

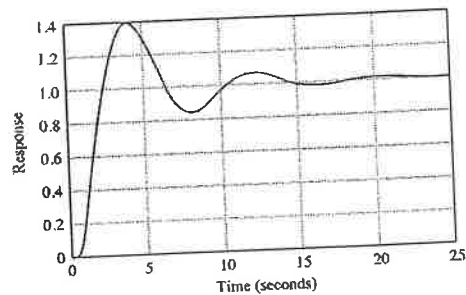
National Examinations December 2019
16-Elec-B2 Advanced Control Systems
3 hours duration

NOTES:

1. This is an open book exam.
2. If a doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
3. Any non-communicating calculator is permitted.
Tables of Laplace are attached.
4. **Any four questions** constitute a complete paper. Only the first four questions as they appear in your answer book will be marked.
5. All questions are of equal value (25%).

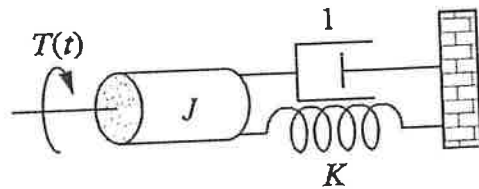
Question 1. Choose the best answer.

- For the unit step response shown in the following figure, what are the %OS and T_p ? [2]
 - 140% and 20 seconds
 - 140% and 4 seconds
 - 40% and 4 seconds
 - 40% and 20 seconds



- For the following rotational system, the system transfer function is given by

$$\frac{\theta(s)}{T(s)} = \frac{\frac{1}{J}}{s^2 + \frac{1}{J}s + \frac{K}{J}}$$



where $\theta(s)$, $T(s)$, J and K represent torque, angular displacement, moment of inertia and spring constant, respectively.

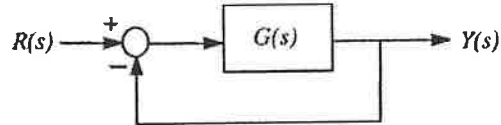
What is the value of J to give a 3 sec settling time? [2]

- 0.5
 - 1
 - 3/8
 - None of the above
- Consider a forward path transfer function $G(s)$ with gain K . Do the zeros of the closed loop system change with changing gain K ? [1]
 - Yes
 - Depends on $G(s)$
 - No
 - Maybe
 - What are the total contributions of four simple poles ($s = 0$) to the Bode phase plot? [1]
 - +45 deg
 - 360 deg
 - 45 deg
 - 90 deg

5. A chemical process is designed to follow a desired path described by (parabola)

$$r(t) = (5 - t + 0.5t^2)u(t)$$

where $r(t)$ is the desired response and $u(t)$ is a unit step function. Consider a unity feedback system.

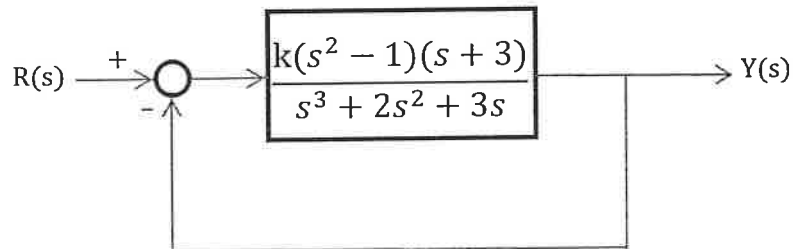


What is the steady-state error $E(s) = R(s) - Y(s)$ with the following open-loop transfer function? [2]

$$G(s) = \frac{10(s + 1)}{s^2(s + 5)}$$

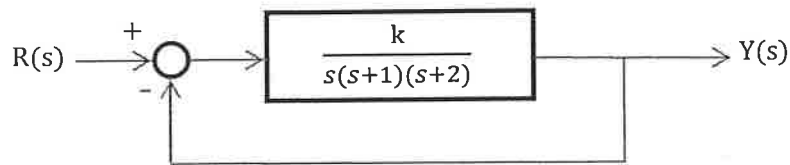
- (a) $ee_{ss} = \lim_{t \rightarrow \infty} e(t) = 0.5$
- (b) $ee_{ss} = \lim_{t \rightarrow \infty} e(t) \rightarrow \infty$
- (c) $ee_{ss} = \lim_{t \rightarrow \infty} e(t) = 1$
- (d) $ee_{ss} = \lim_{t \rightarrow \infty} e(t) = 0$

