Professional Engineers Ontario

National Exams- May, 2017
07-STR-B3

Applications of Finite Elements

3 hours duration

Notes:

1. There are 4 pages in this examination.
2. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made
3. This is a closed book exam but one aid sheet allowed written on both sides.
4. Candidates may use one of the approved non-communicating calculators
5. **Answer only TWO (2) problems out of the three proposed.** The first two problems as they appear in the answer book will be marked.
6. All problems are of equal value.

Problem 1

1.1 Develop the stiffness matrix of a tapered 1D truss element shown in figure 1(a). The length of the element is denoted L, its area varies linearly between A_0 and A_1. The modulus of elasticity, E, is constant.

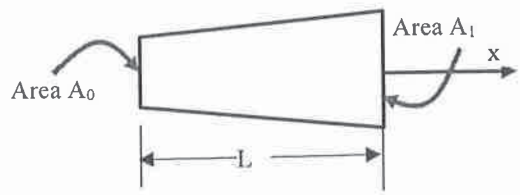

Figure 1(a)

1.2 A bridge deck is supported by several concrete piers. The geometry and loads of a typical pier are shown in Figure 1(b). The 20 kNm^{-2} load represents both the weight of the bridge and an assumed distributed traffic on the bridge. The concrete weights approximately 25 kNm^{-3} and its modulus is $E= 28\times10^6$ $kN.m^{-2}$.

Using two truss elements for the approximation, determine the displacement and stress fields and compare the results with the exact solution.

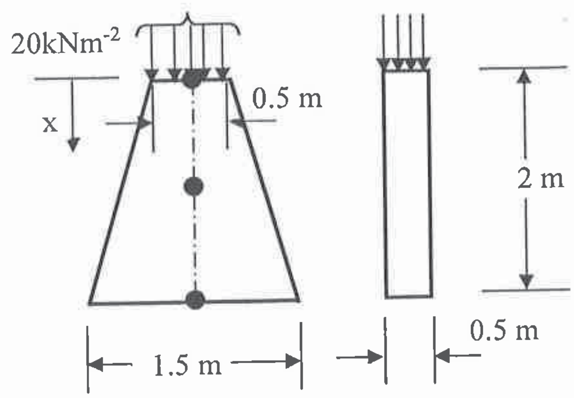

Figure 1(b)

Problem 2

2.1 A square frame subjected to a pair of parting forces, $\boldsymbol{P}$, is shown in Fig 2(a). It is assumed that the frame members are inextensible and that the right angles at the joints are preserved. Draw the deflections shape, shearing forces diagram and bending moment diagram. The stiffness of each member is denoted $\boldsymbol{EI}$.

2.2 The frame is now reinforced with four truss bars as shown in Fig. 2(b), Draw the deflections shape, shearing forces diagram and bending moment diagram and give the extreme values on the shear and moment diagrams. The stiffness of each member of the truss members is given by:

$$EA = \frac{12EI\sqrt{2}}{L^2}.$$

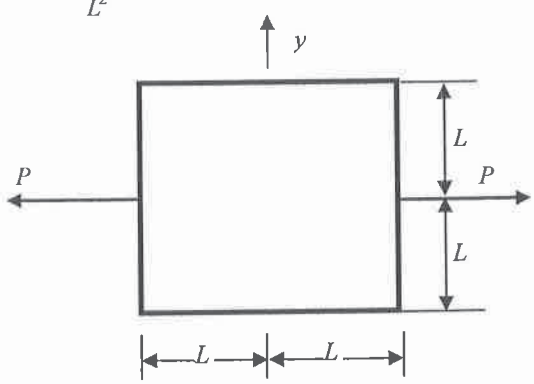

Figure 2(a)

