National Exams May 2016
04-BS-1, Mathematics
3 hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to include a clear statement of any assumptions made along with their answer.
2. Any APPROVED CALCULATOR is permitted. This is a CLOSED BOOK exam. However, candidates are permitted to bring ONE AID SHEET written on both sides.
3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
4. All questions are of equal value.

Marking Scheme:

1. (a) 7 marks, (b) 7 marks, (c) 6 marks
2. 20 marks
3. 20 marks
4. 20 marks
5. 20 marks
6. 20 marks
7. 20 marks
8. 20 marks

1. Find the general solutions of the following differential equations:

   (a) $xy' + y = 2\cos(3x)$,

   (b) $y' + 2xy^2 = 0$,

   (c) $2x^2y'' + 5xy' - 2y = 0$.

   Note that in each case, $'$ denotes differentiation with respect to x.

2. Solve the following initial value problem

$$y'' - 12y' + 45y = 18\cos(3t), \quad y(0) = 0, \quad y'(0) = 0.$$

   Note that, $'$ denotes differentiation with respect to t.

3. Find the general solution to the following system of differential equations.

$$\frac{dx}{dt} = 4x + 2y,$$
$$\frac{dy}{dt} = 3x - y + e^{-2t}.$$

4. Find the minimum value of the function $F(x, y, z) = 2x^2 + y^2 + 3z^2$ subject to the constraint $x + y - z + 1 = 0$

5. Find the line tangent to the intersection of the surfaces

$$3x^2 + 2y^2 - 2z = 1$$

   and

$$x^2 + y^2 + z^2 - 4y - 2z + 2 = 0$$

   at the point $(1, 1, 2)$.

6. Find the volume of the region bounded by the paraboloid $z = \frac{7}{4} + \frac{1}{4}(x^2 + y^2)$ and the plane $z = 4$ that lies outside the cone $z^2 - 4x^2 - 4y^2 = 0$.

7. Find the surface area of that portion of the surface $z = 1 - \sqrt{x^2 + y^2}$ that lies in the first octant.

8. Evaluate the line integral $\oint_C \mathbf{v} \cdot d\mathbf{r}$, where C is the curve formed by the intersection of the cylinder $x^2 + y^2 = 9$ and the plane $z = 1 + y - 3x$, travelled clockwise as viewed from the positive z-axis, and $\mathbf{v}$ is the vector function $\mathbf{v} = 2z^2\mathbf{i} - 2y\mathbf{j} + 2y\mathbf{k}$.