National Exams May 2019

16-Elec-A2, Systems & Control

3 hours duration

NOTES:

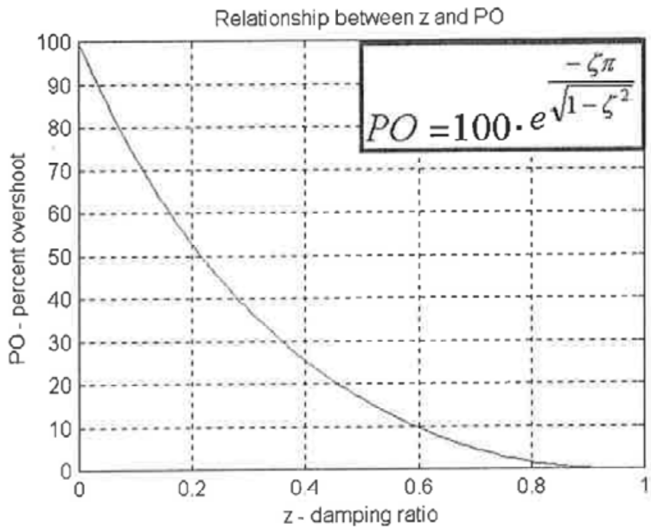
1. This is a CLOSED BOOK EXAM. Only an approved Casio or Sharp calculator is permitted. Candidates are allowed to use a double-sided, **handwritten**, 8.5 by 11" formula and notes sheet. The sheet must be signed and submitted together with the examination paper.
2. If doubt exists as to interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
3. Five (5) questions constitute a complete paper. **YOU ARE REQUIRED TO COMPLETE QUESTION 1 AND QUESTION 2.** Choose three (3) more questions out of the remaining six. Clearly indicate the questions which should be marked - otherwise, only the first five answers provided will be marked. Each question is of equal value. Weighting of topics within each question is indicated. Partial marks will be given. Clearly show steps taken in answering each question.
4. **Use exam booklets to answer the questions - clearly indicate which question is being answered.**

YOUR MARKS		
QUESTIONS 1 AND 2 ARE COMPULSORY:		
Question 1	20	
Question 2	20	
CHOOSE THREE OUT OF THE REMAINING SIX QUESTIONS:		
Question 3	20	
Question 4	20	
Question 5	20	
Question 6	20	
Question 7	20	
Question 8	20	
TOTAL:	100	

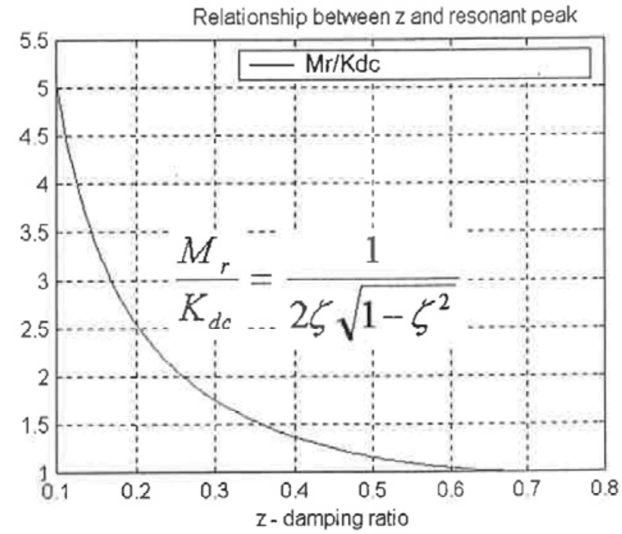
A Short Table of Laplace Transforms

Laplace Transform	Time Function
1	$\sigma(t)$
$\frac{1}{s}$	$1(t)$
$\frac{1}{(s)^2}$	$t \cdot 1(t)$
$\frac{1}{(s)^{k+1}}$	$\frac{t^k}{k!} \cdot 1(t)$
$\frac{1}{s+a}$	$e^{-at} \cdot 1(t)$
$\frac{1}{(s+a)^2}$	$te^{-at} \cdot 1(t)$
$\frac{a}{s(s+a)}$	$(1 - e^{-at}) \cdot 1(t)$
$\frac{a}{s^2+a^2}$	$\sin at \cdot 1(t)$
$\frac{s}{s^2+a^2}$	$\cos at \cdot 1(t)$
$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at} \cdot \cos bt \cdot 1(t)$
$\frac{b}{(s+a)^2+b^2}$	$e^{-at} \cdot \sin bt \cdot 1(t)$
$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$\left(1 - e^{-at} \cdot \left(\cos bt + \frac{a}{b} \cdot \sin bt\right)\right) \cdot 1(t)$
$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \cdot \sin\left(\omega_n\sqrt{1-\zeta^2}t\right) \cdot 1(t)$
$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$\left(1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \cdot \sin\left(\omega_n\sqrt{1-\zeta^2}t + \cos^{-1}\zeta\right)\right) \cdot 1(t)$
$F(s) \cdot e^{-Ts}$	$f(t-T) \cdot 1(t)$
$F(s+a)$	$f(t) \cdot e^{-at} \cdot 1(t)$
$sF(s) - f(0+)$	$\frac{df(t)}{dt}$
$\frac{1}{s}F(s)$	$\int_{0+}^{+\infty} f(t)dt$

