

National Exam December 2019

## 04-BS-6, Mechanics of Materials

Duration: 3 hours

**Notes:**

1. If doubt exists as to the interpretation of any questions, the candidate is urged to submit with the answer paper a clear statement of any assumptions made.
2. Candidates may use one of two calculators, the Casio or Sharp approved models.

This is a CLOSED BOOK exam. However, candidates are permitted to bring the following into the Examination Room:

- ONE Aid Sheet 8.5" x 11" hand-written on both sides containing notes and formulae.  
**Example problems and solutions to problems are NOT ALLOWED!**

3. Any FIVE (5) questions (out of 8 given) constitute a complete paper. Only the first five questions as they appear in your Answer Book(s) will be marked.
4. All questions are of equal value.
5. Information on geometric properties of wide flange or W shaped sections is provided in Appendices (at the end of this Question Paper). There are two pages. Note that this information may not be required.

---

**Note: This Question Paper along with the Aid Sheet (if used) must be handed in with the Answer Book(s).**

*Your Exam will not be marked if you do not hand in the Aid Sheet, unless there is a signed statement by the exam invigilator stating that no Aid Sheet was used in the exam writing.*

---

**Question 1:**

A column consists of a square concrete core of 100 mm x 100 mm dimension and an outer steel sleeve of 2 mm wall thickness, as shown in Figure 1(b). A load of 250 kN is applied to the composite column through a rigid steel bearing plate placed on top of the column (see Figure 1(a)). The structural steel has an allowable stress of 175 MPa and Young's modulus of 200 GPa, while the concrete has an allowable stress of 20 MPa and Young's modulus equal to 16 GPa. Determine:

- [13 marks] 1) The forces carried by the concrete core and sleeve;
- [4 marks] 2) The corresponding displacement of the column; and
- [3 marks] 3) The maximum allowable load that can be carried by the column.

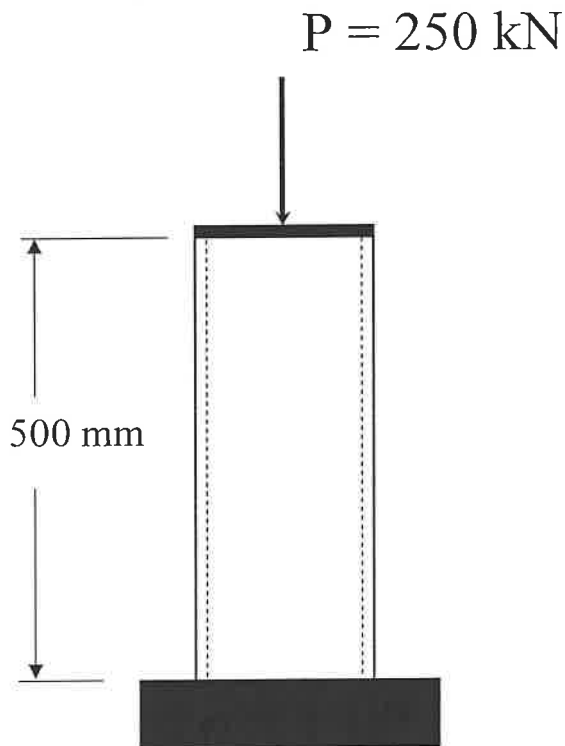


Figure 1(a)

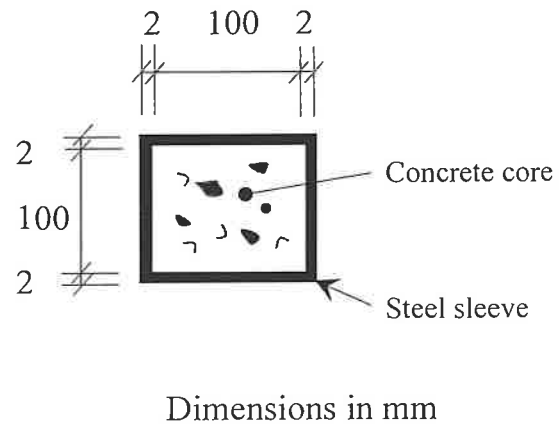


Figure 1(b)

**Question 2:**

Sketch the shear force and bending moment diagrams for the steel beam shown in Figure 2.

