National Exams May 2017

98-Phys-B5, Control

3 hours duration

NOTES:

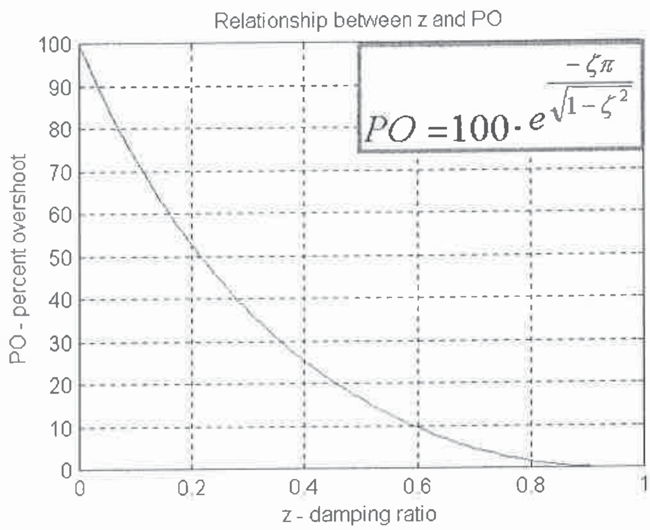
1. This is a CLOSED BOOK EXAM. Only an approved Casio or Sharp calculator is permitted. Candidates are allowed to use a double-sided, **handwritten**, 8.5 by 11" formula and notes sheet. Otherwise, there are no restrictions on the content of the formula sheet. The sheet must be signed and submitted together with the examination paper.
2. If doubt exists as to interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
3. Five (5) questions constitute a complete paper. **YOU ARE REQUIRED TO COMPLETE QUESTION 1 AND QUESTION 2.** Choose three (3) more questions out of the remaining six. Clearly indicate the questions which should be marked - otherwise, only the first five answers provided will be marked. Each question is of equal value. Weighting of topics within each question is indicated. Partial marks will be given. Clearly show steps taken in answering each question.
4. **Use exam booklets to answer the questions - clearly indicate which question is being answered.**

YOUR MARKS		
QUESTIONS 1 AND 2 ARE COMPULSORY:		
Question 1	20	
Question 2	20	
CHOOSE THREE OUT OF THE REMAINING SIX QUESTIONS:		
Question 3	20	
Question 4	20	
Question 5	20	
Question 6	20	
Question 7	20	
Question 8	20	
TOTAL:	100	

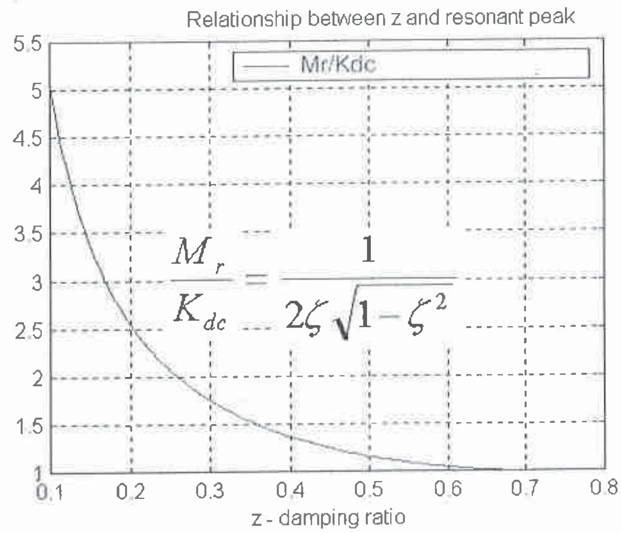
A Short Table of Laplace Transforms

Laplace Transform	Time Function
1	$\sigma(t)$
$\frac{1}{s}$	$1(t)$
$\frac{1}{(s)^2}$	$t \cdot 1(t)$
$\frac{1}{(s)^{k+1}}$	$\frac{t^k}{k!} \cdot 1(t)$
$\frac{1}{s+a}$	$e^{-at} \cdot 1(t)$
$\frac{1}{(s+a)^2}$	$te^{-at} \cdot 1(t)$
$\frac{a}{s(s+a)}$	$(1-e^{-at}) \cdot 1(t)$
$\frac{a}{s^2+a^2}$	$\sin at \cdot 1(t)$
$\frac{s}{s^2+a^2}$	$\cos at \cdot 1(t)$
$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at} \cdot \cos bt \cdot 1(t)$
$\frac{b}{(s+a)^2+b^2}$	$e^{-at} \cdot \sin bt \cdot 1(t)$
$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$\left(1 - e^{-at} \cdot \left(\cos bt + \frac{a}{b} \cdot \sin bt\right)\right) \cdot 1(t)$
$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \cdot \sin\left(\omega_n\sqrt{1-\zeta^2}t\right) \cdot 1(t)$
$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$\left(1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \cdot \sin\left(\omega_n\sqrt{1-\zeta^2}t + \cos^{-1}\zeta\right)\right) \cdot 1(t)$
$F(s) \cdot e^{-Ts}$	$f(t-T) \cdot 1(t)$
$F(s+a)$	$f(t) \cdot e^{-at} \cdot 1(t)$
$sF(s) - f(0+)$	$\frac{df(t)}{dt}$
$\frac{1}{s}F(s)$	$\int_{0+}^{+\infty} f(t)dt$

