National Exams May 2013

07-Elec-A2, Systems & Control

3 hours duration

NOTES:

1. This is a CLOSED BOOK EXAM. Only an approved Casio or Sharp calculator is permitted. Candidates are allowed to use a single-sided, handwritten, 8.5 by 11" formula and notes sheet. Otherwise, there are no restrictions on the content of the formula sheet. The sheet has to be signed and submitted together with the examination paper.
2. If doubt exists as to interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
3. Five (5) questions constitute a complete paper. **YOU ARE REQUIRED TO COMPLETE QUESTION 1 AND QUESTION 2.** Choose three (3) more questions out of the remaining six. Each question is of equal value. Weighting of topics within each question is indicated. Partial marks will be given. Clearly show steps taken in answering each question.
4. **PLEASE WRITE ANSWERS DIRECTLY IN THIS EXAM PAPER – DO NOT USE EXAM BOOKS.** If necessary, you may write on the backside of the pages as long as you write the final answers in the space indicated, and point to where the full calculations are.

YOUR MARKS		
QUESTIONS 1 AND 2 ARE COMPULSORY:		
Question 1	20	
Question 2	20	
CHOOSE THREE OUT OF THE REMAINING SIX QUESTIONS:		
Question 3	20	
Question 4	20	
Question 5	20	
Question 6	20	
Question 7	20	
Question 8	20	
TOTAL:	100	

A Short Table of Laplace Transforms

Laplace Transform	Time Function
1	$\sigma(t)$
$\frac{1}{s}$	$1(t)$
$\frac{1}{s^2}$	$t \cdot 1(t)$
$\frac{1}{s^k}$	$\frac{t^k}{k!} \cdot 1(t)$
$\frac{1}{s+a}$	$e^{-at} \cdot 1(t)$
$\frac{1}{(s+a)^2}$	$t \cdot e^{-at} \cdot 1(t)$
$\frac{a}{s(s+a)}$	$(1-e^{-at}) \cdot 1(t)$
$\frac{a}{s^2+a^2}$	$\sin(at) \cdot 1(t)$
$\frac{s}{s^2+a^2}$	$\cos(at) \cdot 1(t)$
$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at} \cdot \cos(bt) \cdot 1(t)$
$\frac{b}{(s+a)^2+b^2}$	$e^{-at} \cdot \sin(bt) \cdot 1(t)$
$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$\left(1-e^{-at} \cdot \left(\cos bt + \frac{a}{b}\sin bt\right)\right) \cdot 1(t)$
$\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \cdot \sin(\omega_n\sqrt{1-\xi^2}\,t) \cdot 1(t)$
$\frac{\omega_n^2}{s(s^2+2\xi\omega_n s+\omega_n^2)}$	$\left(1-\frac{1}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \cdot \sin(\omega_n\sqrt{1-\xi^2}\,t+\cos^{-1}\xi)\right) \cdot 1(t)$
$F(s) \cdot e^{-Ts}$	$f(t-T) \cdot 1(t)$
$F(s+a)$	$f(t) \cdot e^{-at} \cdot 1(t)$
$sF(s) - f(0+)$	$\frac{df(t)}{dt}$
$\frac{1}{s}F(s)$	$\int_{0+}^{+\infty} f(t)dt$

Question 1 (Compulsory)

Stability Analysis: Frequency Domain vs. s-Domain (Root Locus Analysis and Routh-Hurwitz Criterion).

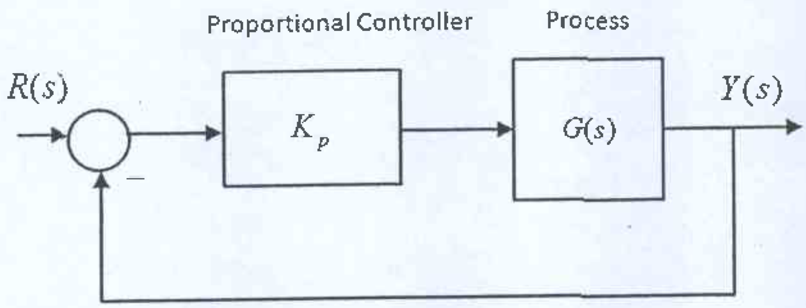


Figure Q1.1

Consider a unit feedback control system in Figure Q1.1, where the process G(s) is described by the following transfer function:

$$G(s) = \frac{100}{(s+2)(s+4)(s+10)}$$

Your task is to investigate the stability of the closed loop system using three different approaches (Bode Plots, Root Locus and Routh-Hurwitz Criterion) by finding:

1. The practical range of safe operating gains for the Proportional Controller in Figure Q1.1;
2. The critical gain, K_{crit}, at which the closed loop system becomes marginally stable, and the corresponding frequency of marginal oscillations, ω_{osc};
3. The Gain Margin of the system, both in V/V units and in decibel units, when the Operating Gain of the Controller is equal to 2.

Part A (7 marks)

Sketch a reasonably accurate Root Locus for the system in the space provided in Figure Q1.2. From your sketch, read off the estimate of the frequency of marginal oscillations, ω_{osc}. Next, calculate the corresponding value of the gain, K_{crit}, using the Magnitude Criterion of the Root Locus. Finally, calculate the Gain Margin. Place your answers in Table Q1.

Part B (7 marks)

In the space provided in Figure Q1.3, sketch a reasonably accurate Bode plot (linear approximation of the frequency response) of the open loop system. From your sketch, read off the estimates of the Gain Margin and the corresponding frequency of cross-over, ω_{cg}. Next, calculate the corresponding value of the critical gain, K_{crit}, and the corresponding frequency of marginal oscillations, ω_{osc}. Place your answers in Table Q1.

Part C (6 marks)

Apply the Routh-Hurwitz criterion of Stability to verify the results of Part A and Part B. Place the answers in Table Q1. Are the results of all three methods consistent?

Table Q1

PART A: Root Locus

Frequency of oscillations as read off the Root Locus sketch	$\omega_{osc} =$	Corresponding value of the gain calculated from Magnitude Criterion	$K_{crit} =$	Gain Margin of the system	$G_{m\,V/V} =$ $G_{m\,dB} =$
Practical range of gains for stable closed loop operation from Part A results:				$< K_p <$	

PART B: Bode Plots

Gain Margin of the system	$G_{m\,dB} =$ $G_{m\,V/V} =$	Corresponding crossover frequency	$\omega_{cg} =$	Practical range of gains for stable closed loop operation from Part B results: $< K_p <$
Critical gain from Bode plot	$K_{crit} =$	Frequency of oscillations from Bode plot	$\omega_{osc} =$	

PART C: Routh-Hurwitz Criterion

Critical gain From Routh-Hurwitz	$K_{crit} =$	Frequency of oscillations from Routh-Hurwitz	$\omega_{osc} =$	Gain Margin of the system	$G_{m\,V/V} =$ $G_{m\,dB} =$
Practical range of gains for stable closed loop operation from Part C results:				$< K_p <$	