6. How many asymptotes of the root locus are there for the system shown below? [1]



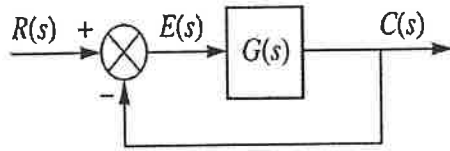
- (a) There are no asymptotes
- (b) There is one asymptote
- (c) There are two asymptotes
- (d) There are three asymptotes

7. What are the angles of the asymptotes in degrees for the system shown below? [1]



- (a) 90 and -90
 (b) 90, 180 and 270
 (c) 60, -60 and 180
 (d) 0, 60 and 180
8. A transfer function has a second order denominator and constant gain as the numerator. Which statement is true for this system? [1]
 (a) The system has one zero at infinity
 (b) The system does not have a zero
 (c) The system has two zeros at origin
 (d) The system has two zeros at infinity
9. What are the two conditions under which the response generated by a pole can be neglected? [1]
 (a) Pole's real part is large compared to the dominant poles and pole is on real axis
 (b) Pole's real part is large compared to the dominant poles and pole is near a zero
 (c) Pole is near imaginary axis and far from dominant poles
 (d) None of the above
10. In a system with complex poles, the imaginary part of a pole generates what part of the system response? [1]
 (a) Time constant of an exponential response
 (b) Amplitude of a sinusoidal response
 (c) Radian frequency of a sinusoidal response
 (d) None of the above
11. For the open loop transfer function $G(s) = \frac{K(s+1)}{s^2+5s+10}$, what is the break-in point? [2]
 (a) $\sigma = -1.35$
 (b) $\sigma = -2.45$
 (c) $\sigma = -3.45$
 (d) $\sigma = -4.65$

12. For the open-loop pole-zero plot in the following figure (with unity feedback):



$$G(s) = \frac{K}{(s+1)(s+2)(s+3)}$$

what is the intercept of the asymptotes? [2]

- (a) $s = -1$
- (b) $s = -2$
- (c) $s = -4$
- (d) $s = -6$

13. In the above problem, what is the range of K for stability? [3]

- (a) $0 < K < 66$
- (b) $10 < K$
- (c) $-6 < K < 60$
- (d) $K < 60$

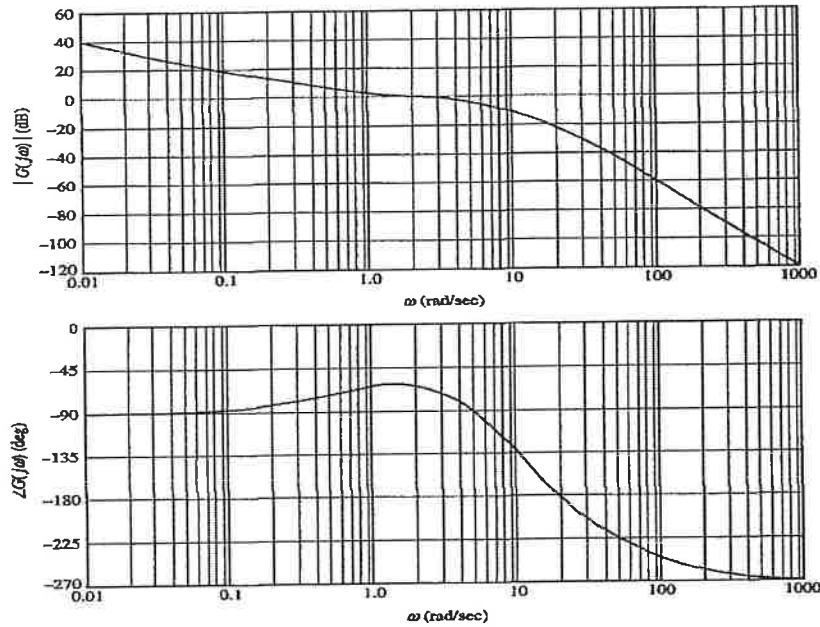
14. For an under-damped system, the resonant frequency peak in the Bode magnitude plot [1]

