + 1} - \frac{\gamma - 1}{\gamma + 1} \qquad M_2^2 = \frac{M_1^2 + \frac{2}{\gamma - 1}}{\frac{2\gamma}{\gamma - 1} M_1^2 - 1}$$

Boundary Layer Equations:

$$\frac{d}{dx}\left(U_o^2 \theta\right) + U_o \frac{dU_o}{dx} \cdot \delta^* = \tau_w / \rho$$

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy$$

$$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy$$

Laminar flow: $$C_{fx} = \frac{\tau_w(x)}{\frac{1}{2}\rho U_\infty^2} = \frac{0.67}{\mathrm{Re}_x^{1/2}}$$

Turbulent flow: $$C_{fx} = \frac{\tau_w(x)}{\frac{1}{2}\rho U_\infty^2} = \frac{0.0266}{\mathrm{Re}_x^{1/7}}$$

Conservation Equations for Cartesian Coordinate system

Continuity Equation:

$$\frac{D\rho}{Dt} + \rho\left(\nabla \bullet \vec{U}\right) = 0$$

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} \quad \text{and} \quad \nabla \bullet \vec{U} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

Linear Momentum:

x-direction: $$\rho\frac{\partial u}{\partial t} + \rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} + \rho w\frac{\partial u}{\partial z} = -\frac{\partial P}{\partial x} + \rho g_x + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}$$

y-direction: $$\rho\frac{\partial v}{\partial t} + \rho u\frac{\partial v}{\partial x} + \rho v\frac{\partial v}{\partial y} + \rho w\frac{\partial v}{\partial z} = -\frac{\partial P}{\partial y} + \rho g_y + \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}$$

z-direction: $$\rho\frac{\partial w}{\partial t} + \rho u\frac{\partial w}{\partial x} + \rho v\frac{\partial w}{\partial y} + \rho w\frac{\partial w}{\partial z} = -\frac{\partial P}{\partial z} + \rho g_z + \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}$$

$$\tau_{xx} = 2\mu\frac{\partial u}{\partial x} - \frac{2}{3}\mu\nabla \bullet \vec{U} \qquad \tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \qquad \tau_{xz} = \tau_{zx} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)$$

$$\tau_{yy} = 2\mu\frac{\partial v}{\partial y} - \frac{2}{3}\mu\nabla \bullet \vec{U} \qquad \tau_{yz} = \tau_{zy} = \mu\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) \qquad \tau_{zz} = 2\mu\frac{\partial w}{\partial z} - \frac{2}{3}\mu\nabla \bullet \vec{U}$$

Potential Flow

Stream functions:

Uniform Flow: $$\Psi = U_o y = U_o r \sin\theta$$

Source Flow: $$\Psi = \frac{m}{2\pi}\tan^{-1}\left(\frac{y-y_o}{x-x_o}\right) = \frac{m}{2\pi}\theta$$

Vortex Flow: $$\Psi = -\frac{\Gamma}{4\pi}\ln\left[(x-x_o)^2+(y-y_o)^2\right] = -\frac{\Gamma}{2\pi}\ln r$$

Doublet Flow: $$\Psi = -\frac{\lambda(y-y_o)}{(x-x_o)^2+(y-y_o)^2} = -\lambda\frac{\sin(\theta)}{r}$$

Potential functions:

Uniform Flow: $$\Phi = U_o x = U_o r\cos\theta$$

Source Flow: $$\Phi = \frac{m}{4\pi}\ln\left[(x-x_o)^2+(y-y_o)^2\right] = \frac{m}{2\pi}\ln r$$

Vortex Flow: $$\Phi = \frac{\Gamma}{2\pi}\tan^{-1}\left(\frac{y-y_o}{x-x_o}\right) = \frac{\Gamma}{2\pi}\theta$$

Doublet Flow: $$\Phi = \frac{\lambda(x-x_o)}{(x-x_o)^2+(y-y_o)^2} = \lambda\frac{\cos(\theta)}{r}$$

Velocity relationships:

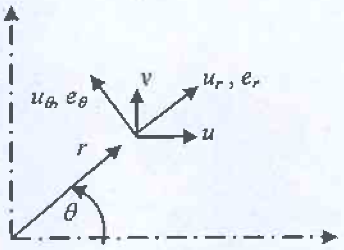
$$u = \frac{\partial\Phi}{\partial x} = \frac{\partial\Psi}{\partial y} \qquad v = \frac{\partial\Phi}{\partial y} = -\frac{\partial\Psi}{\partial x}$$

$$u_r = \frac{\partial\Phi}{\partial r} = \frac{1}{r}\frac{\partial\Psi}{\partial\theta} \qquad u_\theta = \frac{1}{r}\frac{\partial\Phi}{\partial\theta} = -\frac{\partial\Psi}{\partial r}$$

Transformation between Coordinate System

$r = \sqrt{x^2 + y^2}$ $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$x = r\cos\theta$ $y = r\sin\theta$



$\vec{e}_r = \cos\theta\,\vec{i} + \sin\theta\,\vec{j}$ $\vec{e}_\theta = -\sin\theta\,\vec{i} + \cos\theta\,\vec{j}$

$u_r = u\cos\theta + v\sin\theta$ $u_\theta = -u\sin\theta + v\cos\theta$

$u = u_r\cos\theta - u_\theta\sin\theta$ $v = u_r\sin\theta + u_\theta\cos\theta$