

National Exams December 2013

98-Ind-A1

Operations Research

3 hours duration

NOTES:

- 1 If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
- 2 Any non-communicating calculator is permitted. This is an Open Book exam. Note to candidates: You must indicate the type of calculator being used, i.e. write the name and model designation of the calculator, on the first left hand sheet of the exam workbook.
- 3 There may be more questions than you are able to answer in the allotted time. Although the total value of the questions is 200, any marks achieved will be considered toward the 100 total requirements.

- 20 1. A post office has the following workload:

Day of the week	S	M	T	W	T	F	S
Full time equivalent (FTE) required	11	17	13	15	19	14	16

An FTE represents 8 hrs. /day. The post office may meet its daily labour requirements with a combination of full time and part time employees. Both work 5 consecutive days per week. Full-time employees work 8 hrs. /day at a rate of \$15 per hr. and part time employees work 4 hrs. /day at \$10 per hr. Agreements with the union limit the composition of part-time labour to 25% of the labour requirements.

Formulate a Linear Programming model that can be used to minimize the fuel cost incurred in completing the schedule. Do not solve.

2. Consider the following LP problem

$$\text{Max } -3x_1 + 8x_2$$

Subject to

$$4x_1 + 2x_2 \leq 12$$

$$2x_1 + 3x_2 \leq 6$$

$$x_i \geq 0, i = 1, 2.$$

- 10 a. Use the Simplex Method to find the optimal solution.
- 10 b. Suppose that both the "8" in the objective function and the "12" right hand side coefficient are changing together. For every % increase in "8" there is a 2% decrease in "12". For what range of changes does the basis remain optimal? Express changes in solution and objective as a function of % increases to "8".

- 20 3. Joe Cheapskate is a frequent traveler who always finds a way to take best advantage of air travel discounts. He is scheduled for 8 flights between Toronto and New York during the month shown below. As you can see he always flies from Toronto on a weekday and returns back from New York on a weekday later in the same week.

Return tickets between the two cities come in a variety of discount options:

- Full fare of \$500 – no restrictions
- 20% discount if the period between the ticket departure date and the ticket return date includes a weekend
- 30% discount if the period is at least 10 days
- 35% discount if the period is at least 21 days

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
1	2 Leave	3	4	5	6 Return	7
8	9	10 Leave	11	12 Retur	13	14
15	16 Leave	17	18	19	20 Return	21
22	23	24	25 Leave	26 Return	27	28
29	30	31				

Discounts cannot be combined, but the same prices apply if the ticket shows Toronto as the starting point, or New York as the starting point. For example he could buy a Toronto-New York return ticket to cover his flights on the 2nd and 12th and a New York-Toronto return ticket to cover flights on the 6th and 10th.

Formulate a Minimum Cost Network Flow model that can be used to determine the 4 tickets that cover the 8 flights at least cost. Do not solve.

- 20 4 A benefactor has donated to you valuable books of four different heights: 6, 8, 10 and 12 inches. You receive a quote from your handyman for four different shelf heights, which includes both a fixed cost and a variable cost. Shelves to accommodate 6 or 8 inch books cost \$22 to set-up, plus a per foot-length cost of \$2.50 for the 6 inch books and \$3.50 for the 8 inch books. Shelves to accommodate 10 or 12 inch books cost \$25 to set-up, plus \$4.50 per foot-length for the 10 inch books and \$5.50 per foot-length for the 12 inch books. You measure the total amount of shelf length you require for each book size: 12 feet for the 12 inch books, 18 feet for the 10 inch books, 9 feet for the 8 inch books and 10 feet for the 6 inch books. Given that smaller books can be stored on larger shelves, the problem is to determine what combination of shelf lengths minimizes the cost. Formulate a mathematical model that can be used to solve this problem. Do not solve.

- 20 5 An Automated Guided Vehicle (AGV) is used to deliver mail to five departments. The trip starts and ends at the mailroom – location (0, 0). Relative to the mailroom, the department (x, y) coordinates in meters are (10, 30), (10, 50), (30, 10), (40, 40) and (50, 60). The AGV can only move along horizontal and vertical aisles. Formulate an Integer Programming model to find the route which minimizes the round trip. Do not solve.
- 20 6 A machinery supplier has a new machine which a customer is willing to buy for \$10,000, but only if it works immediately after installation. Otherwise the machine will be sent right back, and the supplier will receive nothing. The machine will only work if a key part is sufficiently flat. The supplier can send the machine as is, or first scan the surface for faults before shipment. One scan costs \$500, and two scans \$900. Each scan has a 90% chance of finding a fault if it is faulty, but also has a 40% chance of falsely reporting it's faulty even though it's not. Another option is to overhaul the machine before sending it at a cost of \$2000. If during the overhaul the part is found not be sufficiently flat, it can be fixed for another \$1000 and the supplier is then 100% sure it will work. What course of action will maximize the supplier's expected profit?
- 20 7. A job shop has 4 jobs (A, B, C & D) that must be processed on a single machine. The processing times are respectively 2, 4, 6 and 8 days. The due dates for each job are respectively 4, 14, 10 and 16 days from now. For each day a job is late the shop must compensate the customer \$100. Use dynamic programming to determine how the jobs should be sequenced to minimize the total overdue penalty costs.
- 20 8. Your friend has offered you the opportunity to bet 5\$ on the toss of a coin - heads you receive \$100, tails you receive nothing. You are ready to open your wallet when he warns you not to get mad if the coin turns out to be an unfair coin. Before you walk away you think this may be interesting. Knowing your friend, you figure there is about an 80% chance the coin has tails on both sides, but that there is still a 20% chance the coin is fair and it may be a good bet. You ask your friend to toss the coin once before deciding if you should place your bet. How much money should you be willing to offer him for this toss?

- 20 9. Goods are continually removed from inventory at a steady sales rate of d units per day, at a sales price of r \$ per unit. Every y periods the inventory is replenished with a production run of x units. There is a fixed cost of k \$ for each production run, as well as a variable cost of c \$ per unit produced. While there are units in inventory there is an inventory holding cost of h \$ per unit per day. While there are no units in inventory there is a penalty of l \$ per unit per day for sales not met. Formulate the non-linear objective which will allow management to determine the values of x and y which maximize the average daily profit, as a function of d, r, k, c, h and l .
- 20 10. The Port of Trop can service only one ship at a time. However, there is mooring space for three more ships. Trop is a favorite port of call, but, if no mooring space is available, the ships have to go to the Port of Poop. An average of seven ships arrive each week, according to a Poisson process. The Port of Trop has the capacity to handle an average of eight ships a week, with service times exponentially distributed. What is the expected number of ships waiting for service at the Port of Trop?