National Exams May 2019

16-ELEC-A7, Electromagnetics

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit clear statements of any assumptions made.
2. An approved Casio or Sharp calculator is permitted. This is a **closed book** exam but one 8.5" × 11" aid sheet is allowed with writing on both sides.
3. FIVE (5) questions constitute a complete exam paper. The first five completed questions as they appear in the answer book will be marked.
4. Each question is of equal value. Full marking scheme on page 8.
5. Fully justify your answers. Inadequate justification will lead to only partial marks.

Question 1

A dielectric-dielectric interface shown in Figure 1. The medium in the region $z < 0$ is free space, while the medium in the region $z > 0$ is silica ($\epsilon_r = 3.8$). Both media are lossless. A 1 GHz plane wave is incident as shown, having a polarization in the y-direction and a field strength of $E^i = 2$ V/m. The transmitted (refracted) and reflected electric fields are labelled $\mathbf{E}^t$ and $\mathbf{E}^r$, respectively. The angle of incidence is $\theta_i = 30°$.

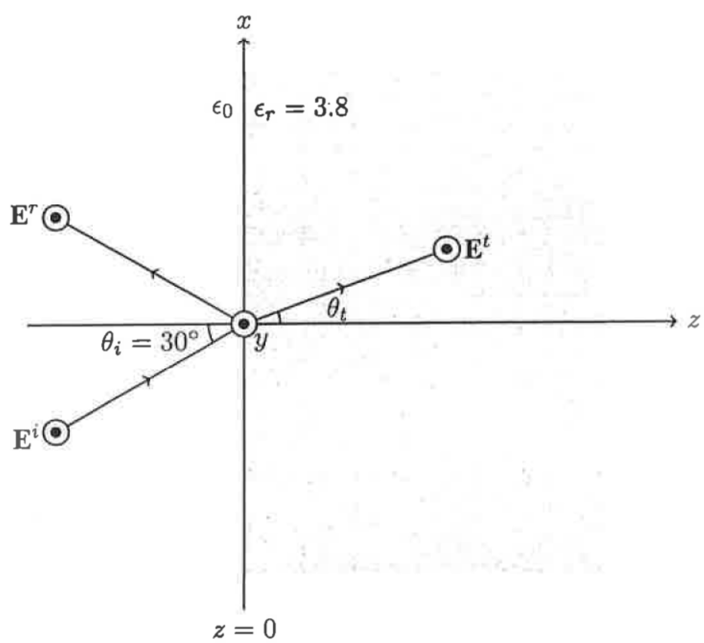

Figure 1 Dielectric media interface and plane waves

(a) Calculate the strength of the magnetic field in the region:

 (i) z < 0; **[4 marks]**

 (ii) ii) z > 0. **[4 marks]**

(b) Determine the angle of refraction, θ_t. **[4 marks]**

(c) Determine the reflection coefficient (E^r/E^i) and transmission coefficient (E^t/E^i) associated with the electric field. **[8 marks]**

Question 2

A complex load impedance $Z_L = 30 + j50\ \Omega$ terminates a 50 Ω transmission line as shown in Figure 2. The line is excited by a voltage source $\tilde{V}_g = 5\angle 0°$. The frequency of the source is 100 MHz. The speed of light along the transmission line is 2×10^8 m/s, and the line is 4 m long.

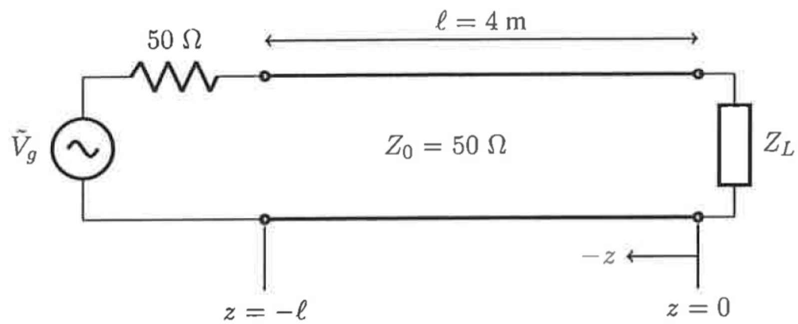

Figure 2 Terminated transmission line

(a) Determine the amplitude of the voltage wave launched at the input of the line at $z = -\ell$. **[6 marks]**

(b) Determine the standing wave ratio along the line, and the location(s) of all maxima, in metres, in the standing wave pattern over the region $-\ell \leq z \leq 0$. Plot the standing wave pattern. **[14 marks]**

Question 3

A lossless transmission line is connected to a generator and load as shown in Figure 3. The line length is 3 m, the transient time through the line is $T = 20$ ns, and its characteristic impedance is 75 Ω.