- (a) Increases by decreasing the damping ratio
- (b) Increases by increasing the damping ratio
- (c) Is independent of the change in damping ratio
- (d) None of the above

15. The amplitude slope for the second order complex poles in a Bode magnitude plot is [1]

- (a) 0 dB/decade
- (b) -6 dB/octave
- (c) -12 dB/octave
- (d) -20 dB/decade

16. The bode diagram of the forward-path transfer function of a unity-feedback control system is shown below. What are the gain margin and phase margin approximately? [2]



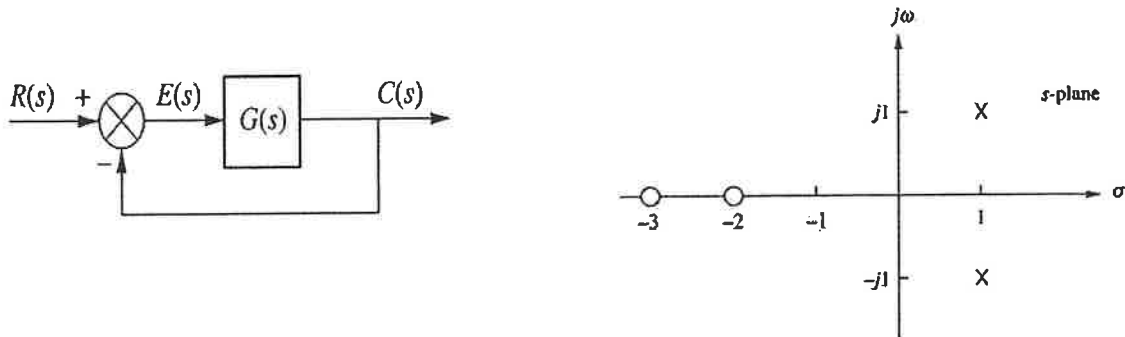
- (a) PM=64 deg, GM=3 dB
- (b) PM=116 deg, GM=3 dB
- (c) PM=116 deg, GM=21 dB
- (d) PM=64 deg, GM=21 dB

17. Which of the following statements is correct for a system with gain margin close to unity or a phase margin close to zero? [1]

- (a) The system is relatively stable
- (b) The system is highly stable
- (c) The system is highly oscillatory
- (d) None of above

Question 2.

For the open-loop pole-zero plot in the following figure (with unity feedback):



- Specify the locations of poles and zeros and write the transfer function $G(s)$. [5]
- Calculate the angle of departure (from poles)? (note: $k= 0, \pm 1, \pm 2, \dots$ in the angle condition of root locus). [5]
- Form the closed loop transfer function and then using Routh table find the $j\omega$ -axis crossing coordinates and corresponding K ? [10]
- Given the break-in coordinate of -2.43 , sketch root locus. [5]

Question 3.

