National Exams December 2013

04-BS-6: Mechanics of Materials

3 hours duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumptions made.

2. Candidates may use one of two calculators, the Casio or Sharp approved models.

   This is a Closed Book exam. However candidates are permitted to bring the following into the examination room:

   - ONE aid sheet 8.5" x 11" hand-written on both sides containing notes and formulae. **Example problems and solutions to problems are not allowed!**

3. Any five questions (out of 8 given) constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.

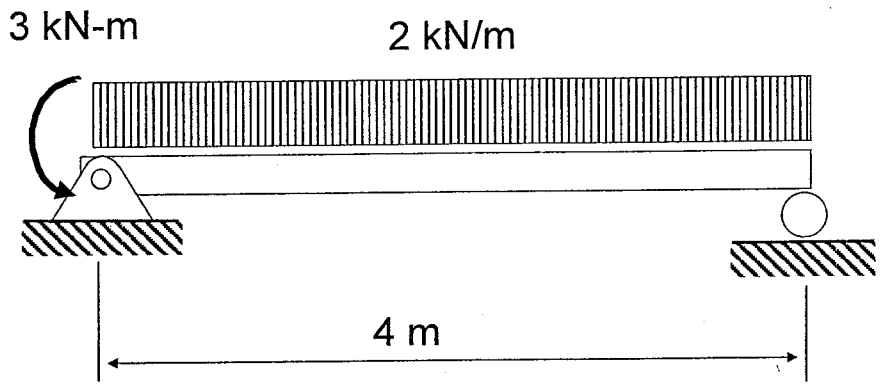
4. All questions are of equal value.

NOTE: The aid sheet must be handed in with the exam!

Your exam will not be marked if you do not hand in an aid sheet, unless there is a signed statement by the exam invigilator stating that no aid sheet was used for the exam.

Question 1: **[20 marks]** A simply supported timber beam supports a uniformly distributed load together with a couple applied at the left support. The timber beam is 100 mm wide x 200 mm high. The wood has an elastic modulus equal to 10 GPa and allowable normal stress of 6 MPa.

Determine the maximum deflection of the beam using the method of integration.



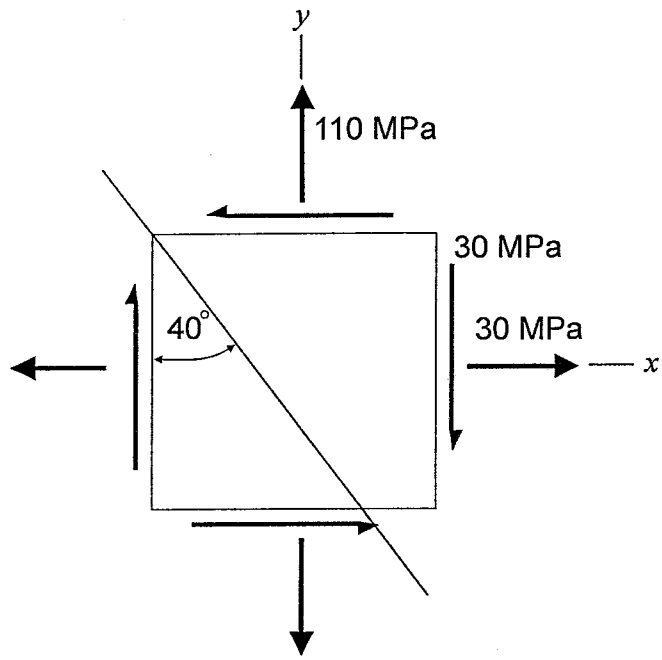
No credit will be given for a solution using the principle of superposition, when combinations of existing solutions are used to find an answer.

Question 2: An element in a state of plane stress is subjected to the normal and shear stresses shown below.

[20 marks] Use the Mohr's circle solution (*not* the transformation equations) to determine the following:

(a) the stress components acting on the inclined plane (orientated 40° from the vertical as shown), showing your answer on a properly oriented element.