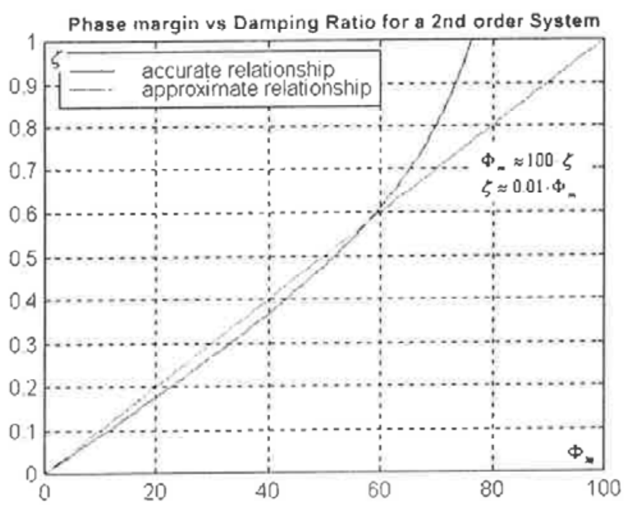
Useful Plots & Formulae



PO vs. Damping Ratio



Resonant Peak vs. Damping Ratio



Second Order Model:

$$G_m(s) = K_{dc}\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

ζ – Damping Ratio (zeta), of the model

ω_n – Frequency of Natural Oscillations of the model

K_{dc} – DC Gain of the model

Definitions for Controllability Matrix, $\mathbf{M_c}$, and Observability Matrix, $\mathbf{M_o}$:

$$\mathbf{M}_c = \begin{bmatrix} \mathbf{B} & \mathbf{AB} & \mathbf{A}^2\mathbf{B} \end{bmatrix} \qquad \mathbf{M}_o = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \end{bmatrix}$$

Definition for Transfer Function from State Space:

$$\mathrm{G(s)} = \mathbf{C} \cdot (\mathrm{s}\mathbf{I} - \mathbf{A})^{-1} \cdot \mathbf{B} + \mathbf{D}$$

Question 1 (Compulsory)

Stability in s-domain: Routh Array and Routh-Hurwitz Criterion of Stability.

Consider a certain hydraulic control system where the input is the reference fluid flow, $R(s)$ and the output is the actual fluid flow, $Y(s)$. The system is to operate under a Proportional + Integral + Derivative (PID) Controller in a parallel form, with the transfer function of the controller, $G_{PID}(s)$, as shown below. The hydraulic process is described by the transfer function, $G(s)$, as shown below.

$$G_{PID}(s) = K_p\left(1 + \frac{1}{\tau_i s} + \tau_d s\right) \qquad G(s) = \frac{10}{(s+1)^2}$$

Controller time constants are as follows: $\tau_i = 0.25$, $\tau_d = 0.1$. Assume the equivalent unit feedback loop. The closed loop system transfer function $G_{cl}(s)$ is defined between the output and the reference signals:

$$G_{cl}(s) = \frac{Y(s)}{R(s)}$$

1) (6 marks) Derive $G_{cl}(s)$ and write it in the TF format (polynomial ratio), as a function of the Proportional gain, K_p.

2) (10 marks) Calculate the value(s) of the Critical Gain, K_{crit}, when the system is marginally stable, as well as the frequency of oscillations, ω_{osc}, resulting when $K_p = K_{crit}$.

3) (4 marks) Determine the range of the Proportional Controller gains, K_p, for a safe, stable operation of the closed loop system.

Question 2 (Compulsory)

Stability in s-domain: Root Locus Analysis, Stability in Frequency Domain: Bode Plots and Gain Margin.

Consider the same hydraulic control system from Question 1.

1) **(5 marks)** The frequency response of the process transfer function, $G(j\omega)$, is shown in Figure Q2.1. Use it to verify the value(s) of the Critical Gain, K_{crit}, and the frequency of oscillations, ω_{osc}, (when $K_p = K_{crit}$), obtained in Question 1.