For the following transfer function

$$F(s) = \frac{4(s + 2)}{s(s + 1)(s + 3)^2}$$

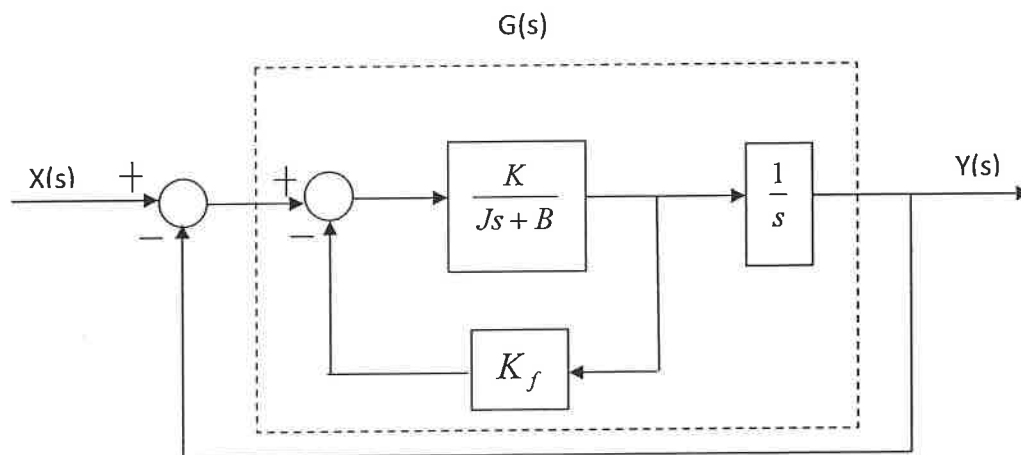
- (a) Form the partial fractions in frequency domain. [15]
- (b) Find the inverse Laplace transform of the function $F(s)$. [10]

Question 4.

Figure below shows the block diagram of a servo motor.

Assume $J = 1\text{kg-m}^2$ and $B = 1\text{N-m-s/rad}$. If the maximum overshoot of the unit-step input and the peak time are 0.1 (or 10%) and 0.2 sec., respectively:

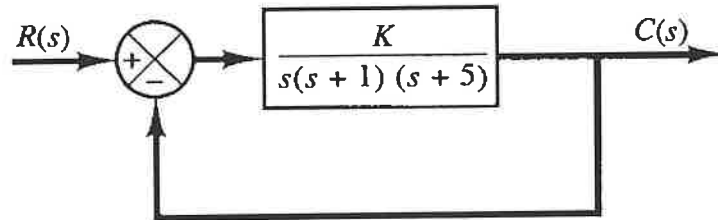
- Find its damping ratio and natural frequency. [4]
- Calculate the settling time. [6]
- Calculate the overall system transfer function and convert it to the standard second-order system format. [5]
- Find the gain K and velocity feedback K_f . [10]



Question 5.

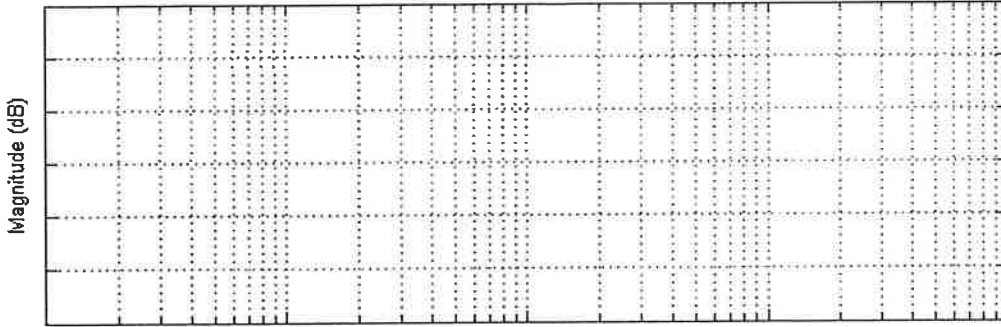
Given the following system with $K = 5$,

- (a) Plot the gain and phase Bode diagram. [20]
- (b) Obtain the gain and phase margin and their frequencies for the system with $K = 5$. [5]