Determine the shear and moment throughout the beam as functions of  $x$ . This means that you need to give formulas for  $V(x)$  and  $M(x)$  along the length of the beam.

Label points of maximum positive and negative bending moment, as well as any inflection points. Show your work by indicating exactly how you obtained your answer.

[20 marks]

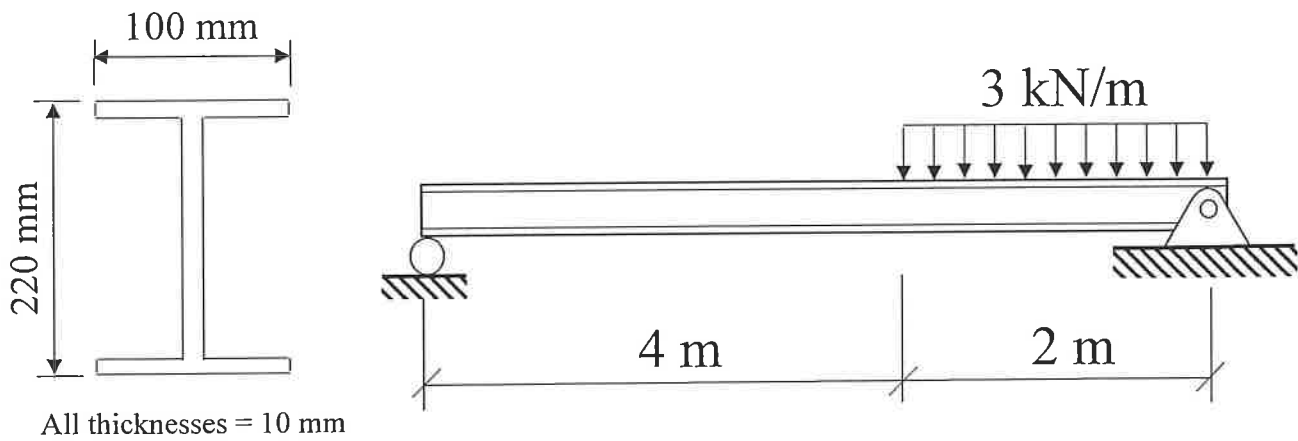


Figure 2

No credit will be given for a solution using the Principle of Superposition, when combination of existing solutions is used to find an answer.

**Question 3:**

For the simply supported beam in Question 2, given that the steel has an allowable normal stress of  $\sigma_{allow} = 240$  MPa, allowable shear stress of  $\tau_{allow} = 85$  MPa, and Young's modulus of  $E_{st} = 200$  GPa, do the following:

- [18 marks]      1) Determine the maximum deflection in the beam using the Method of Integration. Sketch the deflected shape of the structure;
- [2 marks]        2) Does the beam satisfy deflection requirements given that the maximum allowable deflection is equal to  $L/360$  (where  $L$  is the span of the beam)?

**Question 4:**

The grain of a wooden member forms an angle of  $20^\circ$  with the vertical and is subjected to the state of stress as shown in Figure 4.

- [7 marks] 1) Determine the in-plane shearing stress parallel to the grain and the normal stress perpendicular to the grain.
- [4 marks] 2) Show your answer on a Mohr's circle.
- [7 marks] 3) Find the maximum shear stress (and corresponding normal stress) and orientation. Show this on a properly oriented stress element.
- [2 marks] 4) Why would we need to determine the stresses parallel and normal to the grain?