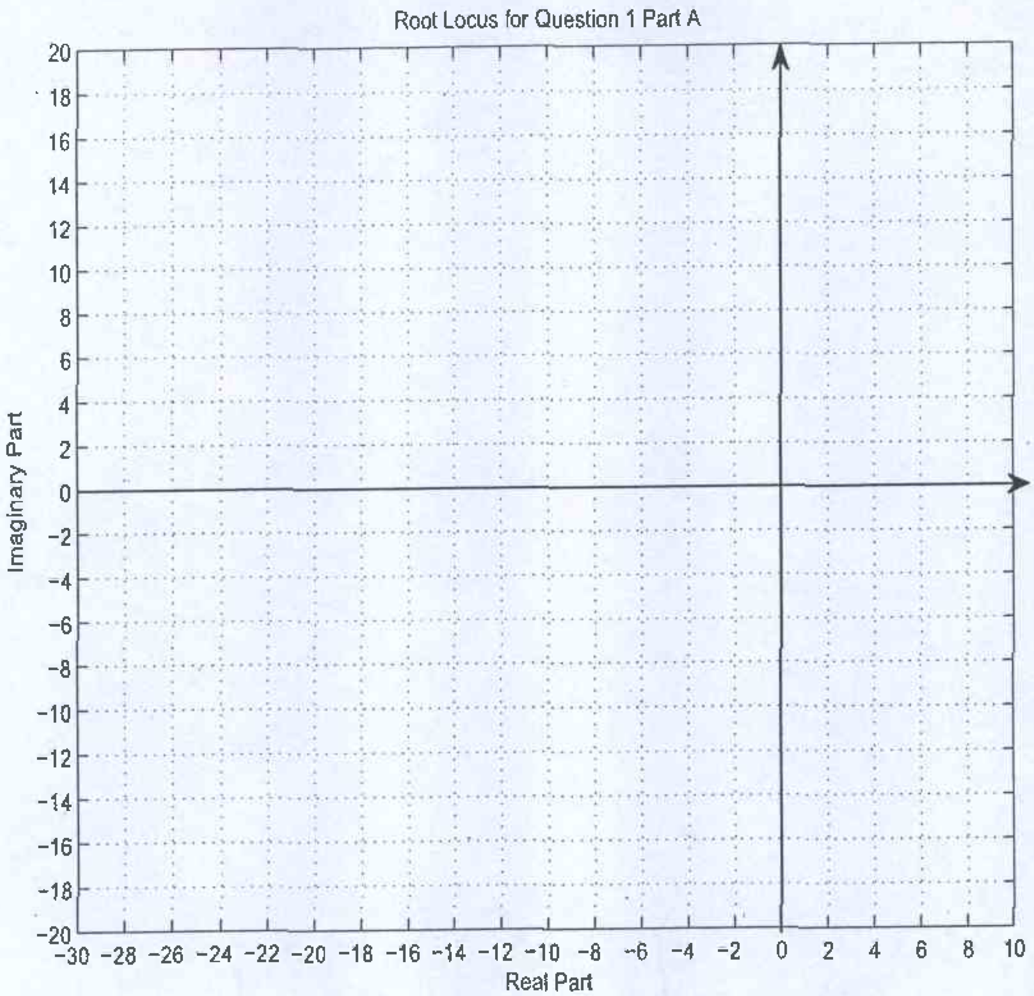

Root Locus for Question 1 Part A
Imaginary Part
Real Part
20
18
16
14
12
10
8
6
4
2
0
−2
−4
−6
−8
−10
−12
−14
−16
−18
−20
−30 −28 −26 −24 −22 −20 −18 −16 −14 −12 −10 −8 −6 −4 −2 0 2 4 6 8 10

Figure Q1.2

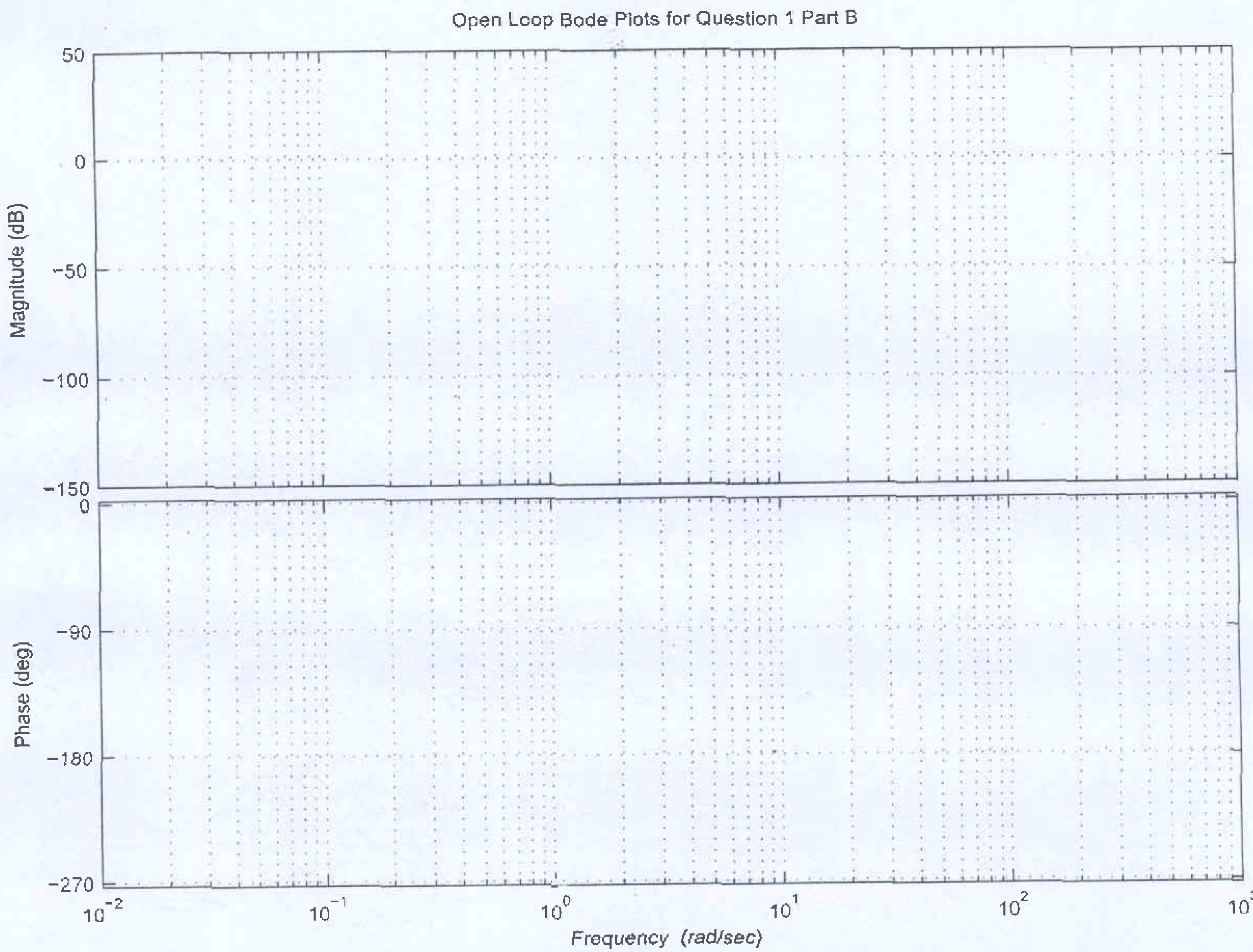

Open Loop Bode Plots for Question 1 Part B
Magnitude (dB)
50
0
−50
−100
−150
Phase (deg)
0
−90
−180
−270
10^−2
10^−1
10^0
10^1
10^2
10^3
Frequency (rad/sec)

Figure Q1.3

Question 1 Continued

Question 1 Continued

Question 2 (Compulsory)

Proportional + Integral + Derivative (PID) Controller Design in s-Domain, Dominant Poles Model, Step Response Specifications.