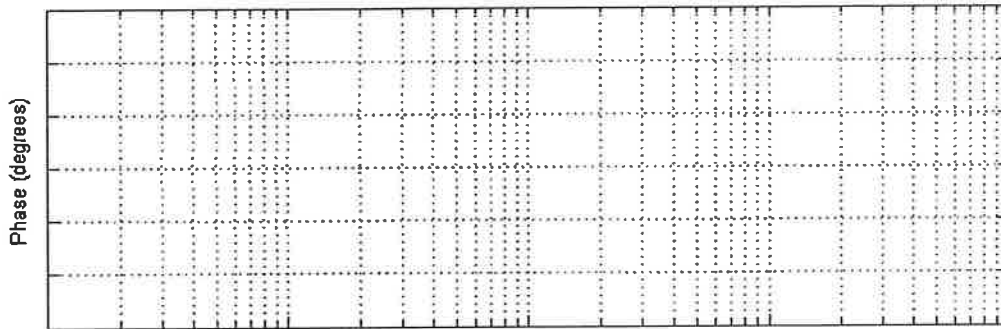


Hint: Normalize the given transfer function. You need to sketch magnitude and phase plots for each component first (i.e., overall gain after normalization, and three poles). Then you need to show the overall Bode diagram.
(Note: Use the semi-log graph provided in the following pages).

