

**National Exams May 2018**

**16-Mec-A3, SYSTEM ANALYSIS AND CONTROL**

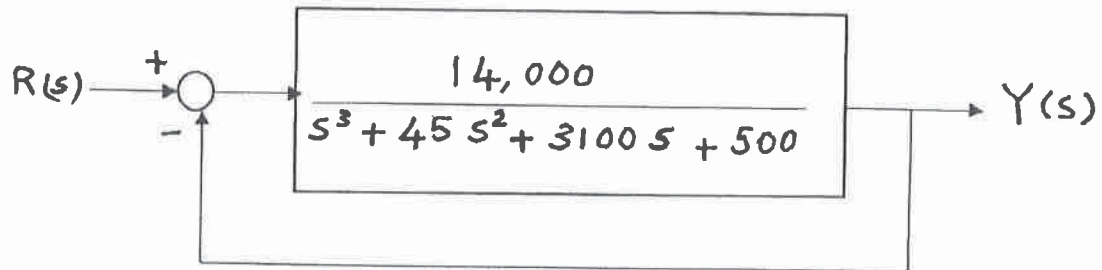
3 hours duration

**Notes:**

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. Candidates may use a Casio or Sharp approved calculator. This is a closed book exam. No aids other than semi-log graph papers are permitted.
3. Any four (4) questions constitute a complete paper. Only the first four (4) questions as they appear in your answer book will be marked.
4. All questions are of equal value.

**Question 1:**

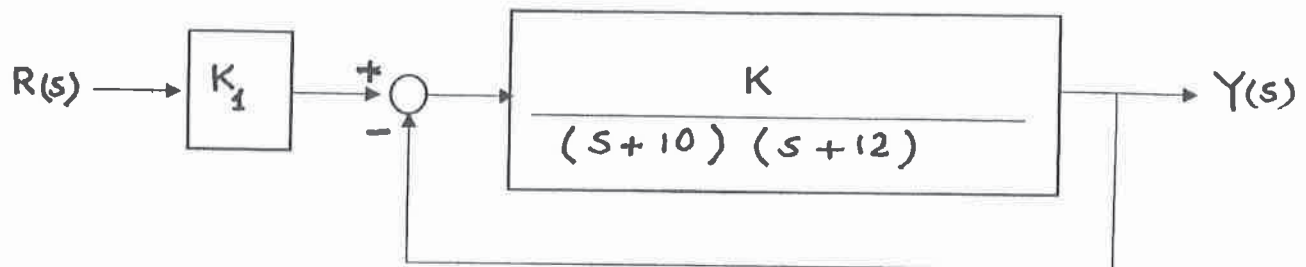
A closed-loop control system is seen below;



- Determine the transfer function  $T(s) = Y(s) / R(s)$ .
- Determine the poles and zeros of  $T(s)$ .
- Use a unit step input,  $R(s) = 1/s$  and obtain the partial fraction expansion for  $Y(s)$ .
- Predict the final value of  $y(t)$  for the unit step input.

**Question 2:**

A system is shown in the figure below;



- Determine the steady – state error for a unit step input in terms of  $K$  and  $K_1$ , where  $E(s) = R(s) - Y(s)$ .

b) Select  $K_1$  so that the steady-state error is zero.

**Question 3:**

A feedback control system has a characteristic equation

$$s^6 + 5s^5 + 14s^4 + 40s^3 + 64s^2 + 80s + 96 = 0$$

Determine whether the system is stable, and determine the values of the roots.

**Question 4:**

A unity negative feedback system has a transfer function

$$\begin{aligned} G(s) &= \frac{K(s^2 + 0.105625)}{s(s^2 + 1)} \\ &= \frac{K(s + j0.325)(s - j0.325)}{s(s^2 + 1)} \end{aligned}$$

Sketch the root locus as a function of  $K$ . Carefully calculate where the segments of the locus enter and leave the real axis.

