National Exams May 2015

98-Phys-A7, Optics

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate should include in the answer clear statements of the interpretation and any assumptions made.

2. This is a CLOSED BOOK EXAM.

3. Candidates may use one of two calculators, the Casio or Sharp approved models.

4. Answers to question 1 plus any four of questions 2 to 6 constitute a complete exam paper.

5. Answer question 1 in the space provided on the exam paper.

6. The first four questions as they appear in the answer book will be marked.

7. Each question is of equal value. Question 1 is mandatory.

1. A phrase, or a diagram and a phrase, is all that is required in most cases. [20 marks total, one mark for each letter]

a) This examination is about light and optics. Define *light*.

b) Define *optics*.

c) Define *geometrical optics*.

d) Define *physical optics*.

e) What problems with *geometrical optics* led to the development of *physical optics*?

f) Define *plane of incidence*.

g) State the *law of reflection*.

h) State the *law of refraction.*

i) Complete any two rows of the following table:

		wavelength range (nm)	frequency range (Hz)
	UV light		
	red light		
	blue light		
	IR light		

j) How is the *ray of geometrical optics* related to the plane wave of physical optics?

k) Define *f*-number.

l) Define *numerical aperture*.

m) Define *dispersion* as it relates to refractive index?

n) What does the concept of *spatial coherence* deal with?

o) What does the concept of *temporal coherence* deal with?.

p) If the vacuum wavelength is λ_o, and the wavelength in a material is λ, what is the refractive index of the material?

q) If the frequency of light in vacuum is ν_o, what is the frequency of the light ν in a material with refractive index $n(\nu)$?

r) What is the difference between *interference* and *diffraction*?

s) What is *Fraunhoffer* diffraction?

t) What is *Fresnel* diffraction?

2. [20 marks total]

a) Using ray tracing, find the image of an extended object that is placed 2 focal lengths in front of:
i) a perfect thin plano convex lens; [3 marks]

ii) a perfect thin plano concave lens. [3 marks]

b) Using ray tracing, find the image of an extended virtual object that is 2 focal lengths behind
i) a perfect thin plano convex lens; [3 marks]

ii) a perfect thin plano concave lens. [3 marks]

c) A small telescope uses an objective of focal length +12 cm and an eyepiece of focal length − 4 cm. Assume that the telescope is aligned for nearing a near point (i.e., a virtual image) of 30 cm.

i) Determine the separation of the lenses of the telescope. [3 marks]

ii) Trace rays through the telescope for an object that is far away. Use the diagram from the ray tracing to derive the angular magnification of the telescope. [3 marks].

iii) The system matrix is used with a column vector of length 2. State clearly what the two elements of the column vector represent. [2 marks]

3. [20 marks total]

a) Write equations for the electric field and magnetic fields for TE and TM polarized plane waves. Let the plane of incidence be the x-y plane and allow the waves to be propagating at an angle of θ with respect to the y axis. Assume that the waves propagate in an isotropic, homogeneous medium with refractive index of n. Remember to draw the coordinate system. [6 marks]

b) Check your answers for the **E** and **H** fields and for the vector **k** for part a) by taking limiting cases and by calculating **E×H**. **EXPLAIN** your reasoning. [4 marks]

c) Allow two coherent, monochromatic plane waves of light that are propagating in the x-y plane to interfere. Assume that one plane wave is propagating at an angle of θ with respect to the y axis and the other plane wave is propagating at an angle of $-\theta$ with respect to the y axis. **DERIVE** (i.e., show your work and explain your reasoning) a simplified expression for the time average of the Poynting vector in the plane $y = 0$ that can be used to determine by inspection (hence the request for a simplified expression) the period of the interference fringes in the $y = 0$ plane. [6 marks]

d) Assume that the fringes in part c) are observed in a material with a refractive index $> n$, and that this material has a planar interface starting at the plane $y = 10$. **DERIVE** (i.e., show your work and explain your reasoning) the period of the interference fringes when observed in the material with refractive index $> n$. [4 marks]