Useful Plots & Formulae



PO vs. Damping Ratio



Resonant Peak vs. Damping Ratio

Second Order Model:

$$G_m(s) = K_{dc}\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

ζ – Damping Ratio (zeta), of the model

ω_n – Frequency of Natural Oscillations of the model

K_{dc} – DC Gain of the model

Definitions for Controllability Matrix, $\mathbf{M_c}$, and Observability Matrix, $\mathbf{M_o}$:

$$\mathbf{M}_c = \begin{bmatrix}\mathbf{B} & \mathbf{AB} & \mathbf{A}^2\mathbf{B}\end{bmatrix} \qquad \mathbf{M}_o = \begin{bmatrix}\mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2\end{bmatrix}$$

Definition for Transfer Function from State Space:

$$\mathrm{G(s)} = \mathbf{C}\cdot(\mathrm{s}\mathbf{I} - \mathbf{A})^{-1}\cdot\mathbf{B} + \mathbf{D}$$

Question 1 (Compulsory)

Signal Flow Diagrams – Mason's Gain Formula. Stability – Routh Array and Routh-Hurwitz Criterion of Stability. Error Analysis.

Consider a servo-positioning system under Proportional + Integral (PI) Control that is represented by the block diagram in Figure Q1.1. The PI Controller has an adjustable Gain, K. The system is to exhibit zero Steady State Error in the response to a unit step input and the Steady State Error in the response to a unit ramp is to be: $e_{ss(ramp)} = 0.1\ V/V$.

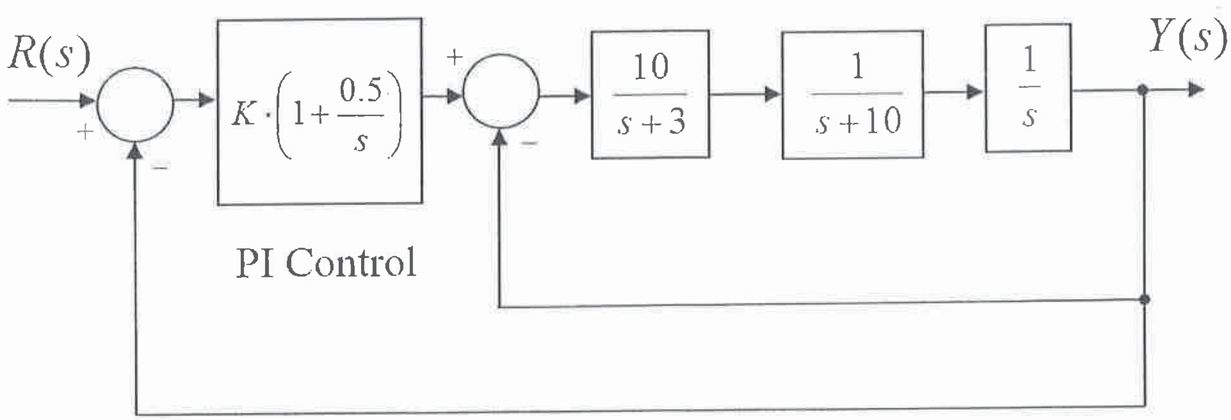


Figure Q1.1

1) **(5 marks)** Calculate the system closed loop transfer function $G_{cl}(s)$. Write it out in the polynomial ratio format, as a function of the Proportional Controller gain K.
2) **(10 marks)** Determine the range of Controller gains for a safe, stable operation of the closed loop system. Specify the critical value(s) of the gain, K_{crit}, as well as the frequency of oscillations, ω_{osc}, resulting when K = K_{crit}.
3) **(5 marks)** Calculate the required operating value for the Proportional Gain (K_{op}) to meet the Steady State Error specifications. When $K = K_{op}$, will the system be still stable? Calculate the corresponding Gain Margin (G_m).

