National Exams December 2016

07-Elec-A2, Systems & Control

3 hours duration

NOTES:

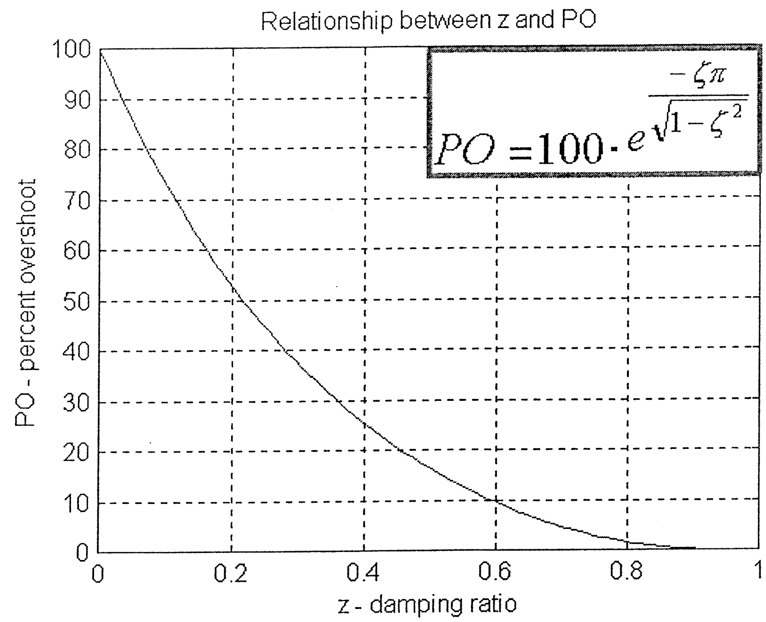
1. This is a CLOSED BOOK EXAM. Only an approved Casio or Sharp calculator is permitted. Candidates are allowed to use a double-sided, **handwritten**, 8.5 by 11" formula and notes sheet. Otherwise, there are no restrictions on the content of the formula sheet. The sheet must be signed and submitted together with the examination paper.
2. If doubt exists as to interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
3. Five (5) questions constitute a complete paper. **YOU ARE REQUIRED TO COMPLETE QUESTION 1 AND QUESTION 2.** Choose three (3) more questions out of the remaining six. Clearly indicate the questions which should be marked - otherwise, only the first five answers provided will be marked. Each question is of equal value. Weighting of topics within each question is indicated. Partial marks will be given. Clearly show steps taken in answering each question.
4. **Use exam booklets to answer the questions - clearly indicate which question is being answered.**

YOUR MARKS		
QUESTIONS 1 AND 2 ARE COMPULSORY:		
Question 1	20	
Question 2	20	
CHOOSE THREE OUT OF THE REMAINING SIX QUESTIONS:		
Question 3	20	
Question 4	20	
Question 5	20	
Question 6	20	
Question 7	20	
Question 8	20	
TOTAL:	100	

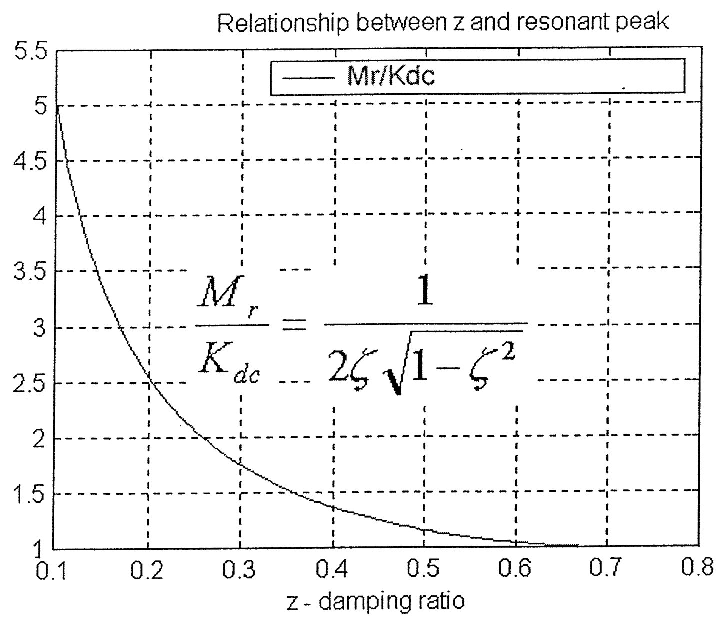
A Short Table of Laplace Transforms

Laplace Transform	Time Function
1	$\sigma(t)$
$\frac{1}{s}$	$1(t)$
$\frac{1}{(s)^2}$	$t \cdot 1(t)$
$\frac{1}{(s)^{k+1}}$	$\frac{t^k}{k!} \cdot 1(t)$
$\frac{1}{s+a}$	$e^{-at} \cdot 1(t)$
$\frac{1}{(s+a)^2}$	$te^{-at} \cdot 1(t)$
$\frac{a}{s(s+a)}$	$(1-e^{-at}) \cdot 1(t)$
$\frac{a}{s^2+a^2}$	$\sin at \cdot 1(t)$
$\frac{s}{s^2+a^2}$	$\cos at \cdot 1(t)$
$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at} \cdot \cos bt \cdot 1(t)$
$\frac{b}{(s+a)^2+b^2}$	$e^{-at} \cdot \sin bt \cdot 1(t)$
$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$\left(1-e^{-at} \cdot \left(\cos bt + \frac{a}{b} \cdot \sin bt\right)\right) \cdot 1(t)$
$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \cdot \sin\left(\omega_n\sqrt{1-\zeta^2}t\right) \cdot 1(t)$
$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$\left(1-\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \cdot \sin\left(\omega_n\sqrt{1-\zeta^2}t+\cos^{-1}\zeta\right)\right) \cdot 1(t)$
$F(s) \cdot e^{-Ts}$	$f(t-T) \cdot 1(t)$
$F(s+a)$	$f(t) \cdot e^{-at} \cdot 1(t)$
$sF(s)-f(0+)$	$\frac{df(t)}{dt}$
$\frac{1}{s}F(s)$	$\int_{0+}^{+\infty} f(t)dt$