4. [20 marks total]

a) Derive the amplitude reflection and transmission coefficients, r and t, for normal incidence on a planar interface between materials with refractive indices of n_1 and n_2. Draw a sketch of the problem, define a coordinate system, and explain your reasoning for each step in the derivation. [10 marks]

b) Assume air and water with a refractive index of 1.33 Sketch carefully r and t for both polarizations as a function of the angle of incidence for internal reflection. Label the salient features and calculate numerical values for the salient features on both the vertical and horizontal axes. [4 marks]

c) Assume air and water with a refractive index of 1.33 Sketch carefully r and t for both polarizations as a function of the angle of incidence for external reflection. Label the salient features and calculate numerical values for the salient features on both the vertical and horizontal axes. [4 marks]

d) Use your results to explain the conditions wherein polarizing sun glasses are most effective. State the orientation of the transmission axis of the sun glasses under normal conditions of use. [2 marks]

5. [20 marks total]

a) Unpolarized light is passed through three polarizers. The angle between the transmission axes of the first and second polarizer is θ_1 and the angle between the transmission axes of the first and third polarizers is θ_2. What percentage of the unpolarized light is transmitted through the three polarizers? [3 marks]

b) A beam of light is known to be a mixture of unpolarized and circularly polarized light. Describe the measurements required to determine the proportions of unpolarized and circularly polarized light in the beam. [3 marks]

c) Mica has refractive indices of 1.600 or 1.5950 for light that is linearly polarized along one of two orthogonal directions and that is propagating along the optical axis of the sheet of mica. A sheet of mica, that is placed between crossed polarizers and that is aligned for maximum transmission of white light through the mica and crossed polarizers, is observed to have a purple colour (i.e., red and blue are transmitted, but not green).

i) Sketch front and side views of the optical system. Choose a coordinate system and indicate the coordinate system on the sketch. Indicate directions and variables of interest on the sketches. Calculate the possible thicknesses of the sheet of mica. [6 marks]

ii) How does the colour that is transmitted change as the sheet of mica is rotated about the optical axis? Provide calculations to support your answer. [4 marks]

iii) How does the colour that is transmitted change as the polarizer on the output side (i.e., the analyzer) is rotated about the optic axis. The mica and polarizer on the input side remain in their original orientations. Provide calculations to support your answer. [4 marks]

6. [20 marks total]

a) Apertures with 2, 10, and 1000 slits are illuminated at normal incidence with monochromatic plane waves with wavelengths of 500 nm and placed directly in front of a lens of focal length of 2 m. The distances between slits are 5 μm and the widths of each slit are 1 μm. For each of the 3 apertures, find:

i) the separation between the zeroth order maximum and the first order maximum [3 marks];

ii) the number of principal maxima that fall between the first zeroes of the diffraction envelope [4 marks]; and,

iii) the widths of the principal maxima in the focal plane of the lens [3 marks].

b) Two narrow slits are illuminated by monochromatic light. If the slit separation is 0.2 mm, and the fifth maximum is located a distance of 15 cm from the zeroth order maximum, what is the wavelength of the light? [3 marks]

c) Assume a plane wave is normally incident on a 2 mm diameter hole in an otherwise opaque screen. If the wavelength of the light is 623.8 nm, at what distance from the screen is the diameter of the beam 4 cm? [3 marks]

d) The headlights on a vehicle are spaced by 2 m. What is the distance to an observing facing the lights if the lights are just resolvable to the observer? Assume the pupils of the observer are 5 mm in diameter and that the headlights are monochromatic with a wavelength of 500 nm. [4 marks]

Question 6 d) is the last question. Some formulae follow.