Question 2 (Compulsory)

2nd Order Dominant Poles Model, System Performance.

Consider the same servo-positioning system under Proportional + Integral (PI) Control from Question 1.

PART A (5 marks)

1. Check the system open loop transfer function $G_{open}(s)$. What is the System Type?
2. If the PI Controller Gain is set to the operating value ($K = K_{op}$) so that the Steady State Error specifications are as required in Question 1, what are the Error Constants, K_{pos}, K_v and K_a?

PART B (15 marks)

When the PI Controller Gain is set to the above operating value ($K = K_{op}$), the closed loop system transfer function, $G_{cl}(s)$, can be shown in a factorized form:

$$G_{cl}(s) = \frac{200(s + 0.5)}{(s + 11.9)(s + 0.505)(s^2 + 0.5925s + 16.64)}$$

1) **(3 marks)** Verify the DC gain of the closed loop system, $G_{cl}(0)$.
2) **(7 marks)** A 2nd order dominant poles model will be applicable for the closed loop system. Determine the model parameters, K_{dc}, ζ, ω_n, and write the model transfer function, $G_m(s)$ – see page 3 for definitions.
3) **(5 marks)** Evaluate the following step response specifications: PO, $T_{rise\ (10\%-90\%)}$, $T_{settle\ \pm 2\%}$, T_{period}, and $e_{ss(step)\%}$.

Question 3

Analytical system response. Partial fractions.

Consider a certain Linear, Time-Invariant (LTI) system, where its transfer function $G(s)$ can be described as:

$$G(s) = \frac{7s(s+3)}{(s+0.5)(s^2+1.6s+16)}$$

1) **(20 marks)** Find an analytical expression for the response of the system when the reference signal is a unit step input: $r(t) = 1(t)$.

Question 4

System Stability in the Frequency Domain: Polar Plots, Nyquist Criterion of Stability.

Consider a unit feedback loop system under Proportional Control (gain K). The transfer function of its open loop is described as follows:

$$G_{open}(s) = K \cdot \frac{(1+s)}{s(s-1)}$$

1) **(10 marks)** Sketch a polar plot of the normalized open loop transfer function, $\frac{G_{open}(j\omega)}{K}$; note that you do not have the frequency response plots available to read off the coordinates of the crossovers with the Imaginary and Real axis, and thus you have to calculate these crossovers using the Fourier Transfer Function $G_{open}(j\omega)$. Clearly indicate the direction of increasing frequency on the resulting polar plot.
2) **(10 marks)** Next, apply the Nyquist criterion of stability to establish the range of values of the Proportional Controller gain, K_p, that will result in a stable closed loop system response. NOTE: clearly show the chosen clockwise (CW) Γ path in the s-plane, and the resulting Nyquist Contour.

Question 5

State Space Model from Transfer Functions, Pole Placement by State Feedback Method, Steady State Errors to Step and Ramp Inputs.

Consider a linear system described by the following transfer function:

$$\frac{Y(s)}{U(s)} = \frac{2s^2 + 5s + 2}{s^3 + 8s^2 + 2s + 30}$$

1) **(10 marks)** Derive a set of state equations in the Controller Canonical Form.

2) **(10 marks)** State-variable feedback is to be applied according to the following equation:

$$u = K \cdot \left(r - \mathbf{k}^T \cdot \mathbf{x}\right)$$

Determine the values of the gain constant K and the state feedback vector $\mathbf{k}$ so that the closed loop system will have poles at: -10 and $-2 \pm j3$, and the steady-state error to a step input will be zero.

Question 6

Controllability and Observability, System Eigenvalues, Transfer Function from State Space Model.

Consider a control system described by the following state space model:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1-2\alpha & -2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \cdot u$$

$$y = \begin{bmatrix} 0 & \alpha & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

1) **(10 marks)** Is the system controllable? Does the Controllability depend on the value of parameter α? If so, find the condition for Controllability. Is the system observable? Does the Observability depend on the value of parameter α? If so, find the condition for Observability. See page 3 for definitions.

2) **(10 marks)** Find the system transfer function, $G(s) = \dfrac{Y(s)}{U(s)}$. HINT: Rather than using the Transfer Function from State Space matrix approach, try using the Mason's Gain Formula on the state space diagram corresponding to the state space equation.

Question 7

Root Locus Analysis and Gain Selection, Stability, Second Order Model, Step Response Specifications.

