National Exams- Dec, 2018
16-CIV-B9

The Finite Element Method

3 hours duration

Notes:

1. There are 4 pages in this examination.
2. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
3. This is a closed book exam but one aid sheet, written on both sides, is allowed.
4. Candidates may use one of the approved Casio or Sharp calculators.
5. All problems are of equal weight.

Problem 1

1.1 Assume that usual beam theory is employed and use the principle of virtual work to evaluate the reactions R_1 and R_2 on the beam shown in **Figure 1(a)**.

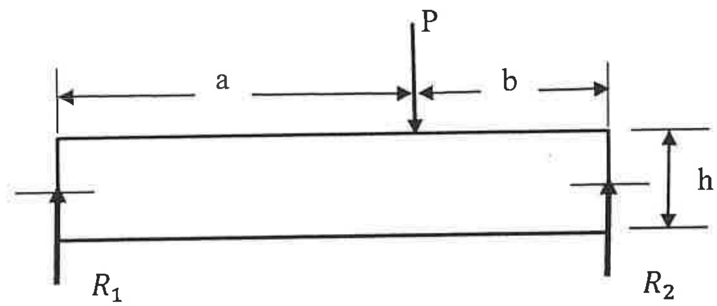

Figure 1(a)

1.2 Now assume that the beam is modeled by a four-node finite element (Figure 1(b)). Show that to be able to evaluate the reaction as in part 1.1 it is necessary that the finite element displacement functions can represent the rigid body mode displacements.

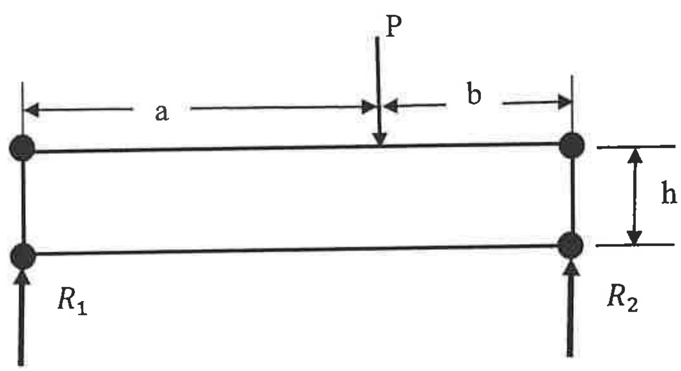

Figure 1 (b)

Problem 2

The beam shown in **Figure 2** is subjected to an uniform load $q = 10\ kN/m$ on span 2-3 (only). As shown, there is a small gap between the support between the middle support 2 and the beam. Draw the bending moment and shear force diagrams of the span 1-2.

Use $L = 6\ m$, $E = 200$ GPa , $\Delta = 5\ mm$.

The cross-section of the beam is rectangular with height $h = 300\ mm$ and width $b = 120\ mm$.

Give the procedure to draw the bending moment and shear force diagrams if the load increases to $q = 12\ kN/m$ (You are not asked to solve the full problem).

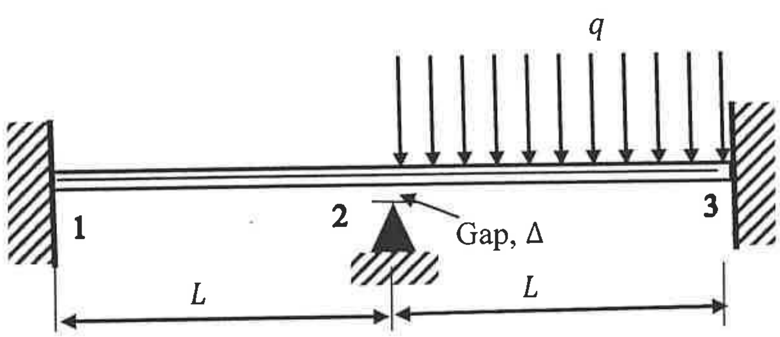

Figure 2

The stiffness matrix of a beam element of length L is given as following:

$$[K] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

The shape functions of two-node beam element are:

$$N_1(s) = \frac{2s^3}{L^3} - \frac{2s^2}{L^2} + 1$$

$$N_2(s) = \frac{s^3}{L^2} - \frac{2s^2}{L} + s$$

$$N_3(s) = -\frac{2s^3}{L^3} + \frac{3s^2}{L^2}$$

$$N_4(s) = \frac{s^3}{L^2} - \frac{s^2}{L}$$

Problem 3

3.1 Develop the shape functions for a rectangular 4 nodes element shown in **Figure 3(a)**.

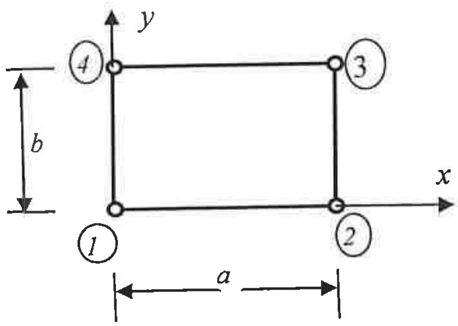

Figure 3(a)

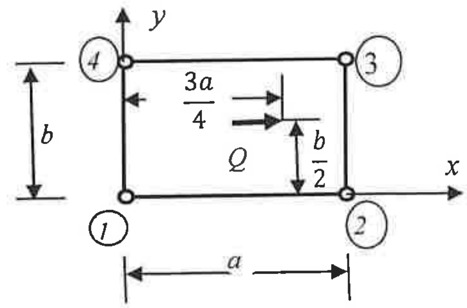

Figure 3(b)

3.2 Show that in the case of plane stress, the stiffness term k_{33} of a four-node rectangular element of dimension a and b (as shown in **Figure 3(a)**) and thickness t is given by:

$$k_{33} = \left(\frac{b}{3a} + \left(\frac{1-\nu}{6}\right)\frac{a}{b}\right) \cdot \frac{Et}{1-\nu^2}$$

3.3 A horizontal concentrated force, shown in **Figure 3(b)** , has a magnitude of Q. Calculate the equivalent forces in the rectangular element shown.

For the plane stress condition, the elasticity matrix is given:

$$[C] = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$