Useful Plots & Second Order Model



PO vs. Damping Ratio



Resonant Peak vs. Damping Ratio

Second Order Model:

$$G_m(s) = K_{dc}\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

ζ – Damping Ratio (zeta), of the model

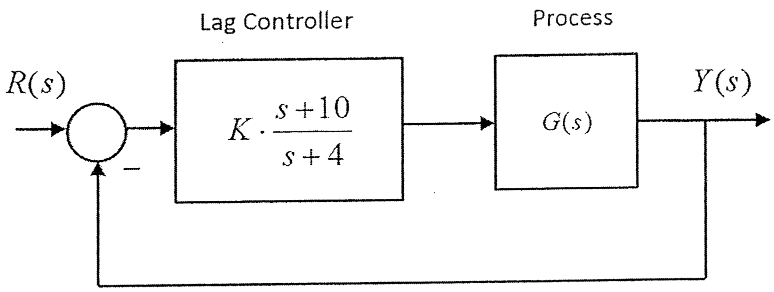
ω_n – Frequency of Natural Oscillations of the model

K_{dc} – DC Gain of the model

Question 1 (Compulsory)

System Stability in the s-Domain and in the Frequency Domain: Bode Plots, Root Locus Plots, Routh-Hurwitz Criterion of Stability.

A unit feedback control system is to be stabilized using a Lag Controller, as shown in Figure Q1.1. The Lag Controller has an adjustable Gain, K.



The process transfer function is described as follows:

$$G(s) = \frac{10}{s^2 + 4s + 8}$$

Figure Q1.1

Your task is to investigate the stability of the compensated closed loop system by finding: a) the value of the Controller Gain at which the closed loop system becomes marginally stable ($K = K_{crit}$) and the corresponding frequency or frequencies of marginally stable oscillations (ω_{osc}), and b) the range or ranges of safe operating gains for the Proportional Controller.

1) **(8 marks)** Apply the Routh-Hurwitz Criterion of Stability to calculate the critical value (or values) of the gain, K_{crit} , the corresponding frequency (or frequencies) of marginally stable oscillations, ω_{osc}, and to establish the range of safe operating gains.

2) **(7 marks)** The Root Locus sketch for the system in question is provided in Figure Q1.2. Use it to read off the frequency or frequencies (ω_{osc}) corresponding to the crossover(s) of the Root Locus with the Imaginary axis. Next, use the Root Locus Magnitude Criterion (shown below) to determine the corresponding values of critical gains (K_{crit}). Finally, use the Root Locus sketch to interpret the corresponding range or ranges of safe operating gains.

Magnitude Criterion: $K = \frac{1}{|G(s^*)|}$ where s^* is a coordinate of the crossover with the Im axis

3) **(5 marks)** The frequency response (Bode plot) for the process G(s) is shown in Figure Q1.3. Use it to verify the above calculations of the critical value (or values) of the gain, K_{crit} , and the corresponding frequency (or frequencies) of marginally stable oscillations, ω_{osc}.