A unit feedback control system is to be stabilized using a Proportional Controller, as shown in Figure Q7.1.

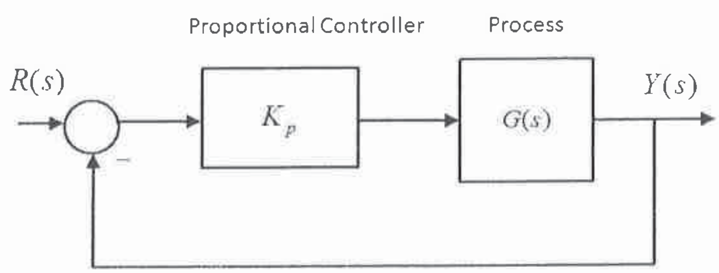


Figure Q7.1

The process transfer function is described as follows:

$$G(s) = \frac{10}{(s+2)(s+4)(s+7)}$$

1) **(10 marks)** Sketch the Root Locus for the system, in the space provided in Figure Q7.2. Calculate all relevant coordinates: asymptotic angles, break-in/away points, the location of the centroid and the coordinates of the crossover with the Imaginary axis, i.e. ω_{osc} and the corresponding value of the critical gain, K_{crit}, at which the system becomes marginally stable.

2) **(7 marks)** It is required that the unit step response of the Closed Loop system exhibits Percent Overshoot of approximately 5%. Determine the corresponding Proportional Gain value, K_{op}, and calculate an estimate of the following specs: Settling Time, $T_{settle(\pm 5\%)}$, Rise Time, $T_{rise(0-100\%)}$, and Steady State Error, $e_{ss(step\%)}$.

3) **(3 marks)** Finally, briefly comment on any possible differences between the expected system response (i.e. of the dominant poles model) and the actual system response.