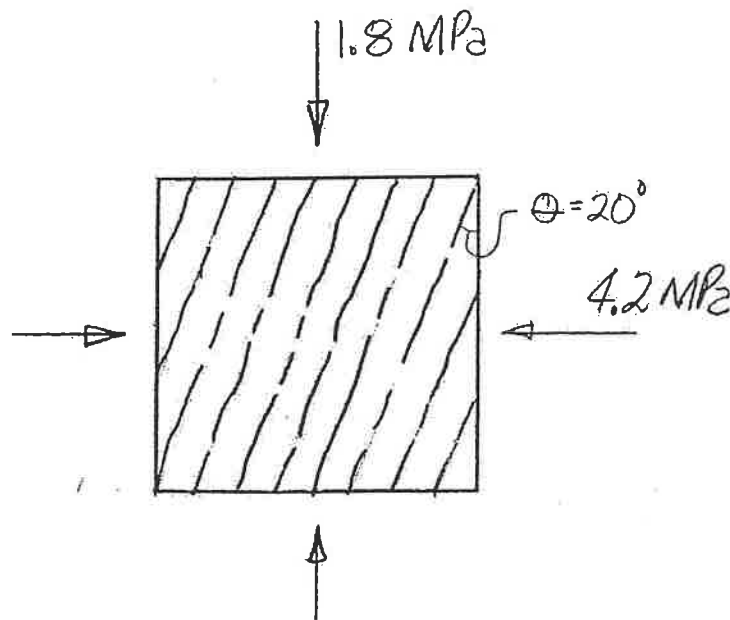


Figure 4

Note that credit will only be **given** for a **solution using Mohr's circle**. This means that you need to draw a Mohr's circle based on the stress components given in this problem. Remember to show numbers on your circle. Your **calculations** must be based on the geometry of your circle. So use your calculator. In other words, you are expected to use trigonometry to construct your Mohr's circle. Do not give a graphical solution that is scaled off! The stress transformation equations can only be used to check your answer.

**Question 5:**

A rigid horizontal bar AB is pin-supported at A and carries a load P at B. The bar is supported at C by a 2.5 m long column (with  $E_{\text{steel}} = 200 \text{ GPa}$  and  $\sigma_Y = 350 \text{ MPa}$ ) having a square cross-section of 50 mm x 50 mm and pinned at both ends, as seen in Figure 5.

Determine the maximum allowable load P that can be applied to the horizontal bar at B without causing the column to buckle. Use a factor of safety with respect to buckling of  $FS = 3$  and consider in-plane buckling only.

[20 marks]

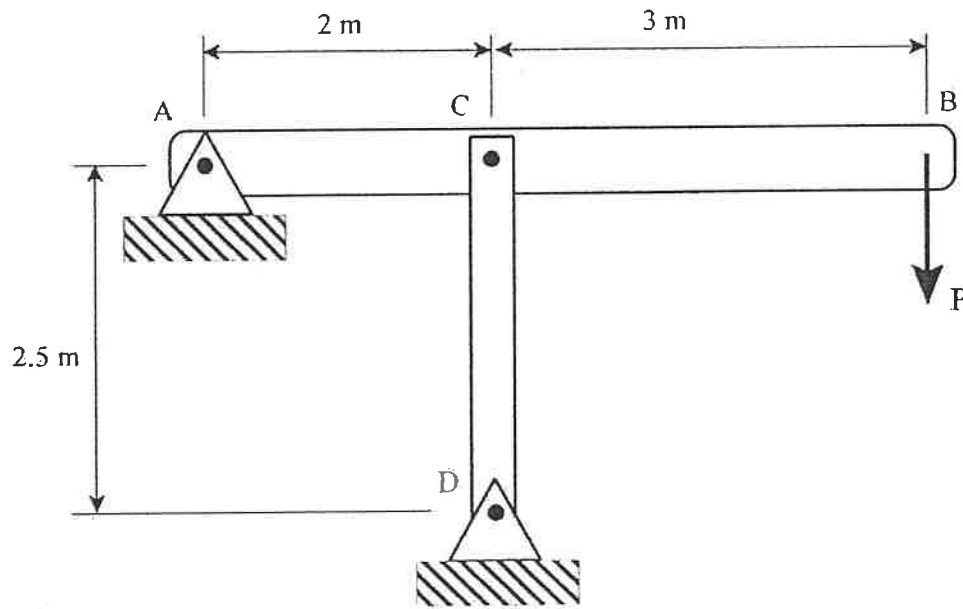


Figure 5

**Questions 6:**

A solid stepped shaft is subjected to three torques (at B, C, and D) acting as shown in Figure 6. The shaft is made of steel with a shear modulus of  $G = 80 \text{ GPa}$  and a yield stress of  $\tau_Y = 150 \text{ MPa}$ . Dimensions along diameter and length of the shaft are shown in figure.