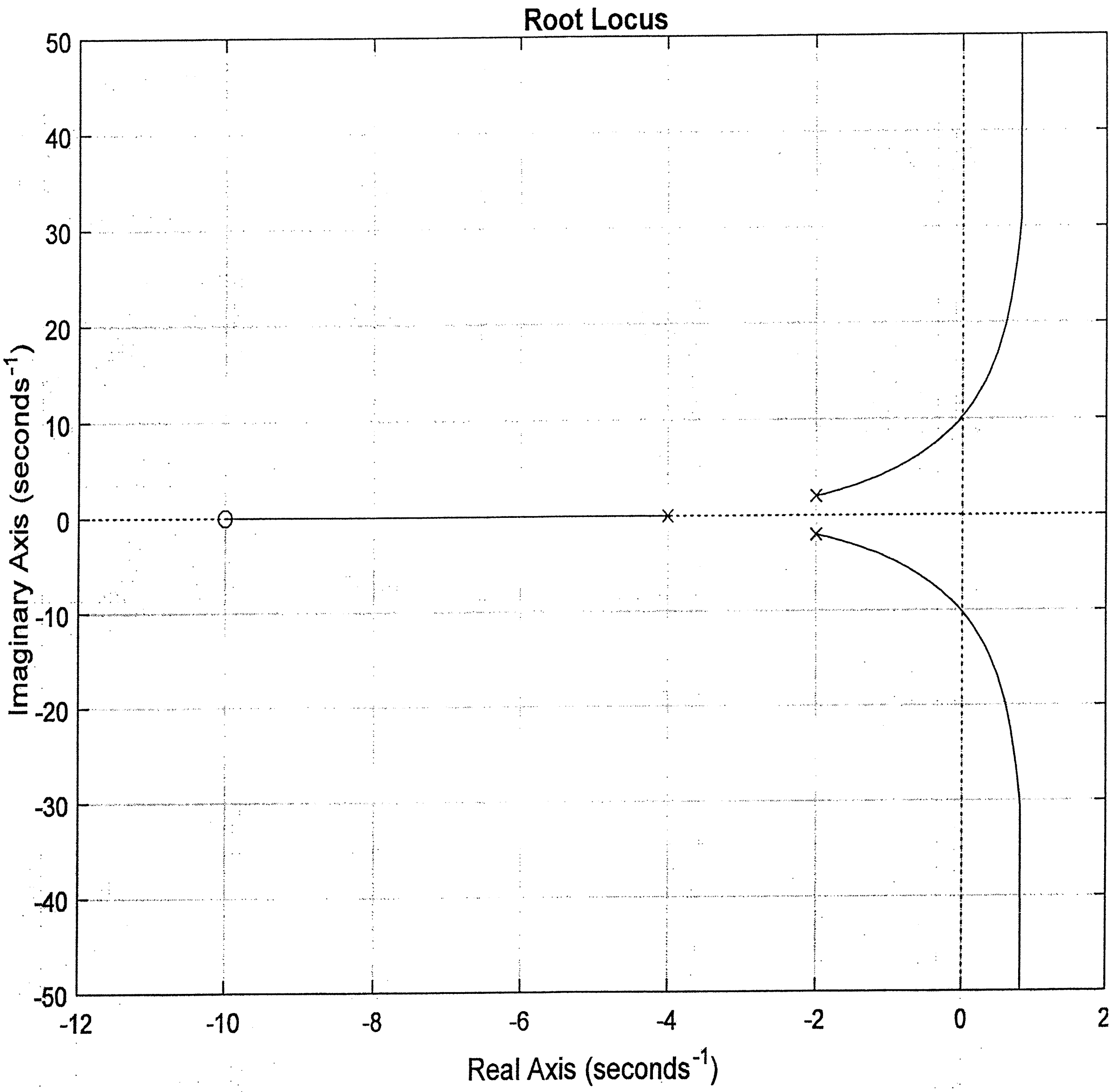


Figure Q1.2 – Root Locus Plot for the System in Question 1

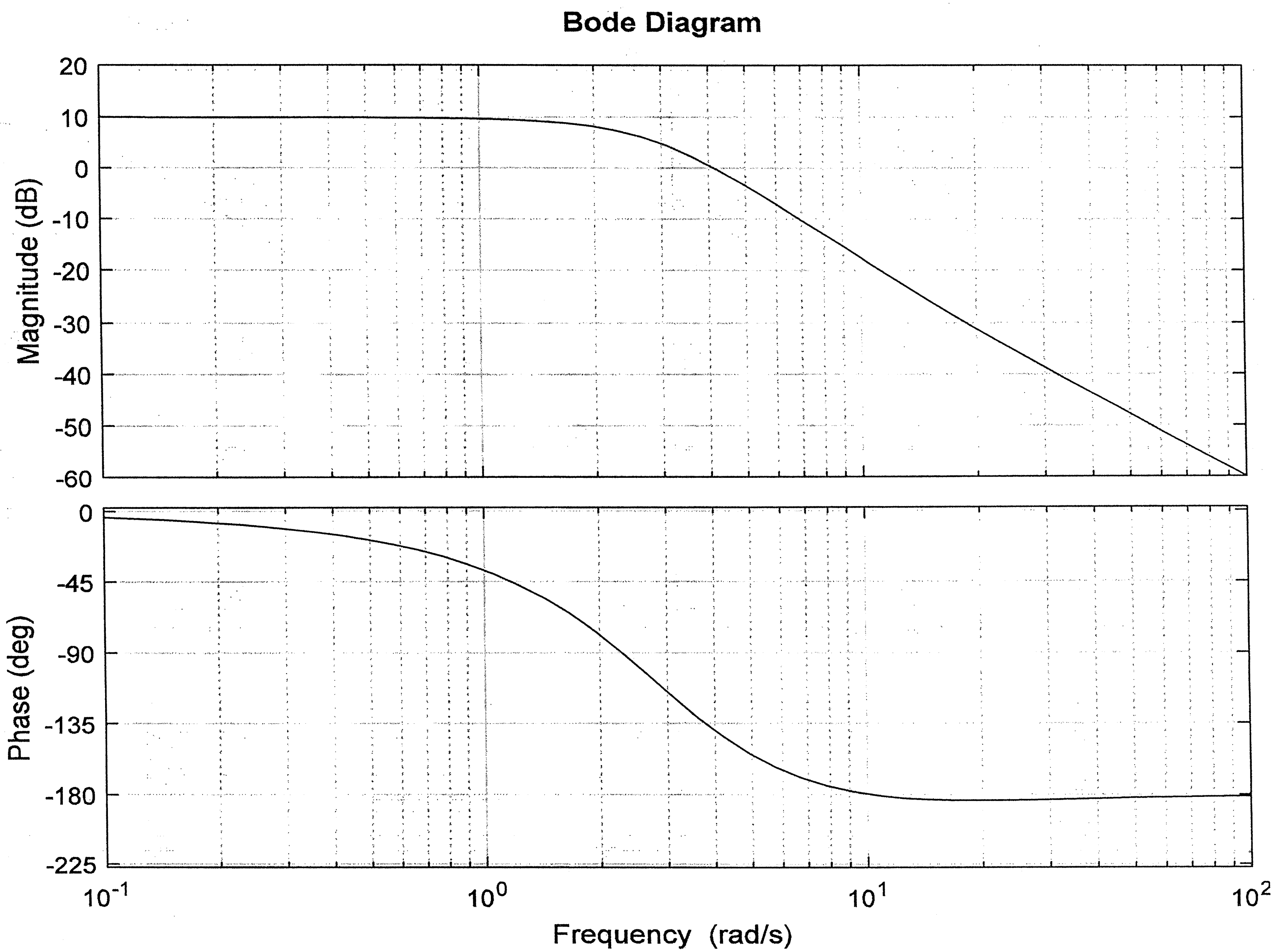


Figure Q1.3 – Frequency Response of G(s) in Question 1

Question 2 (Compulsory)

System Stability in the Frequency Domain: Polar Plots, Nyquist Criterion of Stability.

