National Exams May 2017

98-Ind-A1

Operations Research

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Any non-communicating calculator is permitted. This is an Open Book exam. Note to candidates: You must indicate the type of calculator being used; i.e. write the name and model designation of the calculator, on the first left hand sheet of the exam workbook.

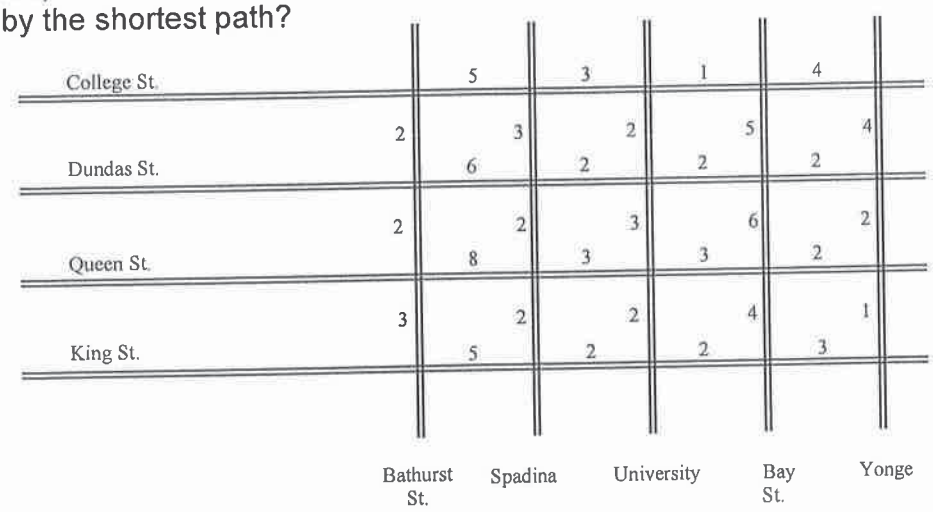
3. All questions are worth 20 marks; therefore, the total value of all 8 questions is 160 marks. Any marks achieved will be considered toward the 100 total requirement.

20

1) A catering company must meet it's daily demand for clean napkins by either buying new napkins, using regular laundry service which requires a full day turnaround, or special overnight service. Supplied with the demand for each of the next 5 days, and the costs for new napkins, regular laundry service, and overnight laundry service, formulate a minimum-cost network flow model, which will minimize the cost of procuring the napkins over the next 5 days. The company currently has no napkins in usable condition, clean or dirty, on the premises or at the laundry.

20

2) A Toronto rat crawls out of its hole at King and Bathurst Streets and decides to raid its favorite garbage can behind a store at College and Yonge Streets. Based on previous experience with cats, traffic and lighting conditions, the rat estimates travel time (in minutes) for each block as shown. Using dynamic programming, how long will the trip take by the shortest path?



Bathurst St. Spadina University Bay St. Yonge

20

3) Environment Canada (EC) is charged with investigating complaints regarding industrial pollution, and issuing citations to prevent recurrences, as it sees fit. Based on the region, it has full discretion to investigate or not, to issue citations or not, based on its analyses. Following are the facts for a particular region. A complaint is investigated by sending a "patrol" of three experts to the site of the alleged offender. By consensus the patrol renders one of three opinions: a low, medium or high level of pollution. Analysis of the data shows that these opinions are 80% correct. The other 20% of the time, each of the two other options have a 10% chance of being true. It costs EC $490,000 to maintain this patrolling system, over the planning horizon. With or without an investigation, EC can choose to ignore the situation, i.e. not issue a citation, in which case the costs over the planning horizon are assessed to be 0M$, 8M$ and 25M$, depending on the level of pollution. If it issues a citation then these costs are changed to 3M$ (up from 0 due to monitoring costs), 5M$ and 10M$. Once a complaint is received, the probabilities of low, medium and high levels of pollution are 0.5, 0.4 and 0.1 respectively. An expected cost criterion is assumed to be valid for this analysis.

a) For which patrol opinions should EC issue a citation?
b) Should EC issue citations without investigation?
c) Should EC maintain the patrol system in this region?