Consider a unit feedback closed loop control system, as shown in Figure Q2.1.

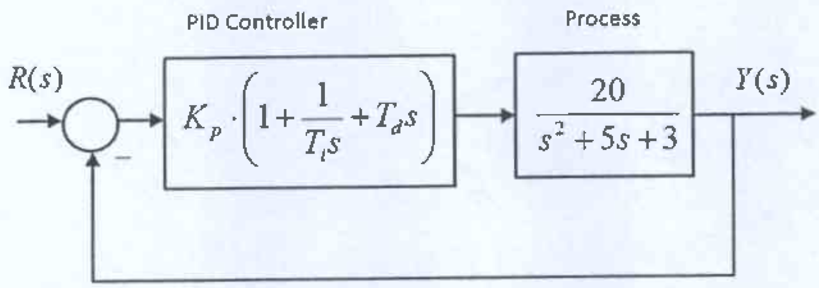


Figure Q2.1

The system is to operate under PID Control in a parallel implementation. Your task is to calculate the PID Controller parameters, K_p, T_i and T_d. In order to do so, please follow the steps described in the following parts.

PART A (5 marks)

We want the compensated closed loop response of our control system to resemble the step response shown in Figure Q2.2 on the following page. Assume that the response can be modeled by a standard second order dominant poles model that is described by the following transfer function:

$$G_m(s) = K_{dc} \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Determine appropriate parameters of the model (i.e. K_{dc}, ω_n, ζ). Write the parameters as well as the model transfer function in Table Q2.

PART B (8 marks)

Derive the closed loop transfer function, $G_{cl}(s) = \frac{Y(s)}{R(s)}$, in terms of K_p, T_i and T_d and write the resulting expression in Table Q2. Next, calculate appropriate numerical values for the PID Controller parameters K_p, T_i and T_d and also place them in Table Q2.

HINT: Assume the dominant poles model for the closed loop system, based on the second order model derived in Part A and then use a factor of 10 times for any additional poles to be placed in the insignificant region of the s-plane.

PART C (7 marks)

Substitute the computed values of the controller parameters, K_p, T_i and T_d into the closed loop transfer function, $G_{cl}(s) = \frac{Y(s)}{R(s)}$ and write the numerical values of its poles and zeros, as well as the DC Gain of the compensated closed loop, in Table Q2.

Sketch a Pole-Zero Map for this transfer function in the space provided in Figure Q2.3. Based on the Pole-Zero Map, discuss briefly whether the actual compensated system response will differ from the expected model response, and if so, describe in what way. Try to be brief but specific.

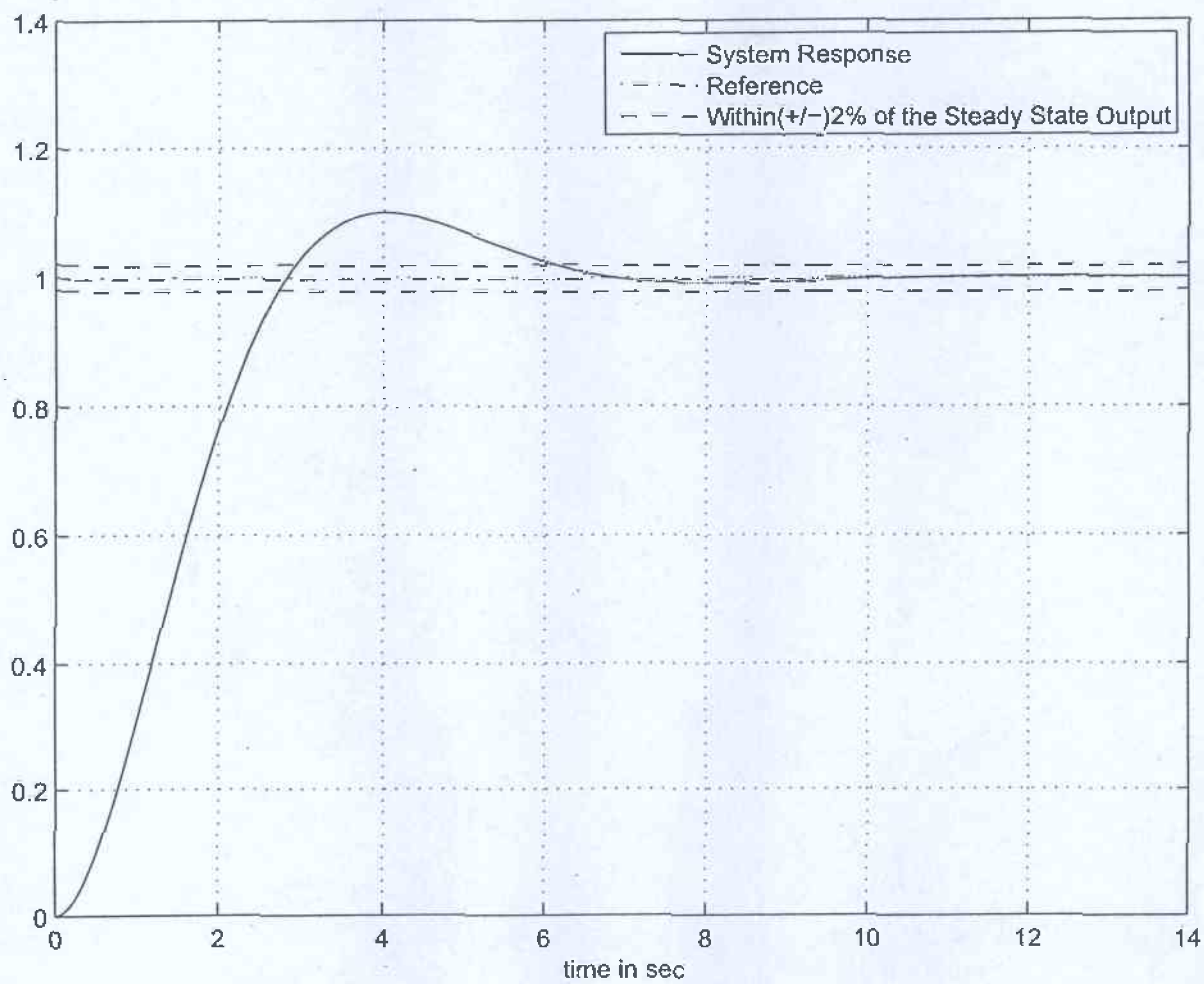


Figure Q2.2

Table Q2

<table>
<tr><td>2nd Order Model DC gain is:
K_{dc} =</td><td>2nd Order Model Natural Frequency is:
ω_n =</td><td>2nd Order Model Damping Ratio is:
ζ =</td></tr>
<tr><td colspan="3">The appropriate 2nd order model for this system is:

$G_m(s) =$ ______________________________</td></tr>
<tr><td colspan="3">The closed loop transfer function of this system, written in terms of the PID Controller parameters, K_p, T_i and T_d, is as follows:

$G_{cl}(s) =$ ______________________________</td></tr>
<tr><td colspan="3">The Controller Parameters are calculates as:</td></tr>
<tr><td>K_p =</td><td>T_i =</td><td>T_d =</td></tr>
<tr><td>The compensated closed loop system DC gain is equal to:

K_{dc} =</td><td>The compensated closed loop system poles are located at:</td><td>The compensated closed loop system zeros are located at:</td></tr>
</table>