$$E(x,y,z) = \frac{ik}{2\pi}\frac{e^{ikz}}{z} e^{i\frac{k}{2z}(x^2+y^2)} \iint E(x_a,y_a,0) e^{i\frac{k}{2z}(x_a^2+y_a^2)} e^{-i\frac{k}{z}(x\,x_a + y\,y_a)} \, dx_a \, dy_a$$

The field in the neighbourhood of the focus of a circular lens of radius a is given in the usual paraxial approximation as

$$E(u,v) = \int_0^1 J_0(2\pi v \rho_a)\, e^{-i\pi u \rho_a^2} \rho_a \, d\rho_a$$

with $\rho_a = \sqrt{(x_a^2 + y_a^2)}$, $\quad u = \frac{1}{\lambda}\frac{a^2}{q(q+\Delta)}\Delta$, $\quad v = \frac{1}{\lambda}\frac{a\sqrt{(x^2+y^2)}}{(q+\Delta)}$.

$J_0(0) = 1$; $J_0(2.4048) = 0$; $J_0(5.5201) = 0$; $J_0(8.6537) = 0$; $J_0(11.7915) = 0$;

$\gamma = \frac{1}{2} k D \sin(\theta)$. The zeros for $J_1(\gamma)$ occur for $\gamma = 0$, 3.832, 5.136, 7.016, 8.417, 10.173, 11.620, 13.324, . . .

$$\frac{d}{dx} x^n J_n(x) = x^n J_{n-1}(x)$$

$$(1+\epsilon)^{\xi} = 1 + \frac{\xi}{1!}\epsilon + \frac{\xi \times (\xi - 1)}{2!}\epsilon^2 + \ldots$$

zone plate radii $R_m = (m\, r_o\, \lambda)^{0.5}$

The intensity in the far-field as a function of the angle θ_m from the normal of a diffraction grating of N lines, line spacing of a, and line width of b, for illumination with a plane wave with $k = 2\pi/\lambda$ and an angle of incidence of θ_i is

$$I(\theta) = I_o \left[\frac{\sin(\beta)}{\beta}\right]^2 \left[\frac{\sin(N\alpha)}{\sin(\alpha)}\right]^2$$

$$\beta = \frac{kb}{2}\left(\sin(\theta_i) + \sin(\theta_m)\right)$$

$$\alpha = \frac{ka}{2}\left(\sin(\theta_i) + \sin(\theta_m)\right)$$

For a blazed grating, $2\theta_b = \theta_i - \theta_m$

The resolution R and the dispersion D for a grating with N lines and order m are

$$R = \frac{\lambda}{\Delta\lambda} = mN \qquad D = \frac{m}{a\cos(\theta)}.$$

double angle formulae:

$$
\begin{aligned}
\sin(A+B) &= \sin(A)\sin(B) + \cos(A)\cos(B)\\
\sin(A-B) &= \sin(A)\sin(B) - \cos(A)\cos(B)\\
\cos(A+B) &= \cos(A)\cos(B) - \sin(A)\sin(B)\\
\cos(A-B) &= \cos(A)\cos(B) + \sin(A)\sin(B)
\end{aligned}
$$

$$
\begin{aligned}
\sin(A) + \sin(B) &= 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\\
\sin(A) - \sin(B) &= 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\\
\cos(A) + \cos(B) &= 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\\
\cos(A) - \cos(B) &= 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)
\end{aligned}
$$

$$
v_p = \frac{\omega}{k} \qquad v_g = \frac{d\omega}{dk}
$$

The translation, refraction at a spherical interface, thin lens, and spherical mirror matrices are listed below.

$$
\begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \quad
\begin{bmatrix} 1 & 0 \\ \dfrac{n_1-n_2}{Rn_2} & \dfrac{n_1}{n_2} \end{bmatrix} \quad
\begin{bmatrix} 1 & 0 \\ \dfrac{-1}{f} & 1 \end{bmatrix} \quad
\begin{bmatrix} 1 & 0 \\ \dfrac{-2}{R} & 1 \end{bmatrix}
$$

THE END