4) A post office has the following workload: 20

Day of the week	S	M	T	W	T	F	S
Full time equivalent (FTE) required	11	17	13	15	19	14	16

An FTE represents 8 hrs/day. The post office may meet its daily labour requirements with a combination of full time and part time employees. Both work 5 consecutive days per week. Full-time employees work 8 hrs/day at a rate of $15/hr, and part time employees work 4 hrs/day at $10/hr. Agreements with the union limit the composition of part-time labour to 25% of the labour requirements. Formulate a linear programming model for finding a minimum cost feasible solution.

5) Consider the following problem 20

Maximize $z = 21x_1 + 9x_2 + 4x_3$ (profit)

Subject to

$2x_1 + x_2 + x_3 \leq 31$ (resource constraint 1)
$3x_1 + 2x_2 + x_3 \leq 60$ (resource constraint 2)
$x_1 + 2x_2 + x_3 \geq 50$ (requirement constraint)
$x_1 \geq 0$
$x_2 \geq 0$

The simplex method yields the following final set of equations

$z + (1/2)x_3 + (2/3)x_4 + x_6 = 291$
$x_1 + (1/3)x_3 + (2/3)x_4 + (1/3)x_6 = 4$
$x_2 + (1/3)x_3 - (1/3)x_4 - (2/3)x_6 = 23$
$x_5 - (2/3)x_3 - (4/3)x_4 + (1/3)x_6 = 2$

where x_4 is the slack variable for resource constraint 1, x_5 is the slack variable for resource constraint 2, and x_6 is the slack variable for the requirement constraint.

a. What is the optimal solution, the maximum profit, the marginal values of resources 1 and 2, and the marginal cost of the requirement?

b. How much can the coefficient of x_2 in the objective function vary without affecting the optimal solution?

c. By how much would the profit be increased if 5 more units of resource 1 where available? What would be the new solution?

20 6) A best-selling textbook sells 5 million copies every fall. Some users keep the book, and some sell it back to the bookstore. Suppose that 90% of all students who buy a new book sell it back, 80% of all students who buy a once-used book sell it back, and 60% of all students who buy a twice-used book sell it back. If a book has been used four or more times, the cover falls off and it cannot be sold back.

a) In the steady state, how many of the new copies of the book will the publisher be able to sell each year?
b) Suppose the bookstore makes a profit of $6 on new books, $3 on once-used books, $2 on twice-used books and $1 on thrice-used books, what will be the average profit per book in steady state?

20 7) An airline flies planes on the following route: Vancouver – Calgary – Toronto – New York – Vancouver. The distances are as follows: Vancouver to Calgary, 687 km, Calgary to Toronto, 2690 km, Toronto to New York 571 km, and New York to Vancouver 3910 km. At each stop the plane may purchase up to 10,000 litres (l) of fuel. The price of fuel varies widely from city to city. In Vancouver it is 0.88 $/l, in Calgary 0.15 $/l, in Toronto 0.95 $/l, and in New York 1.05 $/l. The plane's fuel tank holds 12000 l, and for safety, the plane must carry at least 600 l. upon landing. The amount of fuel used per km on each leg of the flight is [1 + (average amount of fuel in tank on leg of flight/2000)]. Assume that this average amount of fuel is [(1/2) (amount of fuel in tank at start of leg + amount of fuel in tank at end of leg)]. Formulate an LP that can be used to minimize the fuel cost incurred in completing the schedule.

20 8) A mutual fund company must invest $100,000 among three types of investments: bonds, mainstream stocks and speculative stocks. The rates of return are respectively 5%, 7% and 10%. Company policy requires that the investment into any one of the three types cannot exceed 40% and that at least two out of the following three rules must be satisfied:

a. The percentage invested in bonds must be greater than 25%
b. The percentage invested in mainstream stocks must be less than 50%
c. The percentage invested in speculative stocks must be less than 12%

Formulate an Integer Programming model to optimally allocate the $100,000.