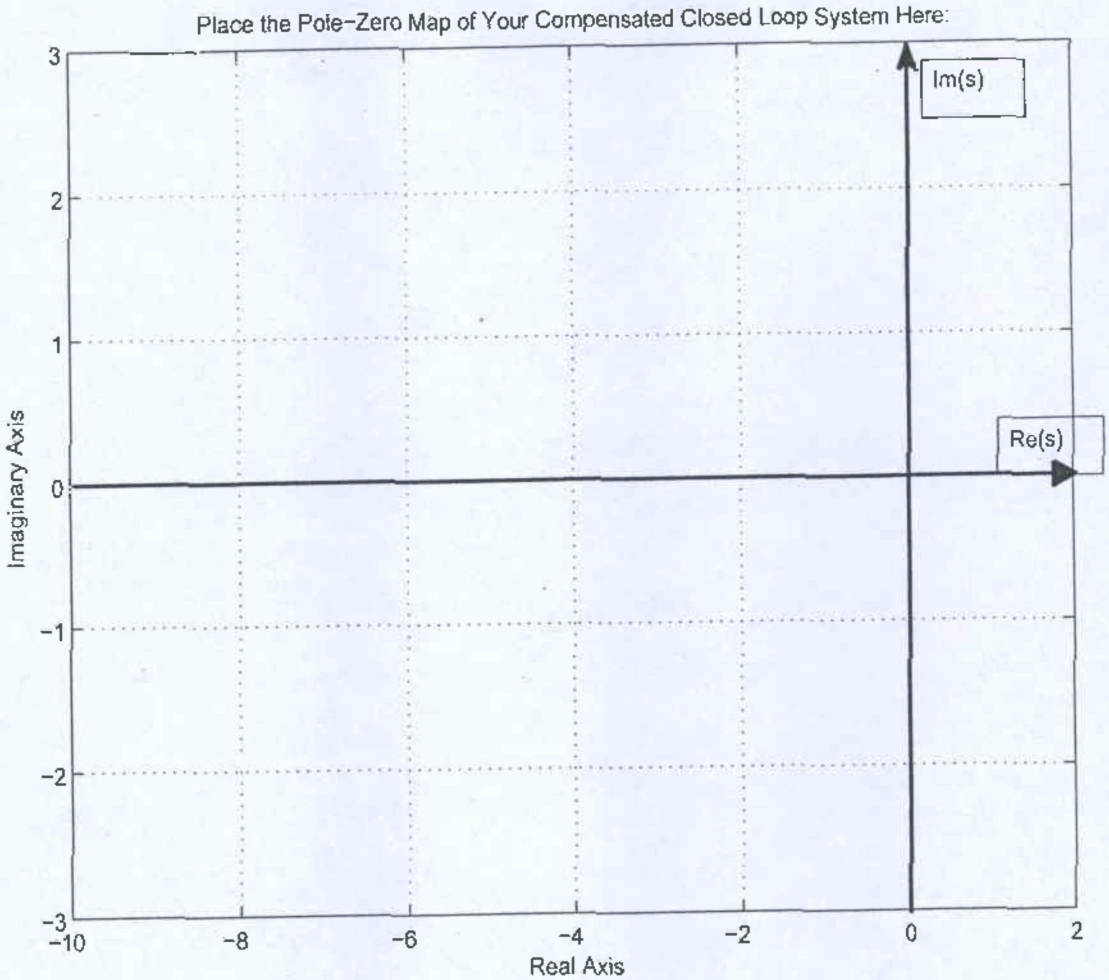


Figure Q2.3

Write your comments for Part C here:

Question 2 Continued

Question 2 Continued

Question 3

Root Locus Analysis and PID Controller Design in s-Domain, Dominant Poles Model, Step Response Specifications.

Consider a unit feedback closed loop control system, as shown in Figure Q3.1.

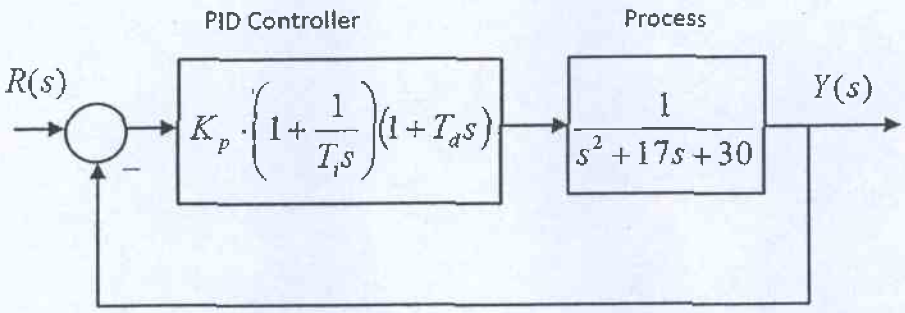


Figure Q3.1

The system is to operate under PID Control in a series implementation, with the Integral Time constant $T_i = 0.1$ seconds and the Derivative Time Constant $T_d = 0.125$ seconds. Your task is to choose the required PID Proportional Gain K_p so that the Settling Time within 2% of the steady state value is $T_{settle(\pm 2\%)} = 0.5$ seconds. To do so, please follow the steps described in the following parts.

PART A (7 marks)

In the space provided in Figure Q3.2, sketch a detailed Root Locus for the system, including the following parameters, if they exist: crossovers with the imaginary axis, break-away/break-in coordinates, asymptotes, centroid, etc., and place their numerical values in Table Q3. Provide only an estimate of a break-away/break-in coordinate(s) if the resulting polynomial equation is of an order higher than 2.

PART B (8 marks)

Based on the Root Locus sketch, calculate the required value of the Proportional Gain K_p so that the Settling Time within 2% of the steady state value is $T_{settle(\pm 2\%)} = 0.5$ seconds. Assuming a second order dominant poles model for the closed loop system, what is the closed loop system equivalent damping ratio ζ? Place your answers in Table Q3.

PART C (5 marks)

Next, based on the Root Locus sketch, discuss whether the assumed dominant poles model represents accurately the dominant dynamic of the actual closed loop system. What differences can be expected and why? Specifically comment briefly on the differences in the settling time, percent overshoot and the rise time of the actual response as compared to the assumed model.

Write your comments for Part C here:

____

____

____

____

____

____

Table Q3

Root Locus centroid is at:	$\sigma =$
Root Locus asymptotic angles are equal to:	$\theta_i =$
The critical value of the Proportional Gain for which the system is marginally stable, and the frequency of resulting oscillations:	$K_{crit} =$ $\omega_{osc} =$
Break-away (leave blank if not applicable) coordinate is at:	$s_b =$
Break-in (leave blank if not applicable) coordinate is at:	$s_b =$
Compensated Closed Loop System	
Closed loop equivalent damping ratio is estimated to be:	$\zeta =$
Proportional PID Controller gain is estimated to be:	$K_p =$