- [5 marks] 1) Draw a torque force diagram for this problem (i.e. show the variation of internal torque along the length of the shaft);
- [10 marks] 2) Determine the maximum shear stress in the shaft, and sketch the corresponding variation of shear stress along the shaft radius at this point (where the stress is maximum);
- [5 marks] 3) Find the angle of twist of the right end of the shaft (point D) relative to the wall (point A). Give your answer in degrees.

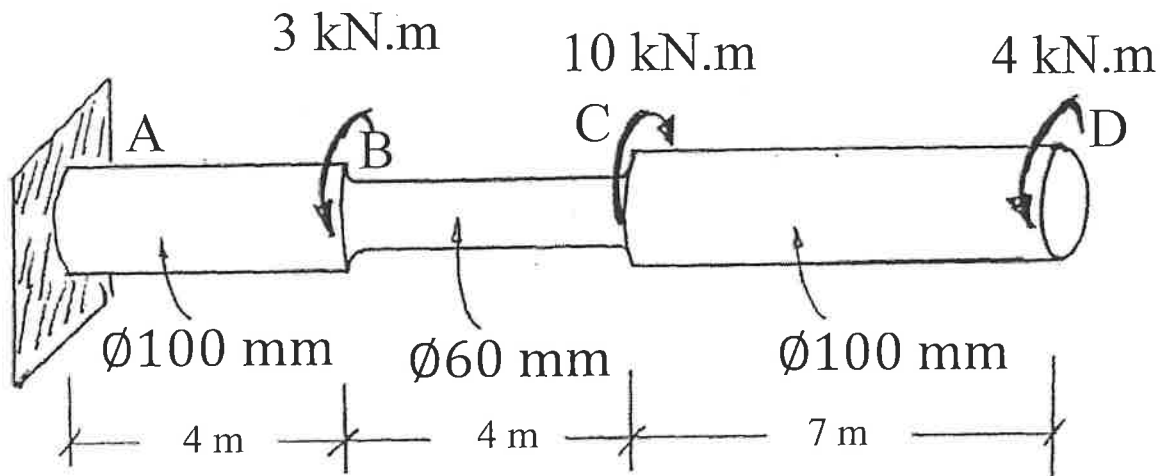


Figure 6

**Questions 7:**

A 144 mm x 240 mm timber beam is strengthened by gluing on a 120 mm x 10 mm steel strap as shown in Figure 7 below. The beam supports a positive bending moment about the horizontal axis of  $M = 20 \text{ kN.m}$  and shear force of  $V = 15 \text{ kN}$ .

$$E_{\text{wood}} = 7 \text{ GPa}$$

$$E_{\text{steel}} = 200 \text{ GPa}$$

[20 marks] Determine the following:

- 1) Maximum bending stress in the wood;
- 2) Maximum bending stress in the steel; and
- 3) Shear stress in the glue joint.