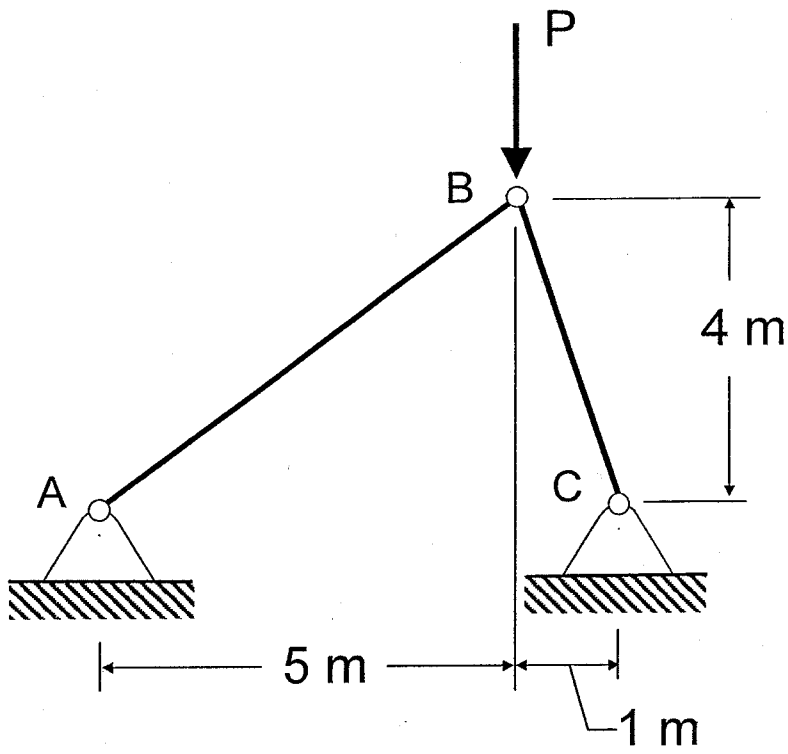
(b) the maximum in-plane shear stress of the element (and associated normal stresses) and orientation of the corresponding planes. Once again, show your answer on a sketch of a properly oriented element.



Note that credit will **only** be given for a **solution using Mohr's circle.** This means that you need to draw a Mohr's circle based on the stress components given in this problem. Remember to show numbers on your circle. Your **calculations** must be based on the geometry of your circle. So use your calculator. In other words, you are expected to use trigonometry to construct your Mohr's circle. Do not give a graphical solution that is scaled off! The stress transformation equations can only be used to check your answer.

Question 3: **[20 marks]** Determine the largest load P that can be applied to the truss structure below given that members AB and BC are made of 100 mm outside diameter hollow steel rods with a 5 mm wall thickness. Both steel rods are pinned at their ends.

Consider in-plane buckling only for the compression members and use a factor of safety of 2 for the Euler buckling load. Do not use a safety factor for yielding of the steel. The steel used in the rods has an allowable yield strength equal to 240 MPa and elastic modulus of 200 GPa.

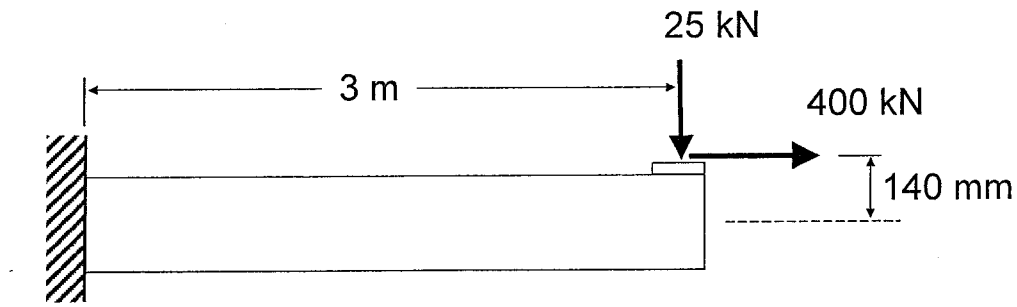


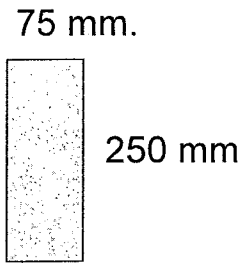
Note: $A_{circle} = \pi r^2$ and $I_{circle} = \pi r^4 / 4$

Question 4: **[20 marks]** A rectangular cantilevered beam is loaded as shown below. The 400 kN horizontal force at the end of the cantilever is offset from the member centroid by 140 mm.

(a) Compute and sketch the distribution of normal stress at the base of the cantilever. Make sure to show maximum and minimum values.

(b) Compute and sketch the shear stress distribution at the base of the cantilever. Make to show maximum and minimum values.