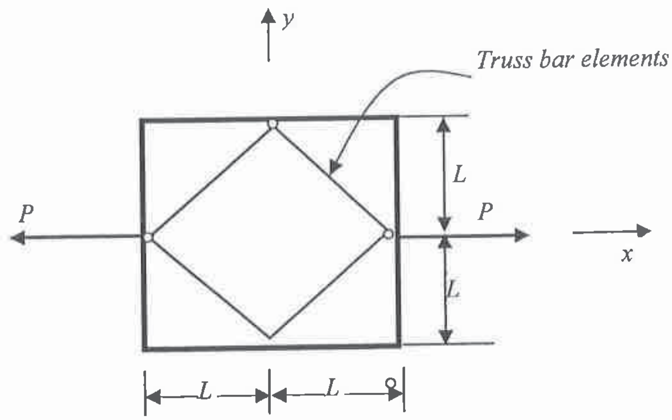

Figure 2(b)

The stiffness matrix of a beam element is given as following:

$$[k] = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ & & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ & & & \frac{EA}{L} & 0 & 0 \\ & \text{SYM} & & & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ & & & & & \frac{4EI}{L} \end{bmatrix}.$$

Problem 3

3.1 Referring to figure 3(a), the interpolation of the displacement field inside the Q4 element is given by:

$$u(x,y) = \sum_{i=1}^{4} N_i(x,y)u_i \quad \text{and} \quad v(x,y) = \sum_{i=1}^{4} N_i(x,y)v_i$$

Show that the shape function N_i are given by:

$$N_1 = \left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right) \qquad N_2 = \frac{x}{a}\left(1-\frac{y}{b}\right)$$

$$N_3 = \frac{xy}{ab} \qquad N_4 = \left(1-\frac{x}{a}\right)\frac{y}{b}$$

3.2 Show that in the case of plane stress, the stiffness term k_{33} of the 4 nodes rectangular element illustrated in figure 3(a) is given by:

$$k_{33} = \left(\frac{b}{3a} + \frac{1-\nu}{6}\frac{a}{b}\right)\frac{Et}{1-\nu^2}$$

where t is the element thickness. The element is made of an elastic material with an elasticity modulus E and Poisson's ratio ν.

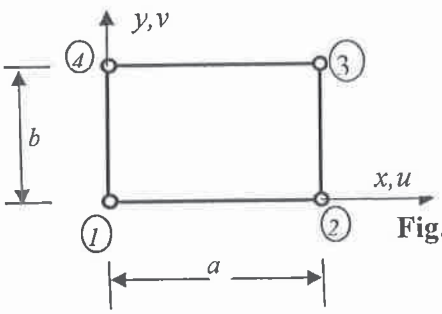

Fig. 3(a)

3.3 Figure 3(b) shows a plate of thickness t reinforced at its center by a bar with constant cross section A. The plate and the bar are made from the same material which has an elastic modulus equal to E. The applied force P is also illustrated in the figure.

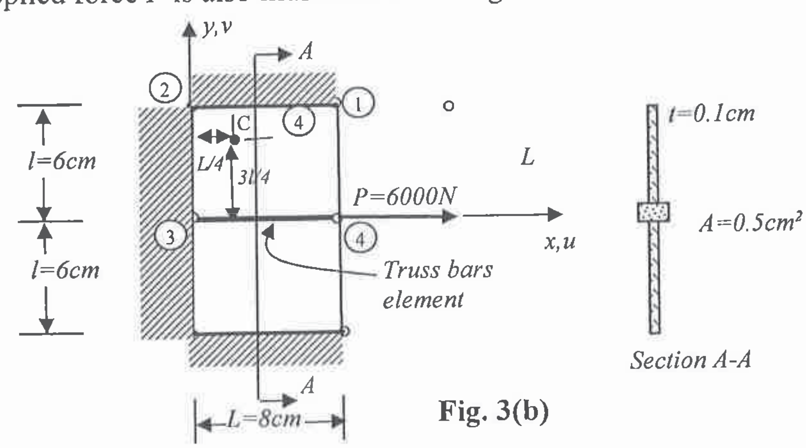

Fig. 3(b)

- Calculate the horizontal displacement u_4
- Calculate the strains ε_x, ε_y *and* γ_{xy} at point C as shown in the Fig. 3(b).

Use $E = 30\times10^6\ N/cm^2$ *and* $\nu = 0.3$