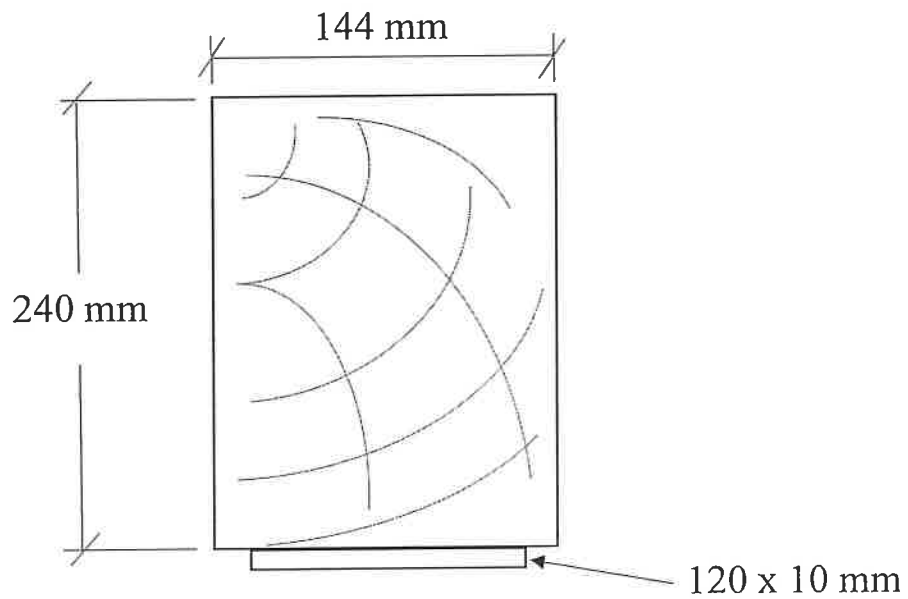


Figure 7



**Question 8:**

The rectangular beam in Figure 8 is subjected to a distributed load of 10 kN/m and an axial load of 200 kN as shown. Determine the distribution of normal stress and shear stress acting over the cross-section of the beam at point B.

Further, represent the result on a volume element sketch.

[20 marks]

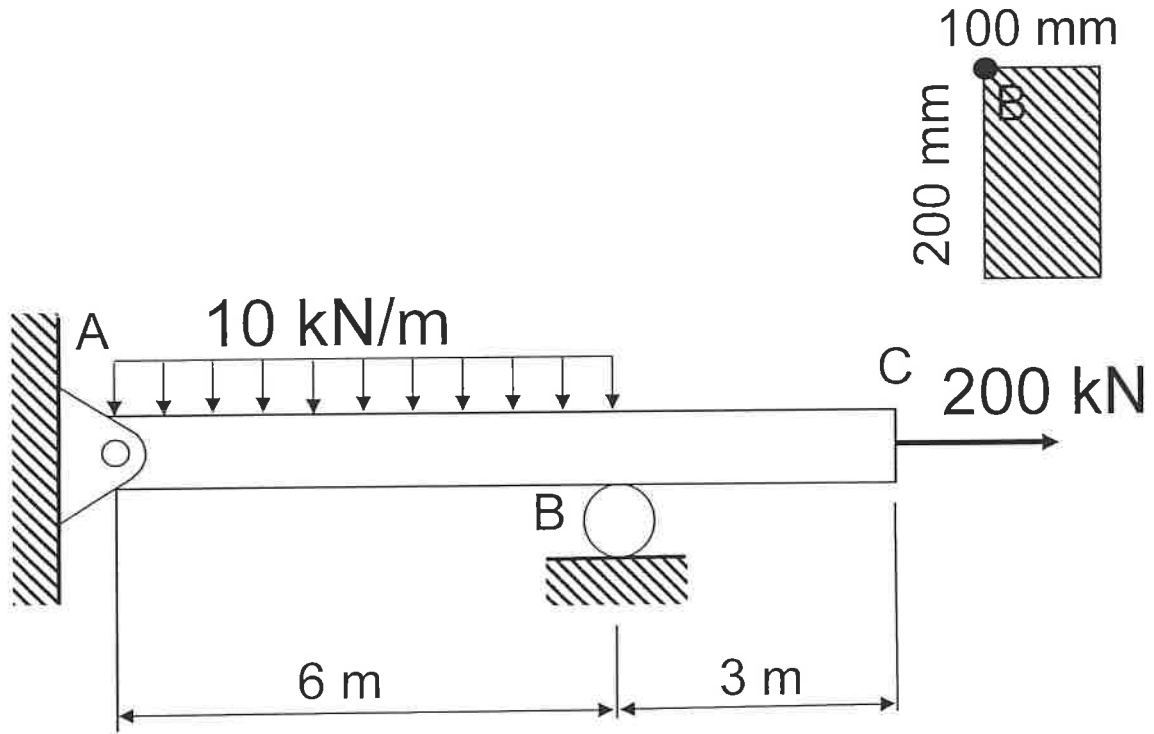
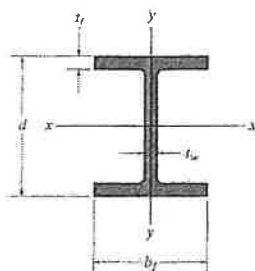