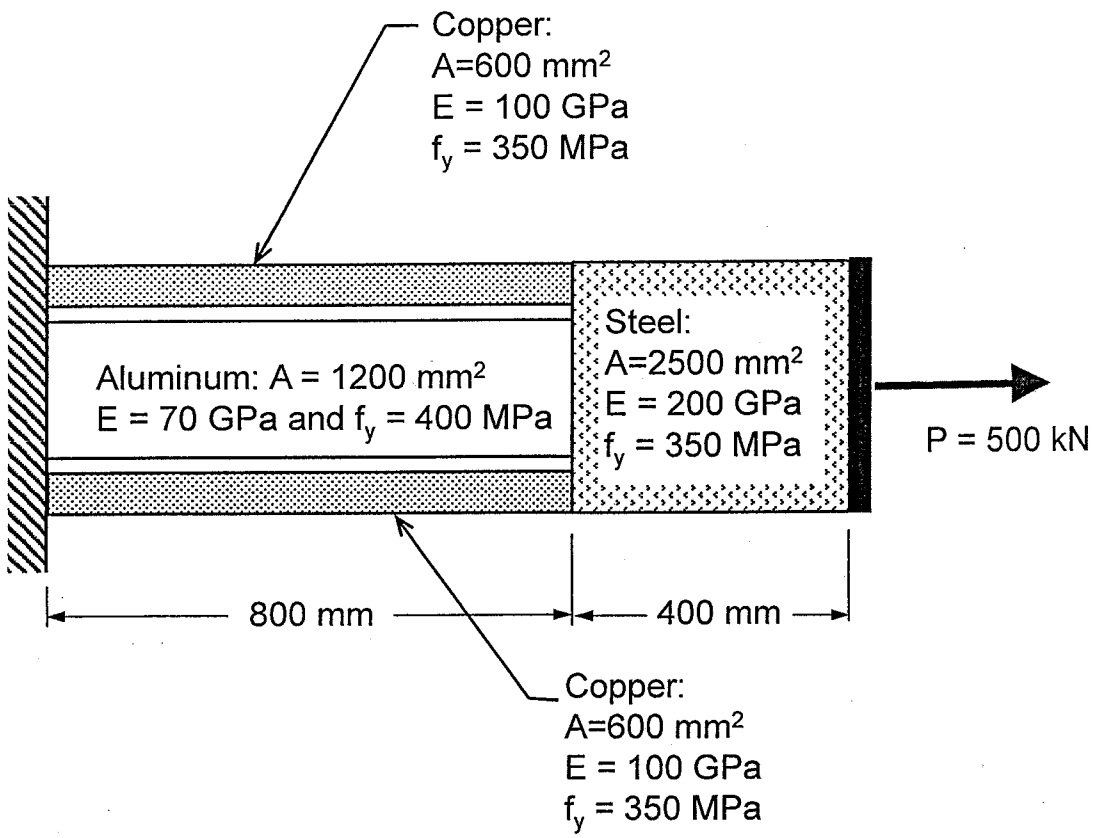


cross section

Question 5: Two copper bars and an aluminum bar are attached to a steel bar as shown. Properties of each bar are as given below.

[14 marks] (a) find the forces developed in each bar

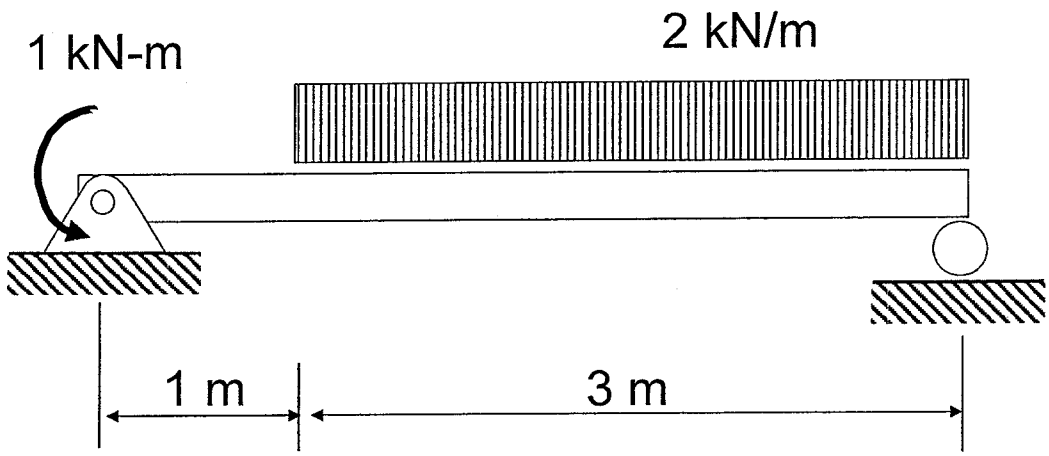
[6 marks] (b) find the total (horizontal) displacement at the end of the member