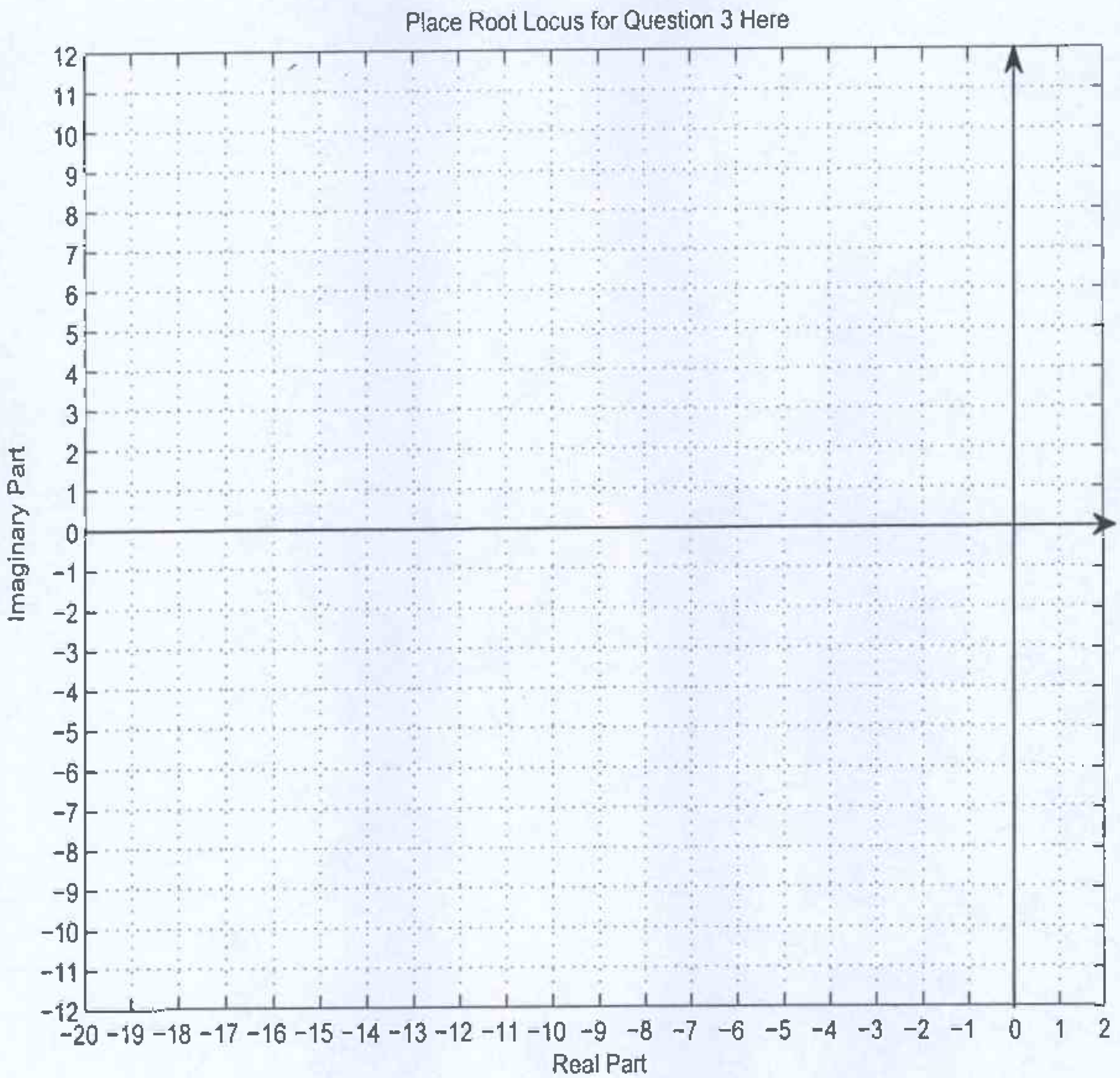

Place Root Locus for Question 3 Here
Imaginary Part
Real Part
12
11
10
9
8
7
6
5
4
3
2
1
0
−1
−2
−3
−4
−5
−6
−7
−8
−9
−10
−11
−12
−20 −19 −18 −17 −16 −15 −14 −13 −12 −11 −10 −9 −8 −7 −6 −5 −4 −3 −2 −1 0 1 2

Figure Q3.2

Question 3 Continued

Question 4

Closed Loop Stability, determining the range of safe operations under Proportional and Proportional + Integral Control - Routh-Hurwitz Criterion of Stability, Gain Margin.

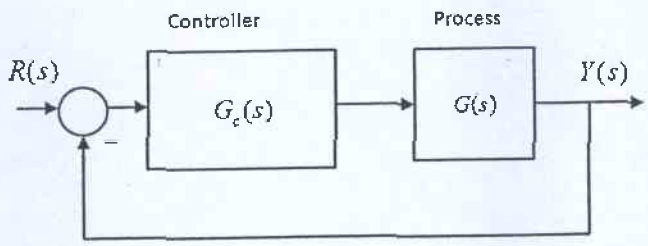


Figure Q4.1

Consider two control systems, both in the same basic unit feedback configuration, as shown in Figure Q4.1.

The first system operates under a Proportional Control:

$$G_c(s) = K_p$$

The second system operates under a Proportional + Integral Control:

$$G_c(s) = K_p \cdot \frac{s+2}{s}$$

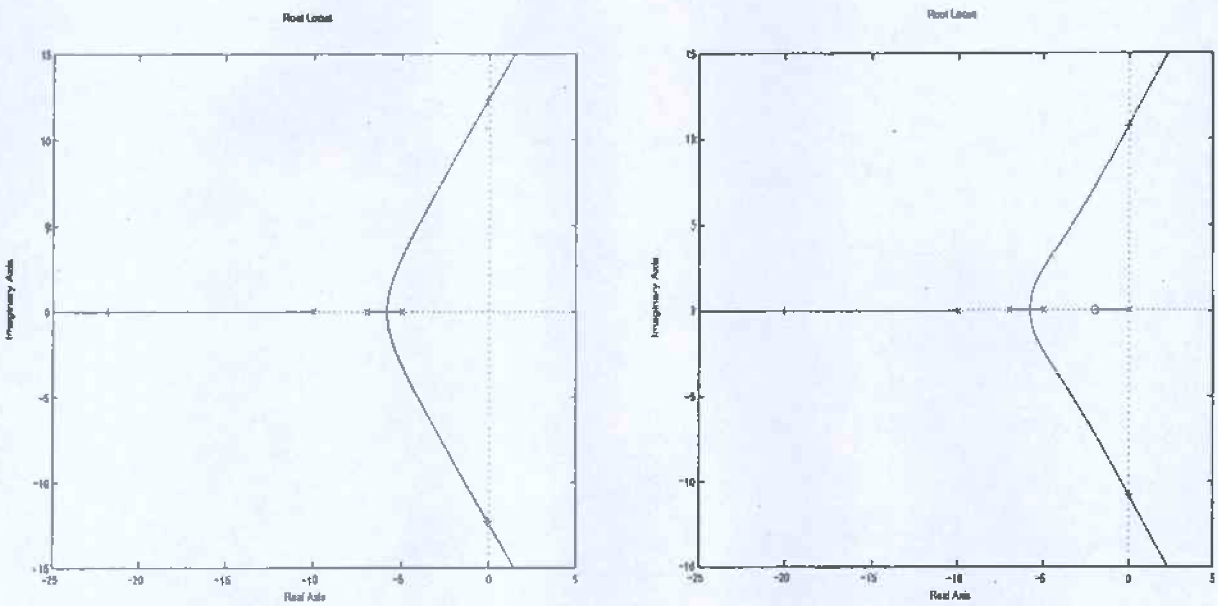
The process in both systems is described by the following transfer function:

$$G(s) = \frac{100}{(s+5)(s+7)(s+10)}$$

Figure Q4.2 shows the Root Loci of the respective control systems. As read off these plots, the frequency of oscillations, ω_{osc}, for the system under Proportional Control is approximately 12.5 rad/sec, and for the system under Proportional + Integral Control it is approximately 10.7 rad/sec.