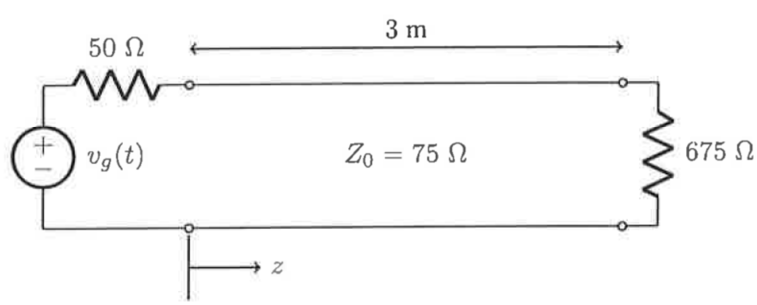

Figure 3 Transmission line circuit

(a) What is the speed of light along the line? **[4 marks]**

(b) Graph the voltage measured at a position $z = 1.5$ m for $0 \leq t \leq 100$ ns if $v_g(t) = 20u(t)$, where $u(t)$ is a unit step function. Indicate the exact times of all discontinuities and the values for all voltages in the waveform. **[16 marks]**

Question 4

A rectangular tunnel with reinforced concrete walls can be modelled as an air-filled ($\epsilon_r = 1$) rectangular waveguide with perfectly conducting walls. The waveguide has width $a = 7$ m and height $b = 4.5$ m.

(a) What is the mode with the lowest cut-off frequency ("dominant") mode of this waveguide? Calculate its cut-off frequency, in MHz. **[4 marks]**

(b) Draw the electric field vector as a function of position of the dominant mode of the waveguide over the cross section of the waveguide. **[8 marks]**

(c) An AM radio station transmitting at $f = 1$ MHz generates a vertical electric field of magnitude $|\mathbf{E}_y| = 0.025$ V/m, measured at the entrance of the tunnel, at $x = a/2$, $y = b/2$. The signal of the radio station is quickly reducing in strength, as one travels down the tunnel. Can you explain why? **[8 marks]**

Question 5

A volumetric charge density in free space is defined in spherical coordinates to be

$$\rho_v(r) = \begin{cases} -\frac{r}{2} + 1, & 0 \le r \le 2,\ 0 \le \theta \le \pi,\ 0 \le \phi \le 2\pi \\ 0 & \text{elsewhere} \end{cases}$$

Determine the following:

(a) An expression for the vector electric field $\mathbf{E}$ in the following regions:

(i) $r \le 2$; **[6 marks]**

(ii) $r > 2$. **[6 marks]**

(b) An expression for the voltage at a point $P(r, \theta, \phi)$ where $r > 2$, if the voltage reference is at $r \to \infty$. **[8 marks]**

Question 6

A cylindrical resistor is formed as shown in Figure 4. The conductor in the region $\rho \leq 1.25$ mm is formed from a conductor with $\sigma = 60.2$ S/m. The cylinder is of length $L = 6.5$ mm. Connection points A and B to the resistor are formed as shown in Figure 4. Assume surfaces at $z = 0$ and $z = L$ are equipotential surfaces.

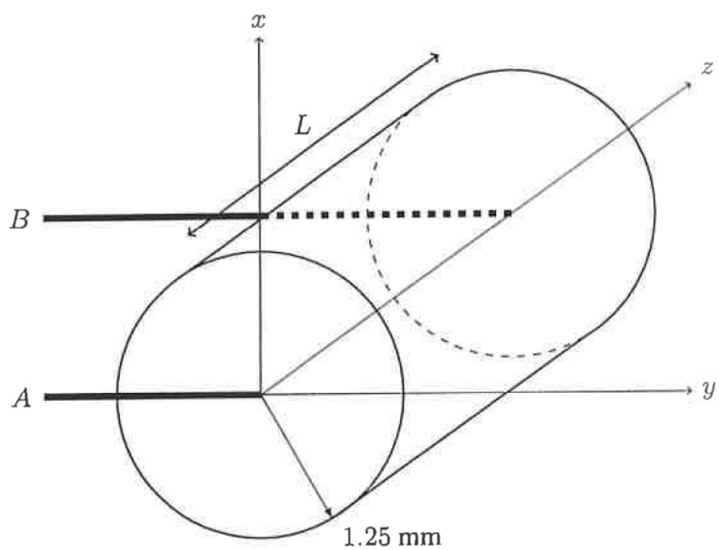

Figure 4 Coaxial resistor

(a) What is the total resistance between points A and B? **[8 marks]**

(b) Determine the vector current density $\mathbf{J}$ and total current I flowing in the region $0 \leq \rho \leq 1.25$ mm, $0 \leq z \leq L$. **[6 marks]**