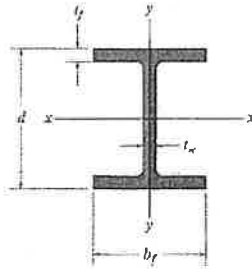
Figure 8

APPENDIX C GEOMETRIC PROPERTIES OF WIDE-FLANGE SECTIONS



Wide-Flange Sections or W Shapes SI Units											
Designation	Area A	Depth d	Web thickness t <sub>w</sub>	Flange		x-x axis			y-y axis		
				width b <sub>f</sub>	thickness t <sub>f</sub>	I	S	r	I	S	r
mm × kg/m	mm <sup>2</sup>	mm	mm	mm	mm	10 <sup>6</sup> mm <sup>4</sup>	10 <sup>3</sup> mm <sup>3</sup>	mm	10 <sup>6</sup> mm <sup>4</sup>	10 <sup>3</sup> mm <sup>3</sup>	mm
W610 × 155	19 800	611	12.70	324.0	19.0	1 290	4 220	255	108	667	73.9
W610 × 140	17 900	617	13.10	230.0	22.2	1 120	3 630	250	45.1	392	50.2
W610 × 125	15 900	612	11.90	229.0	19.6	985	3 220	249	39.3	343	49.7
W610 × 113	14 400	608	11.20	228.0	17.3	875	2 880	247	34.3	301	48.8
W610 × 101	12 900	603	10.50	228.0	14.9	764	2 530	243	29.5	259	47.8
W610 × 92	11 800	603	10.90	179.0	15.0	646	2 140	234	14.4	161	34.9
W610 × 82	10 500	599	10.00	178.0	12.8	560	1 870	231	12.1	136	33.0
W460 × 97	12 300	466	11.40	193.0	19.0	445	1 910	190	22.8	236	43.1
W460 × 89	11 400	463	10.50	192.0	17.7	410	1 770	190	20.9	218	42.8
W460 × 82	10 400	460	9.91	191.0	16.0	370	1 610	189	18.6	195	42.3
W460 × 74	9 460	457	9.02	190.0	14.5	333	1 460	188	16.6	175	41.9
W460 × 68	8 730	459	9.14	154.0	15.4	297	1 290	184	9.41	122	32.8
W460 × 60	7 590	455	8.00	153.0	13.3	255	1 120	183	7.96	104	32.4
W460 × 52	6 640	450	7.62	152.0	10.8	212	942	179	6.34	83.4	30.9
W410 × 85	10 800	417	10.90	181.0	18.2	315	1 510	171	18.0	199	40.8
W410 × 74	9 510	413	9.65	180.0	16.0	275	1 330	170	15.6	173	40.5
W410 × 67	8 560	410	8.76	179.0	14.4	245	1 200	169	13.8	154	40.2
W410 × 53	6 820	403	7.49	177.0	10.9	186	923	165	10.1	114	38.5
W410 × 46	5 890	403	6.99	140.0	11.2	156	774	163	5.14	73.4	29.5
W410 × 39	4 960	399	6.35	140.0	8.8	126	632	159	4.02	57.4	28.5
W360 × 79	10 100	354	9.40	205.0	16.8	227	1 280	150	24.2	236	48.9
W360 × 64	8 150	347	7.75	203.0	13.5	179	1 030	148	18.8	185	48.0
W360 × 57	7 200	358	7.87	172.0	13.1	160	894	149	11.1	129	39.3
W360 × 51	6 450	355	7.24	171.0	11.6	141	794	148	9.68	113	38.7
W360 × 45	5 710	352	6.86	171.0	9.8	121	688	146	8.16	95.4	37.8
W360 × 39	4 960	353	6.48	128.0	10.7	102	578	143	3.75	58.6	27.5
W360 × 33	4 190	349	5.84	127.0	8.5	82.9	475	141	2.91	45.8	26.1

