National Exams May 2016

07-Mec-B12 Robot Mechanics

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit, with the answer paper, a clear statement of any assumptions made.

2. This is a CLOSED BOOK EXAM with one 8.5"x11" formula sheet allowed written on both sides. The formula sheet must not hold any solutions of examples and must be handed in with the exam submission. A non-programmable and approved calculator is permitted.

3. FIVE (5) questions constitute a complete exam paper. The first five questions as they appear in the answer book will be marked.

4. Each question is of equal value.

5. Question value in marks is shown in parentheses at the end of each question part.

6. Logical order, clarity, and organization of the solution steps are important.

Nomenclature:

The unit vectors of coordinate system A are denoted by $\hat{X}_A, \hat{Y}_A, \hat{Z}_A$.

The leading superscript such as A in ${}^{A}V$ indicates the coordinate system to which the vector V is referenced.

The leading subscript and superscript such as A and B in ${}^{B}_{A}T$ indicate the transformation of coordinate frame A relative to B by matrix T.

(20)

1. A position vector is given by ${}^{A}P = [-5 \quad 3 \quad 4]^T$ and a velocity vector by ${}^{B}V = [10 \quad 20 \quad -15]^T$. Given the Homogeneous Transformation

$$
{}^{A}_{B}T = \begin{bmatrix} \sqrt{3}/2 & -0.5 & 0 & 11 \\ 0.5 & \sqrt{3}/2 & 0 & -3 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix};
$$

Compute:

a) ${}^{B}P$ **(12)**

b) ${}^{A}V$ **(8)**

(20)

2. The following frame definitions are given

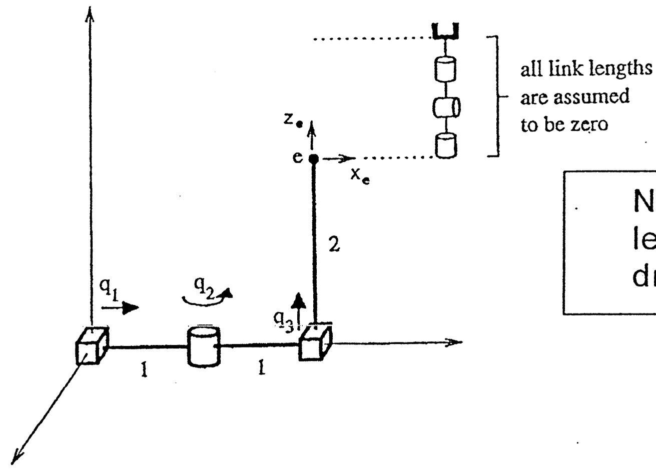
$$
{}^{U}_{A}T = \begin{bmatrix} 1 & 0 & 0 & 11 \\ 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad {}^{B}_{A}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -0.5 & 10 \\ 0 & 0.5 & \sqrt{3}/2 & -20 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

$$
{}^{C}_{U}T = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & \sqrt{2}/2 & -\sqrt{2}/2 & -3 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

a) Draw a frame diagram to show their arrangement graphically. **(5)**

b) Solve for ${}^{B}_{C}T$. **(15)**

(20)
3. Consider the manipulator, shown in Figure 1 below, in its rest position with positive joint motion in the directions indicated.



Note: link lengths are not drawn to scale.

Figure 1

a) Using the axes provided for frames 0 and 3, show your axis assignments on a sketch. **(5)**
b) Derive the DH table for the manipulator without the wrist (i.e. to the end point *e*). **(5)**
c) Calculate ${}^0_3T$. **(10)**

(20)
4. For the manipulator shown in Figure 1 problem 3, without wrist, solve the problem of inverse kinematics, i.e. find all possible closed-form solutions for joint variables q_1, q_2, and q_3 in terms of the desired endpoint coordinates $[x_e \quad y_e \quad z_e]^{\mathrm{T}}$. Note: there is no joint limit.

(20)
5. Attach a spherical wrist to the manipulator shown in Figure 1, problem 3. With all wrist link lengths assumed to be zero, ${}^3_6R$ corresponding to Z-Y-Z Euler angles for the resulting 6-DOF manipulator is shown below:

$$R_6^3 = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 \\ -s_5c_6 & s_5s_6 & c_5 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{6}}{4} & -\frac{\sqrt{6}}{4} \\ -\frac{\sqrt{3}}{2} & \frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$$

Find all solutions $[q_4 \quad q_5 \quad q_6]^{\mathrm{T}}$ to the inverse orientation problem.

(20)
6. For the manipulator, shown in Figure 1 problem 3, without wrist:

   a) Find the Jacobian. **(12)**

Assuming the following motion limits for the joints:

$$0 < q_1 < 1,\ 0 < q_2 < 360°,\ 0 < q_3 < 1,$$

   b) Sketch the reachable workspace of the manipulator without the wrist as projections on the x_0-y_0, y_0-z_0 and x_0-z_0 planes. **(6)**

   c) What can you say about the dexterous workspace of the manipulator without the wrist? **(2)**

(20)
7. A certain two-link planar manipulator has the following Jacobian:

$$^0J(\Theta) = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{bmatrix}.$$

   a) Ignoring gravity, what are the joint torques required so that the manipulator will apply a static force vector $^0F = 20\hat{X}_o$? **(10)**
   b) Given the above Jacobian, investigate the singular configuration(s). **(10)**

Marking Scheme

1.	20 marks total	(12 marks for Part (a) and 8 marks for Part (b))
2.	20 marks total	(5 marks for Part (a) and 15 marks for part (b))
3.	20 marks total	(5 marks for Part (a), 5 marks for Part (b), 10 marks for part (c))
4.	20 marks total	(marks are equally given to the number of solutions)
5.	20 marks total	(marks are equally given to the number of solutions)
6.	20 marks total	(12 marks for Part (a), 6 marks for Part (b), 2 marks for Part (c))
7.	20 marks total	(10 marks for Part (a), (5 marks for each torque), 10 marks for Part (b) (5 for each configuration)