NATIONAL EXAMS
07-Elec-B2 Advanced Control Systems – Dec 2013

3 hours duration

NOTES:

1. If a doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Candidates may use one of two calculators, a Casio FX-991 or a Sharpe EL-540. This is a closed-book examination. Tables of Laplace and z-transforms are attached.

3. Any four questions constitute a complete paper. Only the first four questions as they appear in your answer book will be marked.

4. All questions are of equal value.

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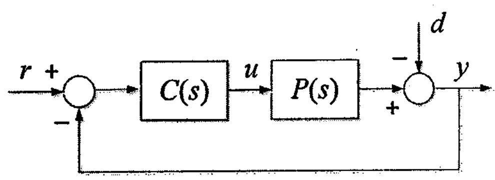
1. Consider the control system below with $P(s) = \dfrac{3(10-s)}{(10+s)(3+2s)}$ and $C(s) = \dfrac{K}{s}$.



(a) The value of K is increased from zero to a value of K_{max} at which the system exhibits sustained oscillation. What is the value of K_{max} and what is the oscillation frequency?

(b) For $K = K_{max}/2$ determine the phase margin.

(c) Define the tracking error, $e(t) = r(t) - y(t)$. Determine the steady state tracking error when $d(t) = 1$, and $r(t)$ is a ramp with unit slope.

(d) Determine the steady state tracking error when $d(t) = 0$, and $r(t) = 2 \sin 3t$.

2. Consider the system, $P(s) = \dfrac{10(\beta s+1)}{s(0.4s+1)^2}$.

(a) Find a state space model for the system.

(b) Justify the conditions under which the system controllable? observable?

(c) The system input and output are uniformly sampled with a sample period of h and the discrete input is applied to a zero order hold device. Determine the poles of the sampled data system as a function of h. Detailed calculations are not necessary.

3. Consider the system,

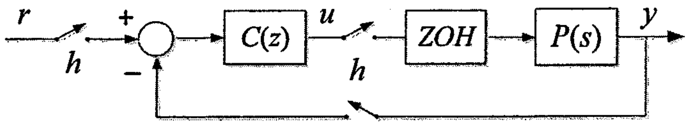
$$\begin{bmatrix} 0 & 1 & 0 \\ & 0 & -1 \\ 0 & 0 & -2 \end{bmatrix} x + B = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x$$

Design a controller of the form, $u(t) = Kx(t) + Lr(t)$ such that the closed loop poles are at $s = -5, -3 \pm j$ and the DC gain, that relates a constant value of y to a constant value of r, is unity.

4. Consider the sampled data system shown on the right. The input to the ZOH, the set-point, r, and the output, y, are uniformly sampled with a sample period of h. $C(z)$ and $P(s)$ are given by,

$$C(z) = K_1 + \frac{K_2}{1-z^{-1}}, \quad P(s) = \frac{1}{s}$$



(a) Determine $C(z)$ such that the closed loop poles are all located at $z = 0$.

(b) Determine the corresponding discrete closed loop transfer function, $T(z)$, that relates $X(z)$ to $R(z)$.

(c) Sketch the associated unit step response at $y(t)$, being careful to show the inter-sample behavior.

5. An experiment is conducted on a continuous time system, $P(s)$, whose output is uniformly sampled with sample period, $h = 1$ second. The discrete input, $u(kh)$, is applied to a zero order hold device which drives $P(s)$. Measurements for $u(kh)$ and $y(kh)$ appear in the Table. Assume that $P(s)$ is a first order continuous system with an unknown time (or transportation) delay of Nh , N being unknown.

kh	$u(kh)$	$y(kh)$
0	0	0
1	1	0
2	1	0
3	1	4.000
4	1	4.800
5	1	4.960
6	1	4.992

(a) Identify a discrete time model, $G(z)$, that relates $y(kh)$ to $u(kh)$.

(b) Identify $P(s)$.

6. (a) Design a proportional feedback controller for the plant, $P(s) = \dfrac{e^{-0.2s}}{s(2s+1)}$, such that the gain margin is 8dB. (b) Determine the associated phase margin. (c) Determine the steady state output when the set point input is a unit step.

Inverse Laplace Transforms	
$F(s)$	$f(t)$
$\dfrac{A}{s+\alpha}$	$Ae^{-\alpha t}$
$\dfrac{C+jD}{s+\alpha+j\beta}+\dfrac{C-jD}{s+\alpha-j\beta}$	$2e^{-\alpha t}\left(C\cos\beta t+D\sin\beta t\right)$
$\dfrac{A}{(s+\alpha)^{n+1}}$	$\dfrac{At^n e^{-\alpha t}}{n!}$
$\dfrac{C+jD}{\left(s+\alpha+j\beta\right)^{n+1}}+\dfrac{C-jD}{\left(s+\alpha-j\beta\right)^{n+1}}$	$\dfrac{2t^n e^{-\alpha t}}{n!}\left(C\cos\beta t+D\sin\beta t\right)$

Inverse z-Transforms	
$F(z)$	$f(nT)$
$\dfrac{Kz}{z-a}$	Ka^n
$\dfrac{\left(C+jD\right)z}{z-re^{j\varphi}}+\dfrac{\left(C-jD\right)z}{z-re^{-j\varphi}}$	$2r^n\left(C\cos n\varphi-D\sin n\varphi\right)$
$\dfrac{Kz}{\left(z-a\right)^r}$, $r=2,3...$	$\dfrac{Kn\left(n-1\right)...\left(n-r+2\right)}{\left(r-1\right)!a^{r-1}}a^n$

Table of Laplace and z-Transforms (*h* denotes the sample period)		
$f(t)$	$F(s)$	$F(z)$
unit impulse	1	1
unit step	$\frac{1}{s}$	$\frac{z}{z-1}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$	$\frac{z}{z-e^{\alpha h}}$
t	$\frac{1}{s^2}$	$\frac{hz}{(z-1)^2}$
$\cos \beta t$	$\frac{s}{s^2+\beta^2}$	$\frac{z(z-\cos \beta h)}{z^2-2z\cos \beta h+1}$
$\sin \beta t$	$\frac{\beta}{s^2+\beta^2}$	$\frac{z\sin \beta h}{z^2-2z\cos \beta h+1}$
$e^{-\alpha t}\cos \beta t$	$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$	$\frac{z(z-e^{-\alpha h}\cos \beta h)}{z^2-2ze^{-\alpha h}\cos \beta h+e^{-2\alpha h}}$
$e^{-\alpha t}\sin \beta t$	$\frac{\beta}{(s+\alpha)^2+\beta^2}$	$\frac{ze^{-\alpha h}\sin \beta h}{z^2-2ze^{-\alpha h}\cos \beta h+e^{-2\alpha h}}$
$t f(t)$	$-\frac{dF(s)}{ds}$	$-zh\frac{dF(z)}{dz}$
$e^{-\alpha t} f(t)$	$F(s+\alpha)$	$F(ze^{\alpha h})$