Recall that this is the frequency corresponding to the critical gain, i.e. the gain at which the closed loop system is marginally stable. What is expected of the Gain Margin for the system under PI Control, as compared to the Gain Margin for the system under P Control? Will it increase or decrease?

Prove your assertion by calculating the safe range for a stable system operation under each of the two control schemes, employing Routh-Hurwitz analysis. Place your answers in Table Q4.



$\omega_{osc} \approx 12.5$ rad/sec

K_{crit}= ?

$\omega_{osc} \approx 10.7$ rad/sec

K_{crit}= ?

Figure Q4.2

Table Q4

The safe range for stable operation of the system under P Control is:	$< K_p <$	
The safe range for stable operation of the system under PI Control is:	$< K_p <$	
Will the Gain Margin of the system under PI Control increase, as compared to the system under P Control? Please circle your answer.	YES	NO

Question 4 Continued

Question 4 Continued

Question 5

Lag Controller Properties, Closed Loop Stability, Second Order Dominant Poles Model from Frequency Response Plots, Step Response Specifications, Steady State Errors and Error Constants.

A certain unit feedback closed loop system is to operate under Lag Control:

$$G_c(s) = K_c \frac{\alpha\tau s + 1}{\tau s + 1} \qquad \alpha < 1$$

The process (i.e. open loop) frequency response plots are shown in Figure Q5.1, and its transfer function is described as follows:

$$\mathrm{G(s)} = \frac{40}{s^3 + 11s^2 + 15s + 5}$$

PART A (10 marks)

Find the Open Loop DC gain, Phase Margin and the corresponding frequency of the crossover for the uncompensated system, and place the appropriate values in Table Q5. Assume a second order dominant poles model for the uncompensated closed loop system, calculate the model parameters (DC gain, K_{dc}, damping ratio, ζ, frequency of natural oscillations, ω_n) and place them in Table Q5. Next, estimate the uncompensated closed loop system step response specifications: Steady State Error, $e_{ss\%}$, Percent Overshoot, PO, and Settling Time, $T_{settle(\pm 2\%)}$, System Type and Error Constants, and also write them in Table Q5.

PART B (10 marks)

Design a Lag Controller for this system so that the compensated closed loop system response has a Steady State Error of 5% and a Percent Overshoot of 10%. **Make sure to show the shape of the compensated open loop frequency response in Figure Q5.1.** Next, place the Controller transfer function in Table Q5, estimate the compensated closed loop system step response specifications: Steady State Error, $e_{ss\%}$, Percent Overshoot, PO, and Settling Time, $T_{settle(\pm 2\%)}$, and also place them in Table Q5.

Open Loop Frequency Plot for Question 5 – Lag Compensation

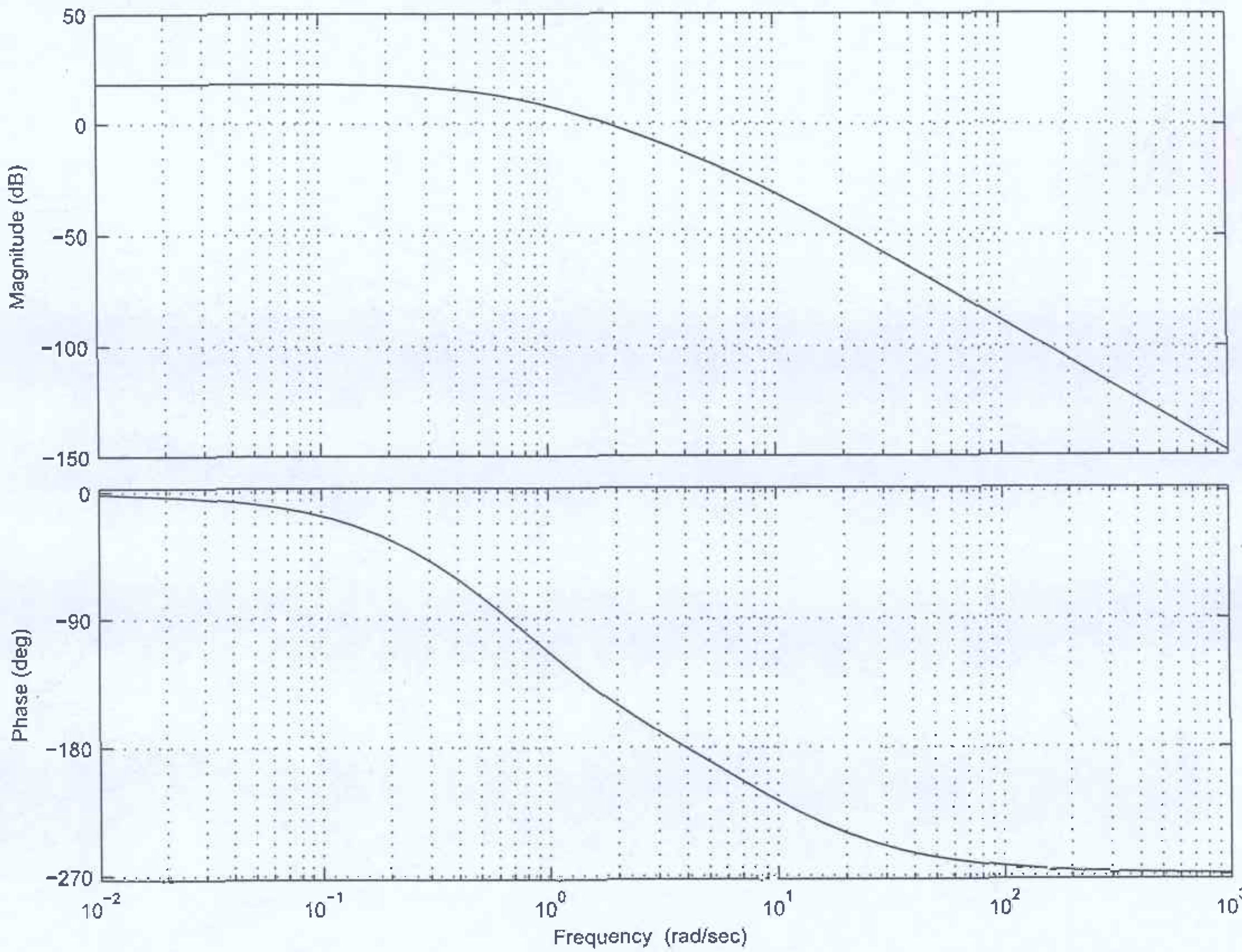


Figure Q5.1

Table Q5

<table>
<tr><td>Open Loop DC Gain:

$K_{dc(open)} =$</td><td>Phase Margin, in degrees:

$\Phi_m =$</td><td>Frequency of the Phase Margin crossover in rad/sec:

$\omega_{cp} =$</td></tr>
<tr><td>Closed Loop DC Gain:

$K_{dc} =$</td><td>Equivalent closed loop damping ratio:

$\zeta =$</td><td>Equivalent closed loop frequency of natural oscillations:

$\omega_n =$</td></tr>
<tr><td colspan="3">Appropriate 2nd order dominant poles model for the closed loop uncompensated system is:

$G_m(s) =$ ____________________</td></tr>
<tr><td>Estimate of the Closed Loop Steady State Error:

$e_{ss\%} =$</td><td>Estimate of the Closed Loop Percent Overshoot:

$PO =$</td><td>Estimate of the Closed Loop Settling Time:

$T_{settle(\pm 2\%)} =$</td></tr>
<tr><td>System Type?</td><td colspan="2">Error Constants:

$K_{pos} =$ $K_v =$ $K_a =$</td></tr>
<tr><td colspan="3">The Lag Controller transfer function is:

$G_c(s) = K_c \frac{\alpha \tau s + 1}{\tau s + 1} =$</td></tr>
<tr><td colspan="3">The Controller Parameters are:

$\alpha =$ $\tau =$ $K_c =$</td></tr>
<tr><td>Estimate of the Closed Loop Steady State Error:

$e_{ss\%} =$</td><td>Estimate of the Closed Loop Percent Overshoot:

$PO =$</td><td>Estimate of the Closed Loop Settling Time:

$T_{settle(\pm 2\%)} =$</td></tr>
</table>

Question 5 Continued

Question 5 Continued

Question 6

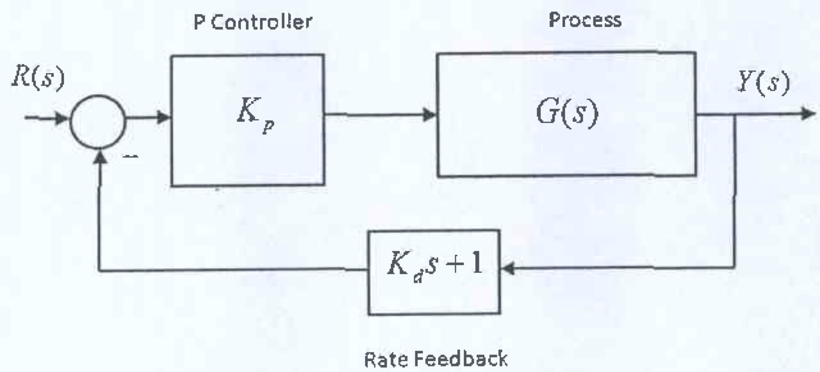
Steady State Errors, Error Constants and System Type. Second Order System Response, Proportional Control and Proportional + Rate Feedback Control, Stability in s-Domain.

PART A (10 marks)

Consider a step response of a certain process $G(s)$ to a unit reference signal, shown in Figure Q6.1 on the next page. Assume a second order dominant poles model for the process G(s), find the model parameters (DC gain, K_{dc}, damping ratio, ζ, frequency of natural oscillations, ω_n) and write the model transfer function $G_m(s)$. Place your answers in Table Q6.

PART B (10 marks)

Assume now that the process G(s) identified in Part A is part of a closed loop control system under Proportional +Rate Feedback Control, as shown below.



Compute the Proportional Controller Gain, K_p, and the Rate Feedback Gain, K_d, such that the closed loop system Steady State Error for a step input, $e_{ss(step\%)}$, is equal to 5%, and the Percent Overshoot PO is equal to 10%. Place your answers in Table Q6. Is the system stable at those values of the Controller Parameters?

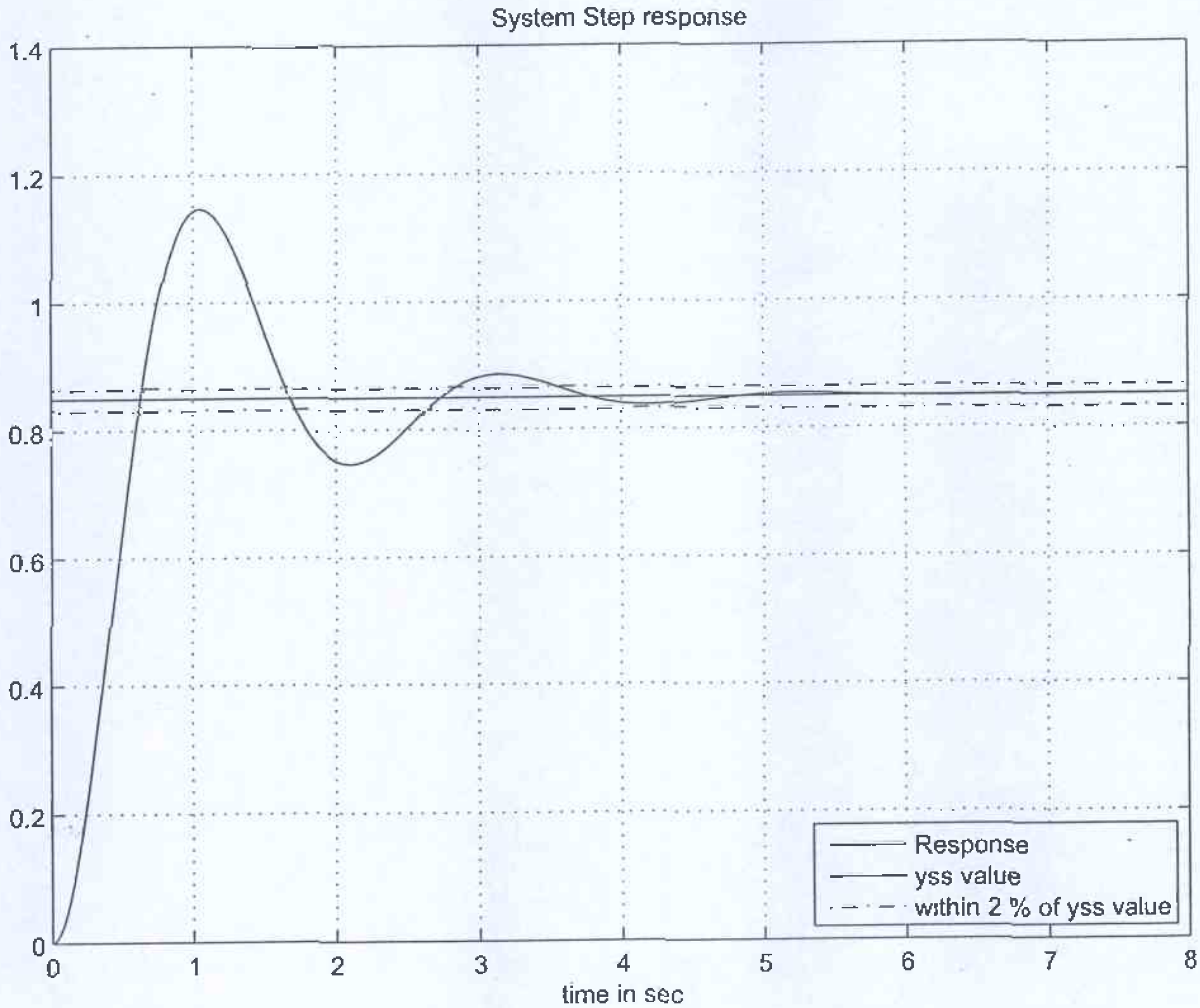

System Step response
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6
7
8
time in sec
Response
yss value
within 2 % of yss value

Figure Q6.1

Table Q6

<table>
<tr><td colspan="6">Appropriate 2nd order dominant poles model for the process G(s) is:

$G_m(s) =$ ____________________</td></tr>
<tr><td colspan="2">$K_{dc} =$</td><td colspan="2">$\zeta =$</td><td colspan="2">$\omega_n =$</td></tr>
<tr><td colspan="6">The Controller Parameters are:</td></tr>
<tr><td colspan="3">$K_p =$</td><td colspan="3">$K_d =$</td></tr>
<tr><td colspan="2">Is the system stable at those values of controller parameters?
Circle your answer:</td><td colspan="2">YES</td><td colspan="2">NO</td></tr>
<tr><td colspan="6">The closed loop compensated step response specifications are estimated as follows:</td></tr>
<tr><td colspan="2">$PO =$</td><td colspan="2">$T_{settle\pm2\%} =$</td><td colspan="2">$e_{ss\%} =$</td></tr>
<tr><td colspan="2">System Type?</td><td colspan="4">Error Constants:

$K_{pos} =$ $K_v =$ $K_a =$</td></tr>
</table>

Question 6 Continued

Question 7

Controllability and Observability, System Eigenvalues, Transfer Function from State Space Model. Controller Design by Pole Placement Method.