Bode Plot

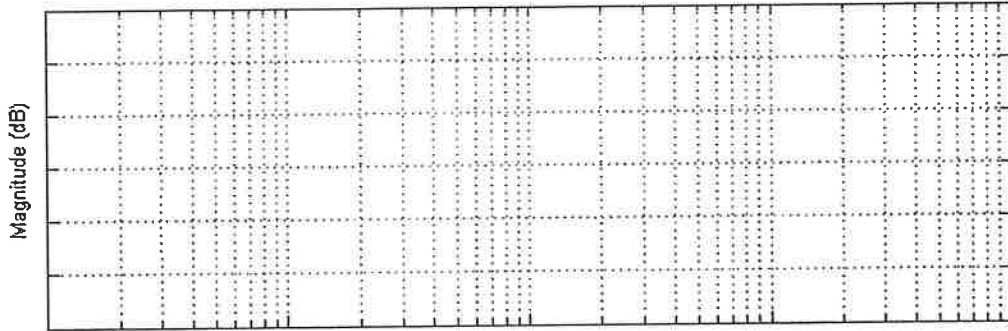


Pole at 0

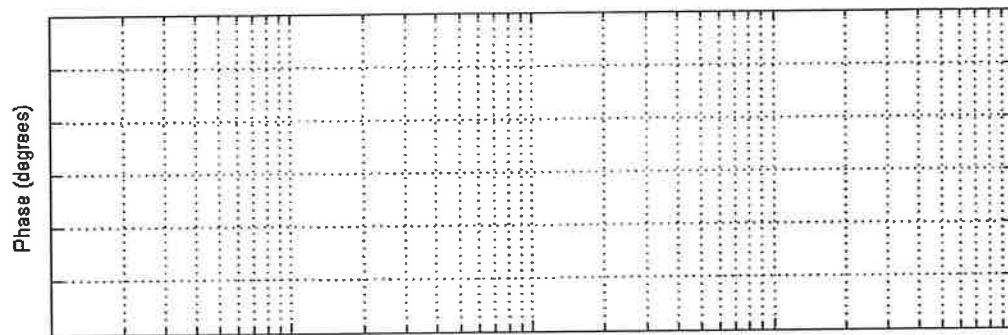


Frequency - log scale

Bode Plot

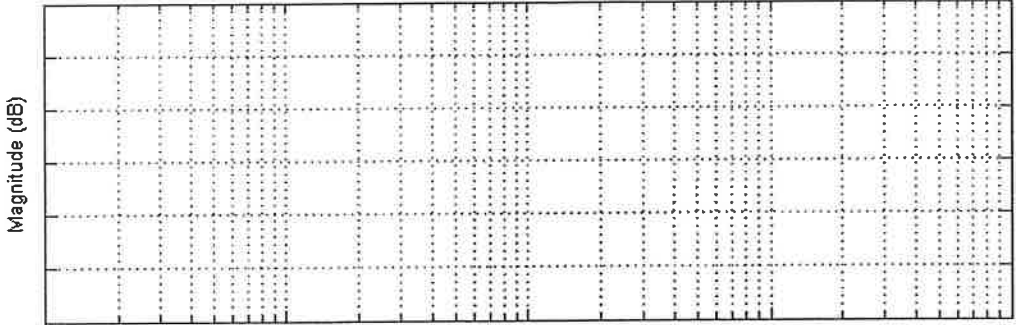


Pole at -1

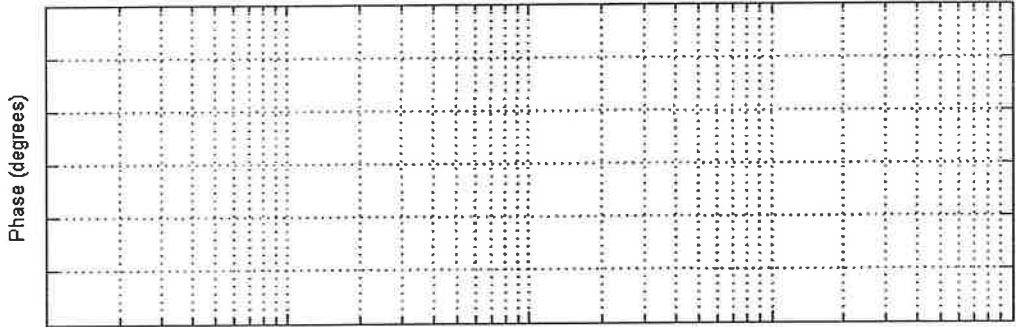


Frequency - log scale

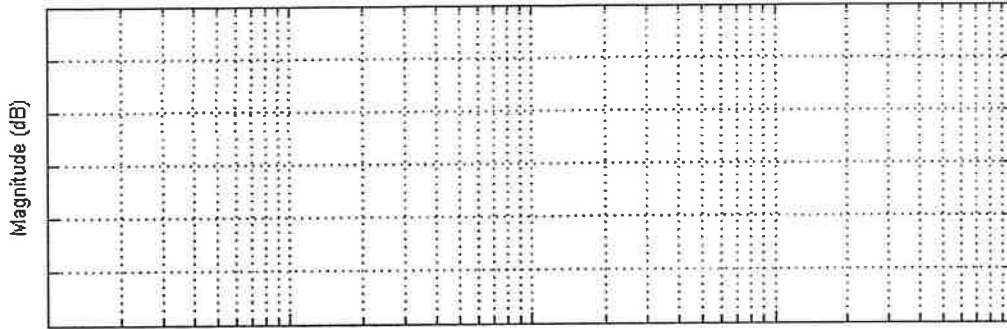
Bode Plot



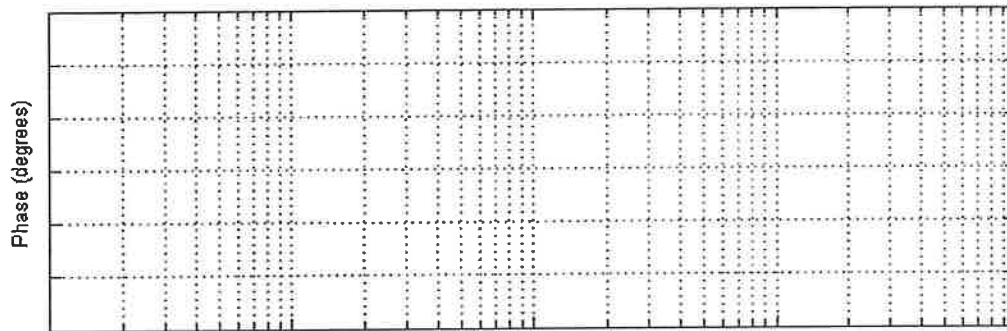
Pole at -5



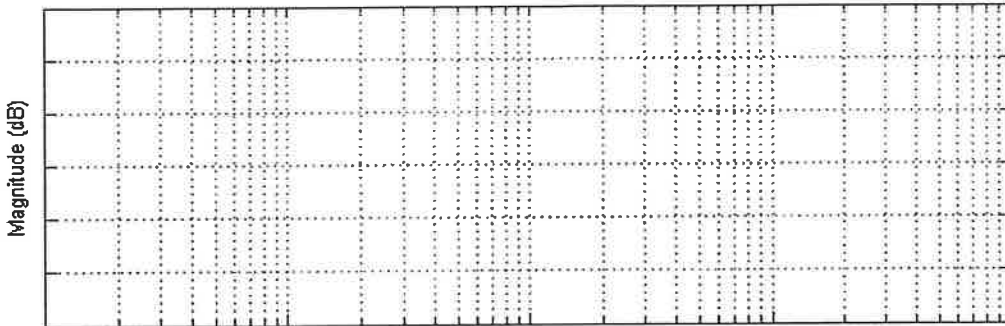
Bode Plot



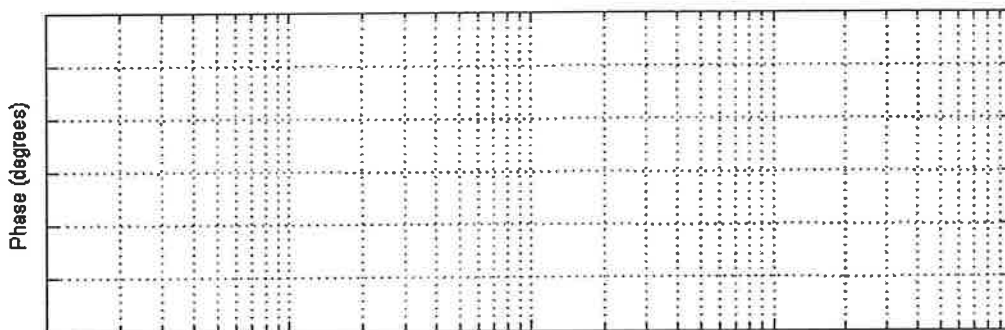
Gain



Bode Plot



$KG(s)$



Frequency - log scale

Inverse Laplace Transforms	
$F(s)$	$f(t)$
$\frac{A}{s+\alpha}$	$Ae^{-\alpha t}$
$\frac{C+jD}{s+\alpha+j\beta} + \frac{C-jD}{s+\alpha-j\beta}$	$2e^{-\alpha t} (C \cos \beta t + D \sin \beta t)$
$\frac{A}{(s+\alpha)^{n+1}}$	$\frac{At^n e^{-\alpha t}}{n!}$
$\frac{C+jD}{(s+\alpha+j\beta)^{n+1}} + \frac{C-jD}{(s+\alpha-j\beta)^{n+1}}$	$\frac{2t^n e^{-\alpha t}}{n!} (C \cos \beta t + D \sin \beta t)$