PART A (10 marks)

Consider the same unit feedback control system shown in Figure Q1.1 in Question 1. Use the information contained in the Bode Plots in Figure Q1.3 to sketch a polar plot of the normalized open loop transfer function, $\frac{G_{open}(j\omega)}{K}$ - read off the crossovers with the Imaginary and Real axis, and indicate these crossovers on the resulting polar plot. As well, clearly indicate the direction of increasing frequency on the plot.

Next, apply the Nyquist criterion of stability to establish the range of values of the Proportional Controller gain, K_p, that will result in a stable closed loop system response. NOTE: clearly show the chosen clockwise (CW) Γ path in the s-plane, and the resulting Nyquist Contour.

PART B (10 marks)

Consider now a unit feedback loop system under Proportional Control (gain K). The transfer function of its open loop is described as follows:

$$G_{open}(s) = K \cdot \frac{(1+s)}{s(s-2)}$$

Sketch a polar plot of the normalized open loop transfer function, $\frac{G_{open}(j\omega)}{K}$; note that, unlike in Part A, you do not have the frequency response plots available to read off the coordinates of the crossovers with the Imaginary and Real axis, and thus you have to calculate these crossovers using the Fourier Transfer Function $G_{open}(j\omega)$. Clearly indicate the direction of increasing frequency on the resulting polar plot.

Next, apply the Nyquist criterion of stability to establish the range of values of the Proportional Controller gain, K_p, that will result in a stable closed loop system response. NOTE: clearly show the chosen clockwise (CW) Γ path in the s-plane, and the resulting Nyquist Contour.

Question 3

State Space vs. Transfer Function Representations, Mason's Gain Formula.

PART A (10 marks)

The transfer function of a single-input single-output (SISO) system is given by:

$$G(s) = \frac{4s^3 + 25s^2 + 45s + 34}{s^3 + 6s^2 + 10s + 8}$$

Sketch one of the possible state realization diagrams for G(s) and obtain the resulting state equations:

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$$
$$\mathbf{y} = \mathbf{Cx} + \mathbf{Du}$$

Clearly indicate values of each matrix in the state equations.

PART B (10 marks)

If the state space description of a certain process is as follows, find its transfer function G(s).
HINT: Sketch the state realization diagram and then use the Mason's Gain Formula.

$$G(s) = \frac{Y(s)}{U(s)}$$

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} -2 & 3 & 2 \\ 0 & -3 & 2 \\ 0 & 0 & -4 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot u$$

$$y = \begin{bmatrix} 2 & 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Question 4

2nd Order Dominant Poles Models in s-domain and in frequency domain.

A certain closed loop control system is shown in Figure Q4.1:

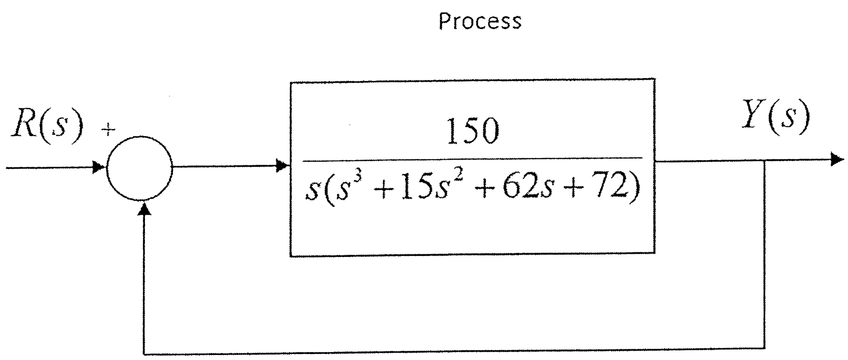


Figure Q4.1 – Control System in Question 4

The closed loop transfer function of the system is already calculated, and factored out, for you:

$$G_{cl}(s) = \frac{Y(s)}{R(s)} = \frac{150}{s^4 + 15s^3 + 62s^2 + 72s + 150}$$
$$= \frac{150}{(s + 8.35)(s + 6.07)(s^2 + 0.58s + 2.96)}$$

1) **(8 marks)** Determine a second order Closed Loop Model that would be appropriate for this system – refer to it as $G_{m1}(s)$.

2) **(8 marks)** Next, Figure Q4.2 shows two frequency response plots – one for the open loop transfer function (as shown in Figure Q4.1), and one for the closed loop transfer function, $G_{cl}(s)$. Label the two plots correctly, and then use them to determine two more second order Closed Loop Models:

   i) One based on the Open Loop frequency response – let's call it $G_{m2}(s)$;
   ii) The other one based on the Closed Loop frequency response– let's call it $G_{m3}(s)$

3) **(4 marks)** Finally, contrast $G_{m1}(s)$ with the two models obtained from the frequency responses. How do these models compare? Comment briefly.