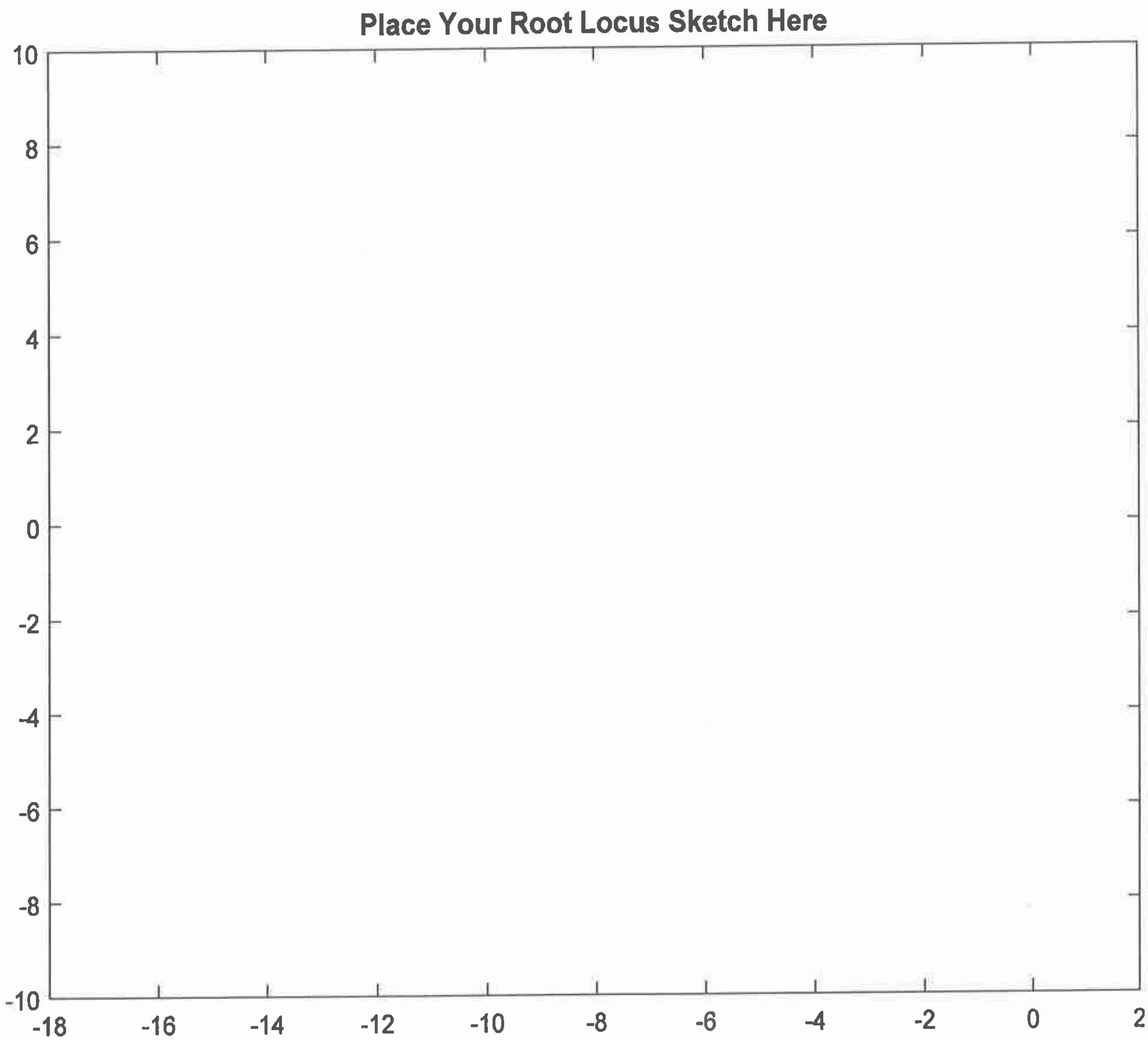


Figure Q7.2 – Root Locus of the System in Question 7

Question 8

PID Controller Design by Pole Placement, Response Specifications, Second Order Model.

Consider a closed loop positioning control system with a PID Controller in a so-called "series" configuration, as shown in Figure Q8.1:

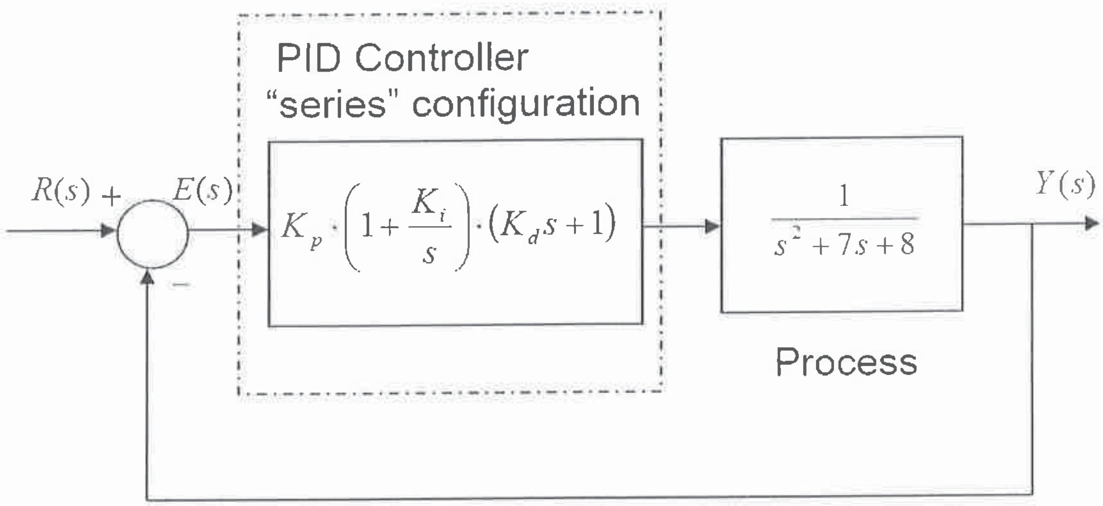


Figure Q8.1

1) **(5 marks)** Derive the Closed Loop system transfer function in terms of Controller Gains K_p, K_d and K_i, and write the system Characteristic Equation, $Q(s) = 0$.

2) **(5 marks)** The compensated Closed Loop step response of this system is to have the following specifications: $PO = 15\%$ and $T_{settle(\pm 2\%)} = 2$ sec. Determine the Closed Loop system damping ratio, ζ, and the frequency of natural oscillations, ω_n, to meet the transient response requirements.

3) **(6 marks)** Choose the pole locations for the Closed Loop system so that system two complex conjugate ("dominant") poles correspond to the desired second order model (above) and the third real pole equals to the value of Integral Gain K_i so that **a pole-zero cancellation** in the Closed Loop transfer function occurs. Compute the required Controller gains K_p, K_d, and K_i.

4) **(4 marks)** Note that you are expected to solve a quadratic equation to find the gains in item 3), which means you will have two sets of solutions. Choose ONLY ONE set for your final answer – clearly identify it, and justify your choice by briefly commenting on any possible differences between the expected system responses (i.e. of the dominant poles model) and the actual system responses.