Table of Laplace and z-Transforms (<i>h</i> denotes the sample period)		
<i>f(t)</i>	<i>F(s)</i>	<i>F(z)</i>
unit impulse	1	1
unit step	$\frac{1}{s}$	$\frac{hz}{z-1}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$	$\frac{z}{z-e^{-\alpha h}}$
<i>t</i>	$\frac{1}{s^2}$	$\frac{hz}{(z-1)^2}$
$\cos \beta t$	$\frac{s}{s^2+\beta^2}$	$\frac{z(z-\cos \beta h)}{z^2-2z\cos \beta h+1}$
$\sin \beta t$	$\frac{\beta}{s^2+\beta^2}$	$\frac{z \sin \beta h}{z^2-2z\cos \beta h+1}$
$e^{-\alpha t} \cos \beta t$	$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$	$\frac{z(z-e^{-\alpha h} \cos \beta h)}{z^2-2ze^{-\alpha h} \cos \beta h+e^{-2\alpha h}}$
$e^{-\alpha t} \sin \beta t$	$\frac{\beta}{(s+\alpha)^2+\beta^2}$	$\frac{ze^{-\alpha h} \sin \beta h}{z^2-2ze^{-\alpha h} \cos \beta h+e^{-2\alpha h}}$
<i>t f(t)</i>	$-\frac{dF(s)}{ds}$	$-zh \frac{dF(z)}{dz}$
$e^{-\alpha t} f(t)$	$F(s+\alpha)$	$F(ze^{\alpha h})$