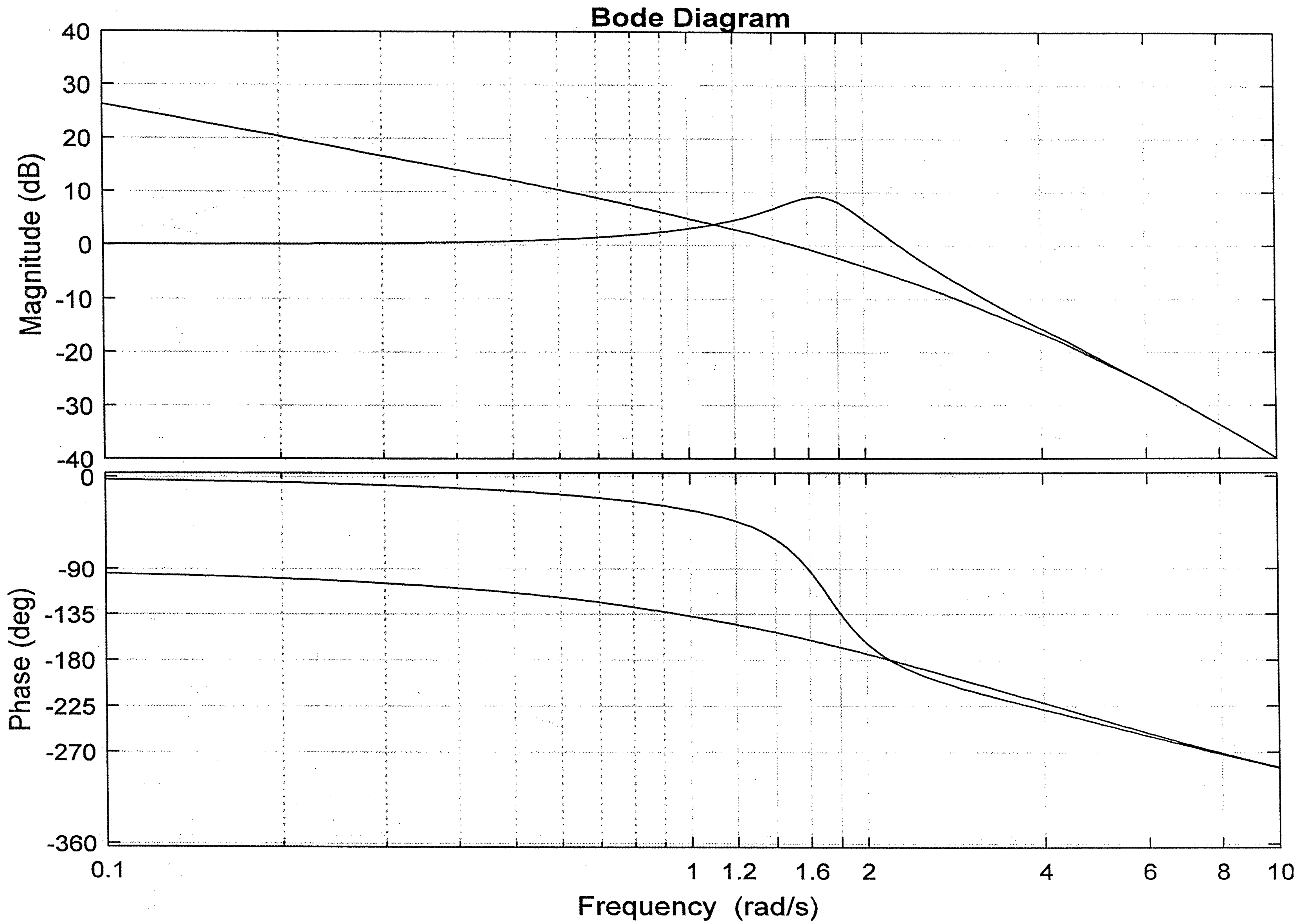


Figure Q4.2 - Open and Closed Loop Frequency Response Plots of the System in Question 4

Question 5

Root Locus Analysis and Gain Selection, Stability, Second Order Model, Step Response Specifications.

A unit feedback control system is to be stabilized using a Proportional Controller, as shown in Figure Q5.1.

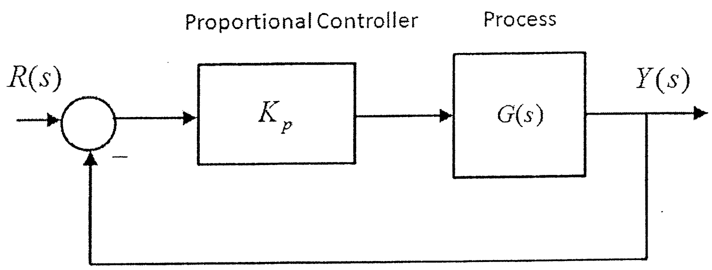


Figure Q5.1

The process transfer function is described as follows:

$$G(s) = \frac{50}{s(s+5)(s+10)}$$

1) **(10 marks)** Sketch the Root Locus for the system, in the space provided in Figure Q5.2. Calculate all relevant coordinates: asymptotic angles, break-in/away points, the location of the centroid and the coordinates of the crossover with the Imaginary axis, i.e. ω_{osc} and the corresponding value of the critical gain, K_{crit}, at which the system becomes marginally stable.

2) **(7 marks)** It is required that the unit step response of the Closed Loop system exhibits Percent Overshoot of approximately 5%. Determine the corresponding Proportional Gain value, K_{op}, and calculate an estimate of the following specs: Settling Time, $T_{settle(\pm 5\%)}$, Rise Time, $T_{rise(0-100\%)}$, and Steady State Error, $e_{ss(step\%)}$.

3) **(3 marks)** Finally, briefly comment on any possible differences between the expected system response (i.e. of the dominant poles model) and the actual system response.