**Question 5:**

A unity feedback control system has a transfer function

$$G(s) = \frac{K(s^2 + 6s + 12)}{s^2(s + 1)}$$

We desire the dominant roots to have a damping ratio equal to 0.707. Find the gain  $K$  when this condition is satisfied. Show that the complex roots are  $s = -2.3 \pm j2.3$  at this gain.

**Question 6:**

Sketch the Bode Diagram representation of the frequency response for the transfer functions;

$$\text{a) } GH(s) = \frac{s - 10}{s^2 + 6s + 10}$$

$$\text{b) } GH(s) = \frac{30(s + 8)}{s(s + 2)(s + 4)}$$

## Table of Laplace Transforms

$f(t)$	$\mathcal{L}\{f(t)\} = F(s)$		$f(t)$	$\mathcal{L}\{f(t)\} = F(s)$	
1	$\frac{1}{s}$	(1)	$\frac{ae^{at} - be^{bt}}{a - b}$	$\frac{s}{(s-a)(s-b)}$	(19)
$e^{at} f(t)$	$F(s-a)$	(2)	$te^{at}$	$\frac{1}{(s-a)^2}$	(20)
$\mathcal{L}\{t-a\}$	$\frac{e^{-as}}{s}$	(3)	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$	(21)
$f(t-a)\mathcal{L}\{t-a\}$	$e^{-as}F(s)$	(4)	$e^{at} \sin kt$	$\frac{k}{(s-a)^2 + k^2}$	(22)
$\delta(t)$	1	(5)	$e^{at} \cos kt$	$\frac{s-a}{(s-a)^2 + k^2}$	(23)
$\delta(t-t_0)$	$e^{-st_0}$	(6)	$e^{at} \sinh kt$	$\frac{k}{(s-a)^2 - k^2}$	(24)
$t^n f(t)$	$(-1)^n \frac{d^n F(s)}{ds^n}$	(7)	$e^{at} \cosh kt$	$\frac{s-a}{(s-a)^2 - k^2}$	(25)
$f'(t)$	$sF(s) - f(0)$	(8)	$t \sin kt$	$\frac{2ks}{(s^2 + k^2)^2}$	(26)
$f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0) - \dots - f^{(n-1)}(0)$	(9)	$t \cos kt$	$\frac{s^2 - k^2}{(s^2 + k^2)^2}$	(27)
$\int_0^t f(x)g(t-x)dx$	$F(s)G(s)$	(10)	$t \sinh kt$	$\frac{2ks}{(s^2 - k^2)^2}$	(28)
$t^n$ ( $n = 0, 1, 2, \dots$ )	$\frac{n!}{s^{n+1}}$	(11)	$t \cosh kt$	$\frac{s^2 - k^2}{(s^2 - k^2)^2}$	(29)
$t^x$ ( $x > -1 \in \mathbb{R}$ )	$\frac{\Gamma(x+1)}{s^{x+1}}$	(12)	$\frac{\sin at}{t}$	$\arctan \frac{a}{s}$	(30)
$\sin kt$	$\frac{k}{s^2 + k^2}$	(13)	$\frac{1}{\sqrt{\pi t}} e^{-a^2/4t}$	$\frac{e^{-a\sqrt{s}}}{\sqrt{s}}$	(31)
$\cos kt$	$\frac{s}{s^2 + k^2}$	(14)	$\frac{a}{2\sqrt{\pi t^3}} e^{-a^2/4t}$	$e^{-a\sqrt{s}}$	(32)
$e^{at}$	$\frac{1}{s-a}$	(15)	$\operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right)$	$\frac{e^{-a\sqrt{s}}}{s}$	(33)
$\sinh kt$	$\frac{k}{s^2 - k^2}$	(16)			
$\cosh kt$	$\frac{s}{s^2 - k^2}$	(17)			
$\frac{e^{at} - e^{bt}}{a - b}$	$\frac{1}{(s-a)(s-b)}$	(18)			