Consider a control system described by the following state space model:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & -2 & -2 & -1\end{bmatrix}\cdot\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 0\\ 1\end{bmatrix}\cdot u$$

$$y=\begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}$$

Part A (10 marks)

Find the system transfer function $G(s)=\dfrac{Y(s)}{U(s)}$

HINT: Because the system equations are in a Canonical Form, you should be able to do so by inspections, without calculating the Transfer Function Matrix;

Next, find the system eigenvalues, check the system controllability and observability and identify the uncontrollable or unobservable mode(s), if any.

Part B (10 marks)

The system is to operate as a regulator. Assume the state feedback:

$$u = K\cdot \mathbf{x}$$

where:

$$\mathbf{K}=\begin{bmatrix}k_1 & k_2 & k_3 & k_4\end{bmatrix}$$

and use pole placement by state feedback to place the compensated system eigenvalues at: -1, -2, -3 and -4. What are the gain values in the vector **K** ? Discuss any implementation issues that may be relevant. Are there any concerns? Would the situation be any different if the closed loop eigenvalues were to be placed at: -2, -3, -4 and -5? Explain.

Question 7 continued

Question 7 continued

Question 8

Stability Analysis in Frequency Domain - Nyquist Stability Criterion. Polar plots, Complex Numbers Algebra.

Consider again the same system as in Figure Q1.1, operating under Proportional Control. The process transfer function in frequency domain is described as:

$$G(j\omega) = \frac{100}{(j\omega + 2)(j\omega + 4)(j\omega + 10)}$$

Your task is to investigate the stability of the closed loop system using the Nyquist Stability Criterion.

PART A (8 marks)

Figure Q8.1 shows a Nyquist contour for $G(j\omega)$ - apply the Nyquist Criterion of stability to find the critical gain, K_{crit}, at which the closed loop system becomes marginally stable and the practical range of safe operating gains for the Proportional Controller.

Find the Gain Margin of the system, both in V/V units and in decibel units, when the Operating Gain of the Controller is equal to 2 - use Figure Q8.1 to read the required values off the graph.

PART B (12 marks)

Find analytical solutions for the crossovers of the Nyquist contour shown in Figure Q8.1 with the Real and Imaginary axis, and based on that the corresponding frequency of marginal oscillations, ω_{osc}, and the critical gain, K_{crit}, at which the closed loop system becomes marginally stable - are your answers for the stability range consistent with the results from Part A? Place your answers in Table Q8.

Table Q8.1

Crossovers with Real Axis:		Crossovers with Imaginary Axis:	
Corresponding frequencies:		Corresponding frequencies:	

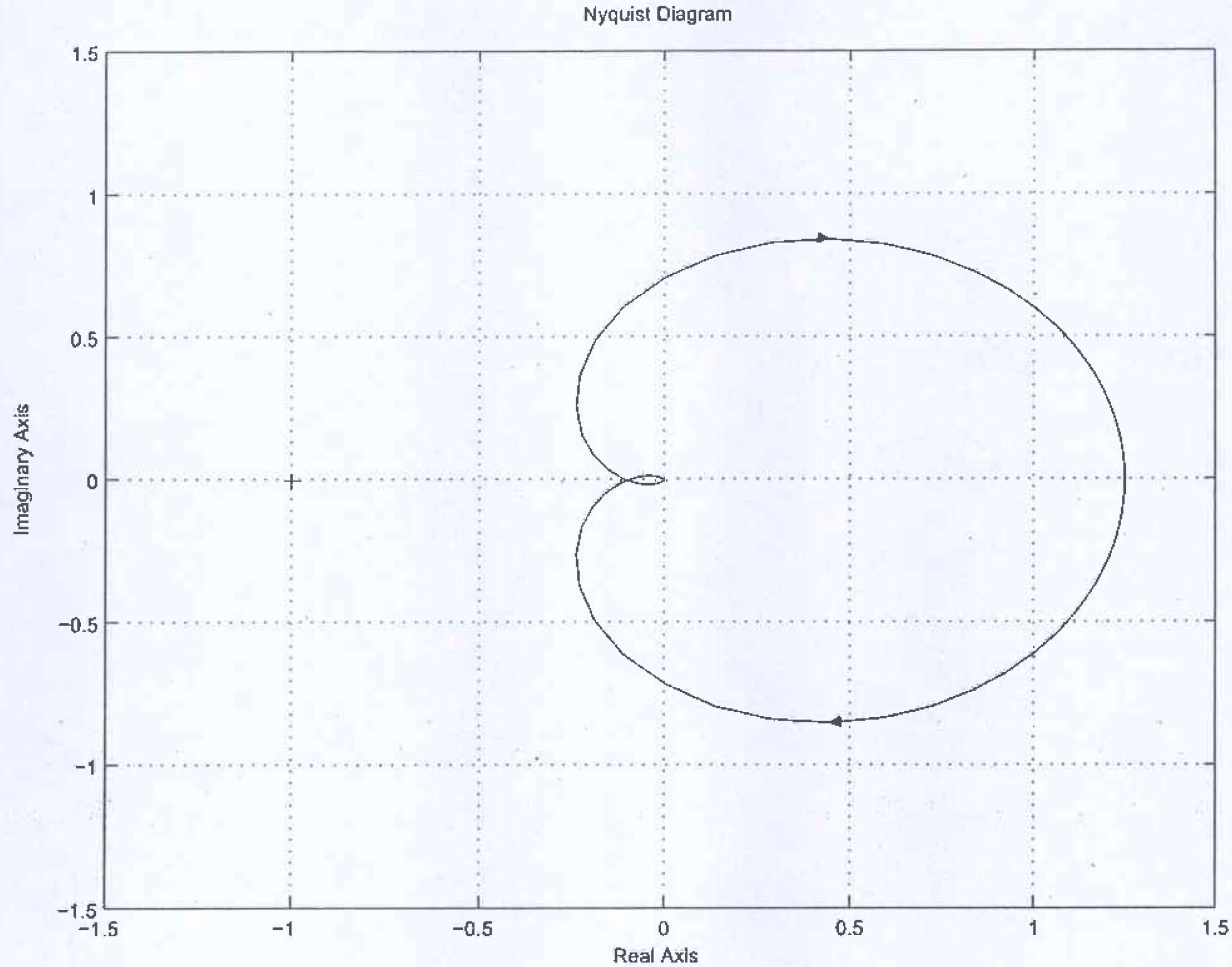


Figure Q8.1

Are your analytical results in Part B consistent with the graphical results in Part A?

Question 8 Continued

Question 8 Continued