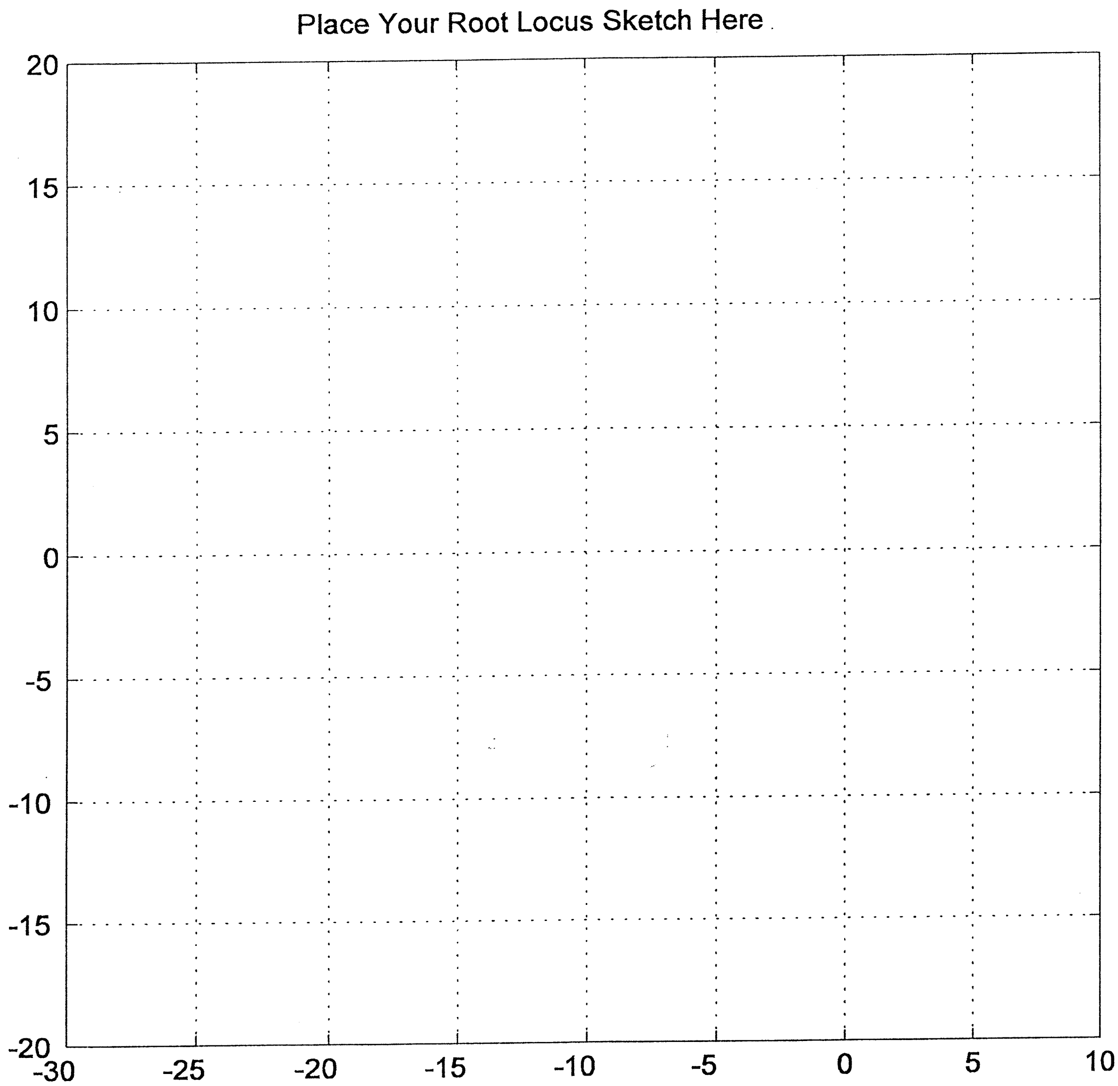


Figure Q5.2 – Root Locus of the System in Question 5

Question 6

Frequency Response vs. Transfer Function Models – properties of Bode Plots, corner frequencies, standard slopes, and their application to identifying the dynamic of processes.

A frequency response of a certain industrial process was obtained experimentally, and its magnitude plot (in dB) is shown in Figure Q6.1. It is known that the process dynamics include three poles (a complex conjugate pair and a real one) as well as one zero, all in LHP. Thus, the process model, $G_m(s)$, can be described by the following equation:

$$G_m(s) = K_{dc} \cdot \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \cdot \frac{s+z}{z} \cdot \frac{p}{s+p}$$

Use the plot in Figure Q6.1 to determine the corner frequencies corresponding to the real pole ($-p$)and the real zero ($-z$). Next, determine the damping ratio and the frequency of natural oscillations (ζ, ω_n) of the complex conjugate pair of poles, as well as the overall DC gain of the system (K_{dc}). Provide a rationale for your choices of parameters, as well as calculations of ζ, ω_n, based on the location and magnitude of the resonant peak.

Next, write the final transfer function of the process model, $G_m(s)$, in a polynomial ratio form:

$$G_m(s) = \frac{a_1 s + a_0}{s^3 + b_2 s^2 + b_1 s + b_0}$$

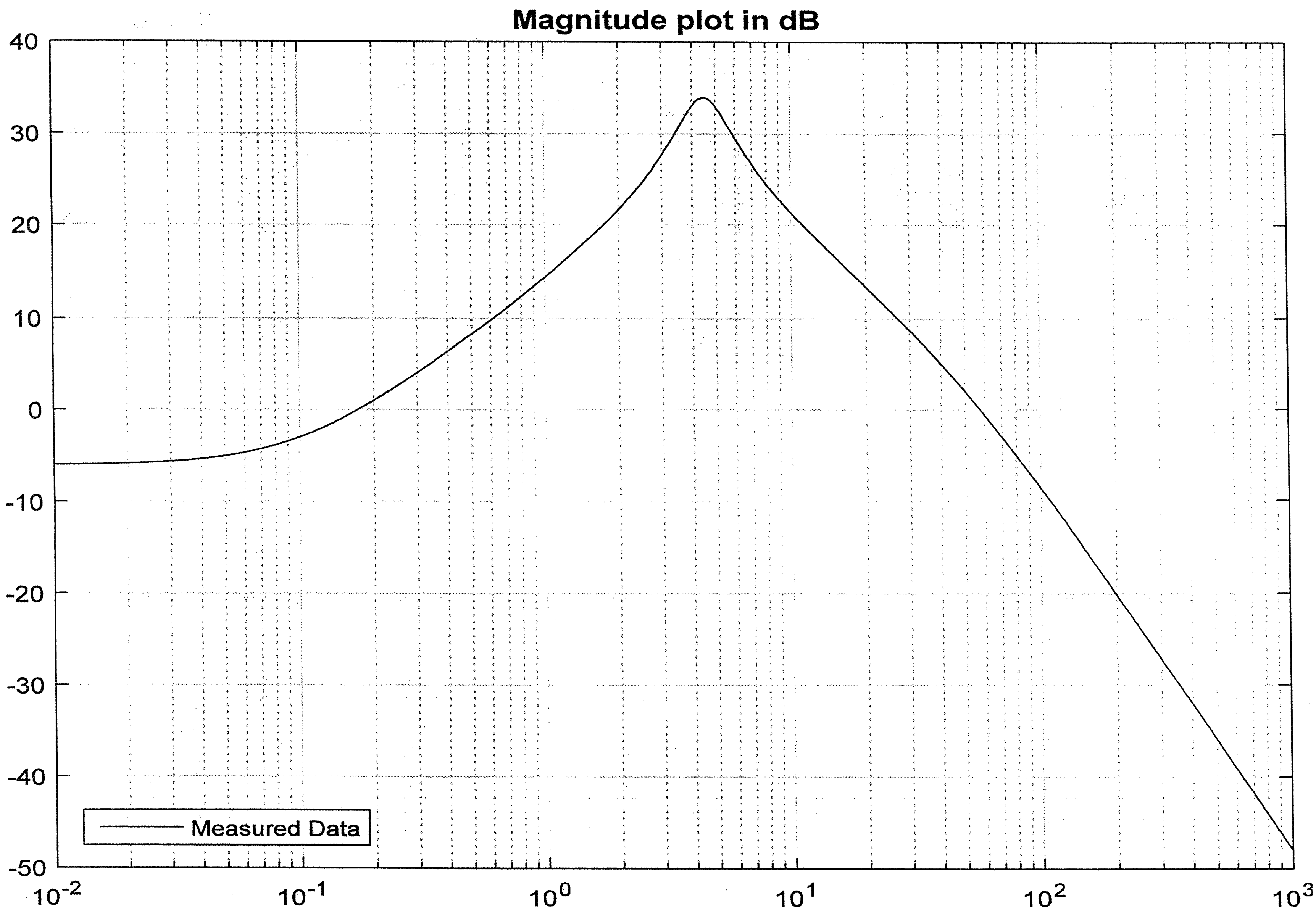


Figure Q6.1 – Magnitude Plot of Frequency Response in Question 6

Question 7

Controller design in Frequency Domain – Lag Controller, Step Response Specifications.

Consider a unit feedback closed loop control system, as shown in Figure Q7.1.

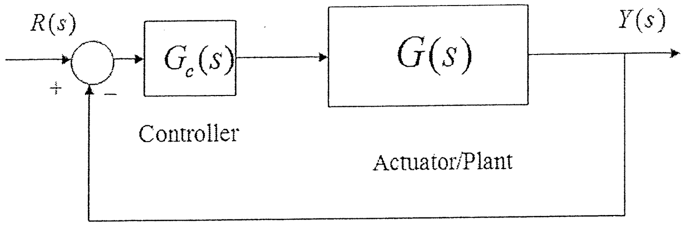


Figure Q7.1

The process transfer function is described as follows:

$$G(s) = \frac{30(s+2)}{(s+0.1)^2(s+20)^2}$$

The system is to operate under **Lag Control.** The Lag Controller transfer function is as follows:

$$G_c(s) = K_c \cdot \frac{\tau\alpha s+1}{\tau s+1} = \frac{a_1 s + a_0}{b_1 s + 1}$$

Where τ is the so-called Lag Time Constant and $\alpha < 1$. Open loop frequency response plots of G(s) are shown in Figure Q7.2. The closed loop performance requirements are:

- The Steady State Error for the unit step input for the compensated closed loop system is one half of the Steady State Error for the uncompensated system.
- Percent Overshoot is approximately 10%.

1) **(5 marks)** Read off what are the current values of the Phase margin, Φ_m, and the Crossover Frequency, ω_{cp}. Based on these, estimate the uncompensated closed loop step response specs: Percent Overshoot, PO, Steady State Error, $e_{ss(step\%)}$, and Settling Time, $T_{settle(\pm 2\%)}$.

2) **(3 marks)** Based on the specifications, calculate the required values of the Phase Margin for the compensated system, Φ_{mc}, and the DC gain of the controller, K_{dc}.

3) **(10 marks)** Design the Lag Controller such that it meets the closed loop response requirements.

4) **(2 marks)** Estimate the compensated closed loop step response specs: Percent Overshoot, PO, Steady State Error, $e_{ss(step\%)}$, and Settling Time, $T_{settle(\pm 2\%)}$.