Question 6: **[20 marks]** A simply supported beam is subjected to a uniformly distributed load over part of the span in addition to a moment at the left hand support. See below.

Determine the shear and moment throughout the beam as functions of x. This means that you need to give formula(s) for **V(x)** and **M(x)** along the length of the beam.

Next construct the shear force and bending moment diagrams. Remember to label points of maximum positive and negative bending moment, as well as any inflection points. Show your work by indicating exactly how you obtained your answer.



Remember that it is important you set this problem up correctly by calculating the correct reaction forces at the support(s).

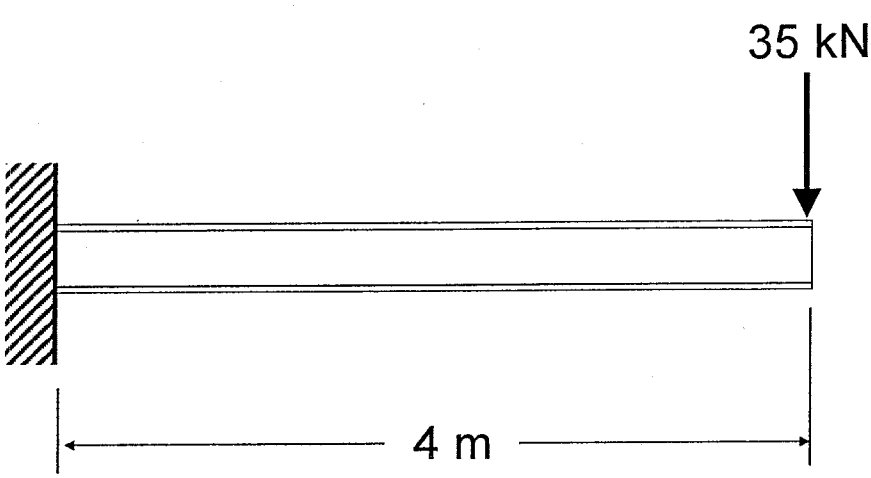
No credit will be given for a solution using the principle of superposition, when combinations of existing solutions are used to find an answer.

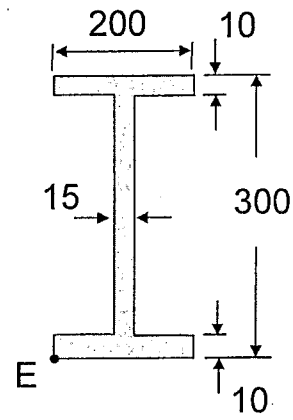
Question 7: [20 marks] A cantilevered beam (fixed at the left) supports a concentrated load of 35 kN acting 4 m from the fixed support. The beam has the cross-section shown below and is made from steel having an allowable normal stress of 240 MPa and allowable shear stress of 60 MPa. The elastic modulus of the steel used equals 200 GPa.

Do the following:

(a) determine the maximum (absolute) normal stress and maximum shear stress in the beam.

(b) determine the shear stress at the tip of the flange (point E) for the beam section at the base of the cantilever. To receive marks you must give reasons to justify your answer.





Beam cross-section
(all dimensions in mm)

Question 8: A circular stepped shaft is fixed at the right end (point E) and subjected to three torques (points A, B, and C) acting as shown below. Part of the shaft (DE) is hollow and the entire shaft is made of aluminum with G = 25 GPa and a yield stress of 200 MPa. Dimensions (diameter and length) and magnitude of the torques are given in the diagram.

[12 marks] (a) determine the maximum shear stress in the stepped shaft and sketch the corresponding variation of shear stress along the shaft radius at this point.

[6 marks] (b) find the angle of twist at the end of the shaft (point A) and give your answer in degrees.

[2 marks] (c) What would happen if the loads on the shaft were doubled?