WIDE-FLANGE SECTIONS OR W SHAPES FPS UNITS



Wide-Flange Sections or W Shapes SI Units											
Designation	Area A	Depth d	Web thickness t <sub>w</sub>	Flange		x-x axis			y-y axis		
				width b <sub>f</sub>	thickness t <sub>f</sub>	I	S	r	I	S	r
W310 × 129	16 500	318	13.10	308.0	20.6	308	1940	137	100	649	77.8
W310 × 74	9 480	310	9.40	205.0	16.3	165	1060	132	23.4	228	49.7
W310 × 67	8 530	306	8.51	204.0	14.6	145	948	130	20.7	203	49.3
W310 × 39	4 930	310	5.84	165.0	9.7	84.8	547	131	7.23	87.6	38.3
W310 × 33	4 180	313	6.60	102.0	10.8	65.0	415	125	1.92	37.6	21.4
W310 × 24	3 040	305	5.59	101.0	6.7	42.8	281	119	1.16	23.0	19.5
W310 × 21	2 680	303	5.08	101.0	5.7	37.0	244	117	0.986	19.5	19.2
W250 × 149	19 000	282	17.30	263.0	28.4	259	1840	117	86.2	656	67.4
W250 × 80	10 200	256	9.40	255.0	15.6	126	984	111	43.1	338	65.0
W250 × 67	8 560	257	8.89	204.0	15.7	104	809	110	22.2	218	50.9
W250 × 58	7 400	252	8.00	203.0	13.5	87.3	693	109	18.8	185	50.4
W250 × 45	5 700	266	7.62	148.0	13.0	71.1	535	112	7.03	95	35.1
W250 × 28	3 620	260	6.35	102.0	10.0	39.9	307	105	1.78	34.9	22.2
W250 × 22	2 850	254	5.84	102.0	6.9	28.8	227	101	1.22	23.9	20.7
W250 × 18	2 280	251	4.83	101.0	5.3	22.5	179	99.3	0.919	18.2	20.1
W200 × 100	12 700	229	14.50	210.0	23.7	113	987	94.3	36.6	349	53.7
W200 × 86	11 000	222	13.00	209.0	20.6	94.7	853	92.8	31.4	300	53.4
W200 × 71	9 100	216	10.20	206.0	17.4	76.6	709	91.7	25.4	247	52.8
W200 × 59	7 580	210	9.14	205.0	14.2	61.2	583	89.9	20.4	199	51.9
W200 × 46	5 890	203	7.24	203.0	11.0	45.5	448	87.9	15.3	151	51.0
W200 × 36	4 570	201	6.22	165.0	10.2	34.4	342	86.8	7.64	92.6	40.9
W200 × 22	2 860	206	6.22	102.0	8.0	20.0	194	83.6	1.42	27.8	22.2
W150 × 37	4 730	162	8.13	154.0	11.6	22.2	274	68.5	7.07	91.8	38.7
W150 × 30	3 790	157	6.60	153.0	9.3	17.1	218	67.2	5.54	72.4	38.2
W150 × 22	2 860	152	5.84	152.0	6.6	12.1	159	65.0	3.87	50.9	36.8
W150 × 24	3 060	160	6.60	102.0	10.3	13.4	168	66.2	1.83	35.9	24.5
W150 × 18	2 290	153	5.84	102.0	7.1	9.19	120	63.3	1.26	24.7	23.5
W150 × 14	1 730	150	4.32	100.0	5.5	6.84	91.2	62.9	0.912	18.2	23.0