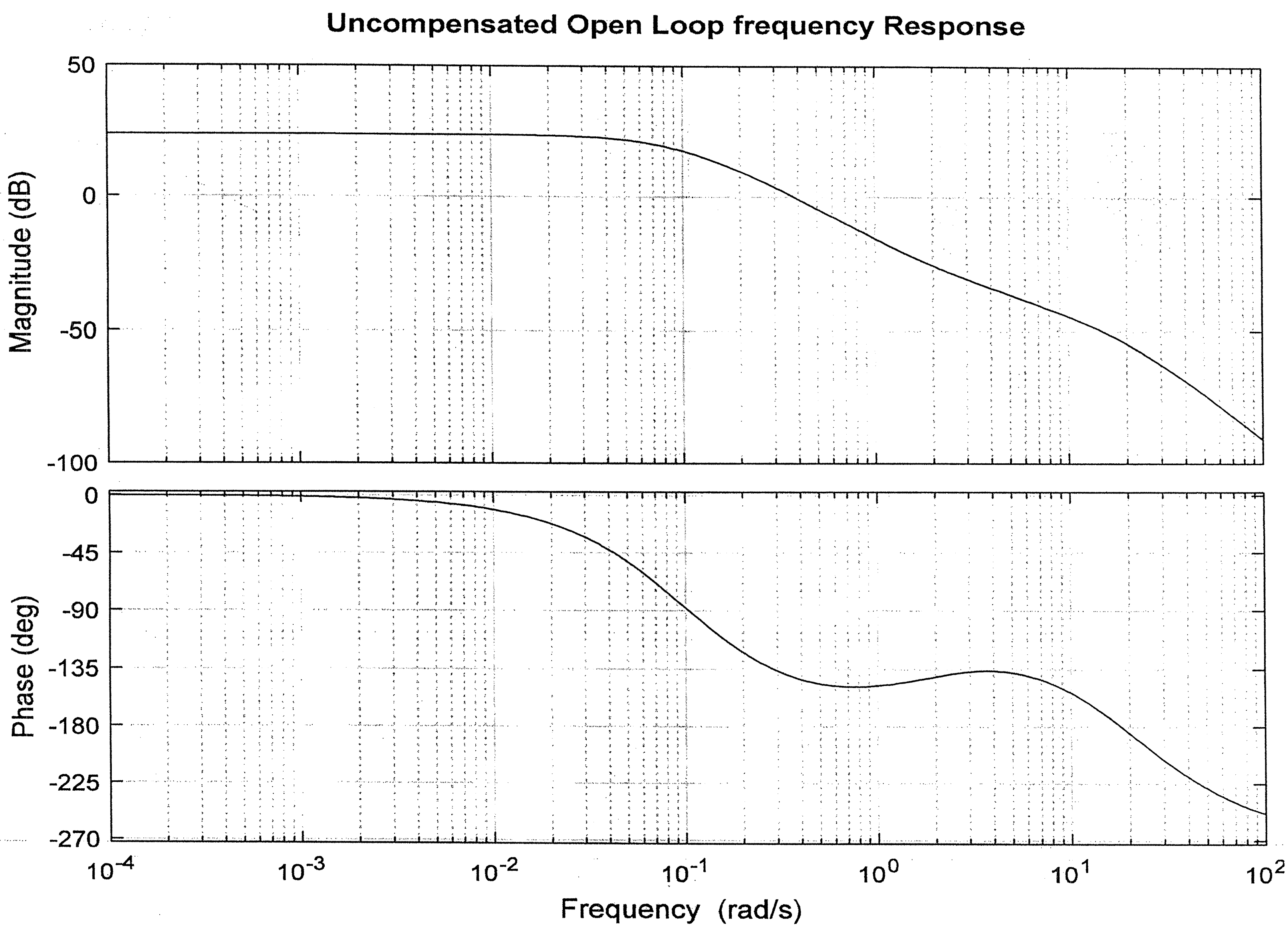


Figure Q7.2 –Uncompensated Frequency Response in Question 7

Question 8

PID Controller Design by Pole Placement, Response Specifications, Second Order Model.

Consider a closed loop positioning control system with a PID Controller in a so-called "series" configuration, as shown in Figure Q8.1:

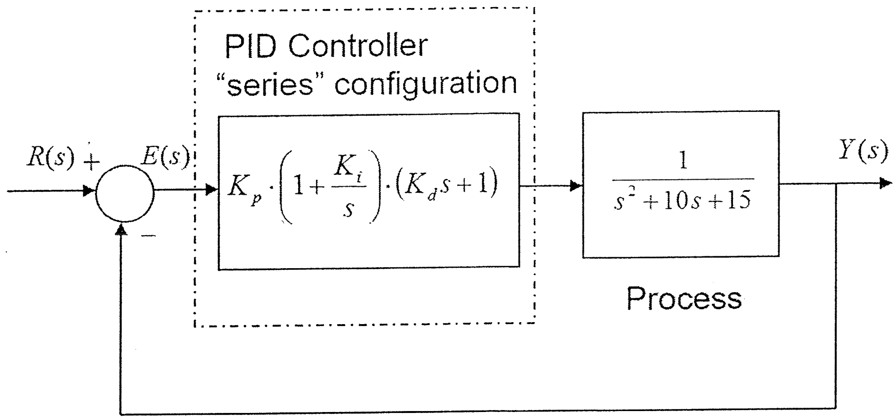


Figure Q8.1

1) **(5 marks)** Derive the Closed Loop system transfer function in terms of Controller Gains K_p, K_d and K_i, and write the system Characteristic Equation, $Q(s) = 0$.

2) **(5 marks)** The compensated Closed Loop step response of this system is to have the following specifications: $PO = 10\%$ and $T_{settle(\pm 2\%)} = 0.9$ sec. Determine the Closed Loop system damping ratio, ζ, and the frequency of natural oscillations, ω_n, to meet the transient response requirements.

3) **(6 marks)** Choose the pole locations for the Closed Loop system so that system two complex conjugate ("dominant") poles correspond to the desired second order model (above) and the third real pole equals to the value of Integral Gain K_i so that **a pole-zero cancellation** in the Closed Loop transfer function occurs. Compute the required Controller gains K_p, K_d, and K_i.

4) **(4 marks)** Note that you are expected to solve a quadratic equation to find the gains in item 3), which means you will have two sets of solutions. Choose ONLY ONE set for your final answer – clearly identify it, and justify your choice by briefly commenting on any possible differences between the expected system responses (i.e. of the dominant poles model) and the actual system responses.