(c) Using Joule's Law, determine the power dissipated by the resistor. **[8 marks]**

Question 7

This problem is in free space. An infinite line charge exists along the z-axis with a linear charge density of $\rho_l = 10$ nC/m. A point charge $Q_1 = 200$ nC is placed at at a position described by the position vector $\mathbf{r}_1 = 5\mathbf{a}_x + 3\mathbf{a}_y + 2\mathbf{a}_z$.

(a) Derive and calculate, using Gauss' Law, the vector electric flux density $\mathbf{D}$ produced by the line charge alone at a field point P at $\mathbf{r} = 3\mathbf{a}_x + 4\mathbf{a}_y$. Express your answer in terms of Cartesian coordinates and unit vectors. **[10 marks]**

(b) Determine the total electric flux at point P caused by the line charge and point charge. What is the total electric field at point P? **[10 marks]**

Question 8

A cylindrical wire of radius a is centred along the z-axis. The wire carries a nonuniform current described by

$$\mathbf{J} = \frac{12\rho}{a}\mathbf{a}_z \text{ [A/m}^2\text{]}$$

where ρ is the radial distance from the wire.

(a) Determine the total current flowing in the wire. **[6 marks]**

(b) Determine the vector magnetic field $\mathbf{H}$ for $a \leq \rho < \infty$. **[6 marks]**

(c) Determine the vector magnetic field $\mathbf{H}$ for $0 \leq \rho \leq a$. **[8 marks]**

Question 9

A current-carrying loop is placed in the xy-plane as shown in Figure 5. The loop has a radius of 3 m and a carries a current of 1 A flowing in the $\mathbf{a}_\phi$ direction.

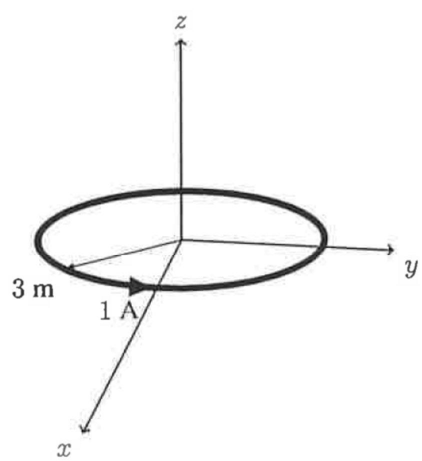

Figure 5 Wire Loop

(a) Derive the vector magnetic field $\mathbf{H}$ produced by the loop at a field point along the z-axis, as a function of z. Evaluate the magnetic field at the point $(0, 0, 0.5 \text{ m})$. **[12 marks]**

(b) Calculate the magnetic flux produced over the area of the loop by the current, assuming the magnetic field over its area is constant. **[8 marks]**

Question 10

A 1 MHz uniform current flows in a vertical wire antenna of length 15 m. The wire radius is 2 cm and the wire is made of steel ($\sigma = 6.2 \times 10^6$ S/m). Determine:

1. The radiation resistance of the antenna. **[6 marks]**
2. The radiation efficiency of the antenna. **[6 marks]**
3. The electric field intensity at a distance of 20 km from the antenna if the radiated power is 1 kW. **[8 marks]**

Marking Scheme

1. a) i) 4 marks ii) 4 marks b) 4 marks c) 8 marks
2. a) 6 marks b) 14 marks
3. a) 4 marks b) 16 marks
4. a) 4 marks b) 8 marks c) 8 marks
5. a) i) 6 marks ii) 6 marks b) 8 marks
6. a) 8 marks b) 6 marks c) 8 marks
7. a) 10 marks b) 10 marks
8. a) 6 marks b) 6 marks c) 8 marks
9. a) 12 marks b) 8 marks
10. a) 6 marks b) 6 marks c) 8 marks