2) **(12 marks)** Sketch the Root Locus for the system, in the space provided in Figure Q2.2. Calculate all relevant coordinates: asymptotic angles, break-in/away points, the location of the centroid and the coordinates of the crossover with the Imaginary axis, i.e. ω_{osc} and the corresponding value of the critical gain, K_{crit}, at which the system becomes marginally stable.

3) **(3 marks)** Do your calculations based on Root Locus Construction Rules confirm the results from Question 1? What is the range of the Proportional Controller gains, K_p, for a safe, stable operation of the closed loop system?

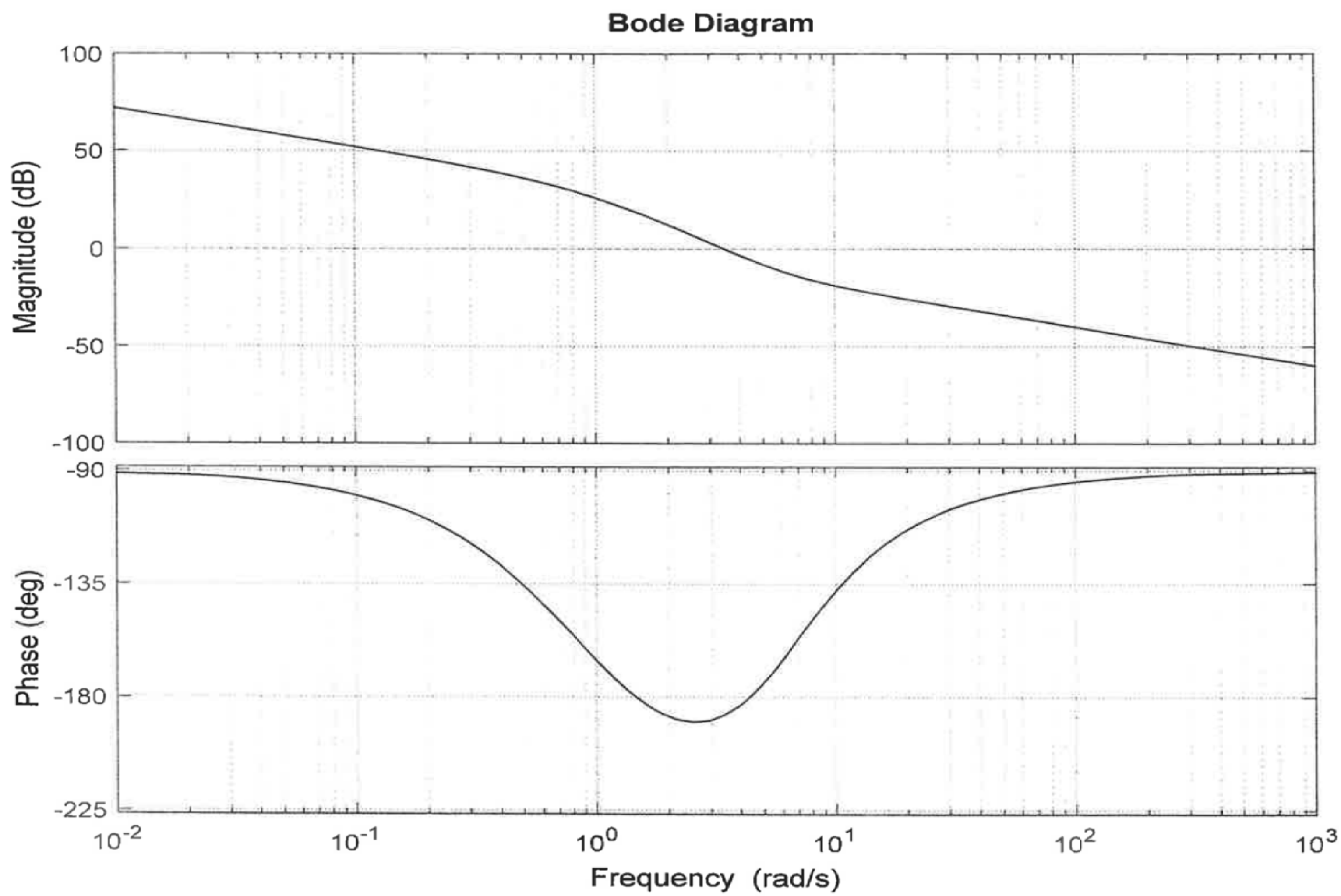


Figure Q2.1 - Frequency Response Plots of the Process $G(j\omega)$ in Question 2

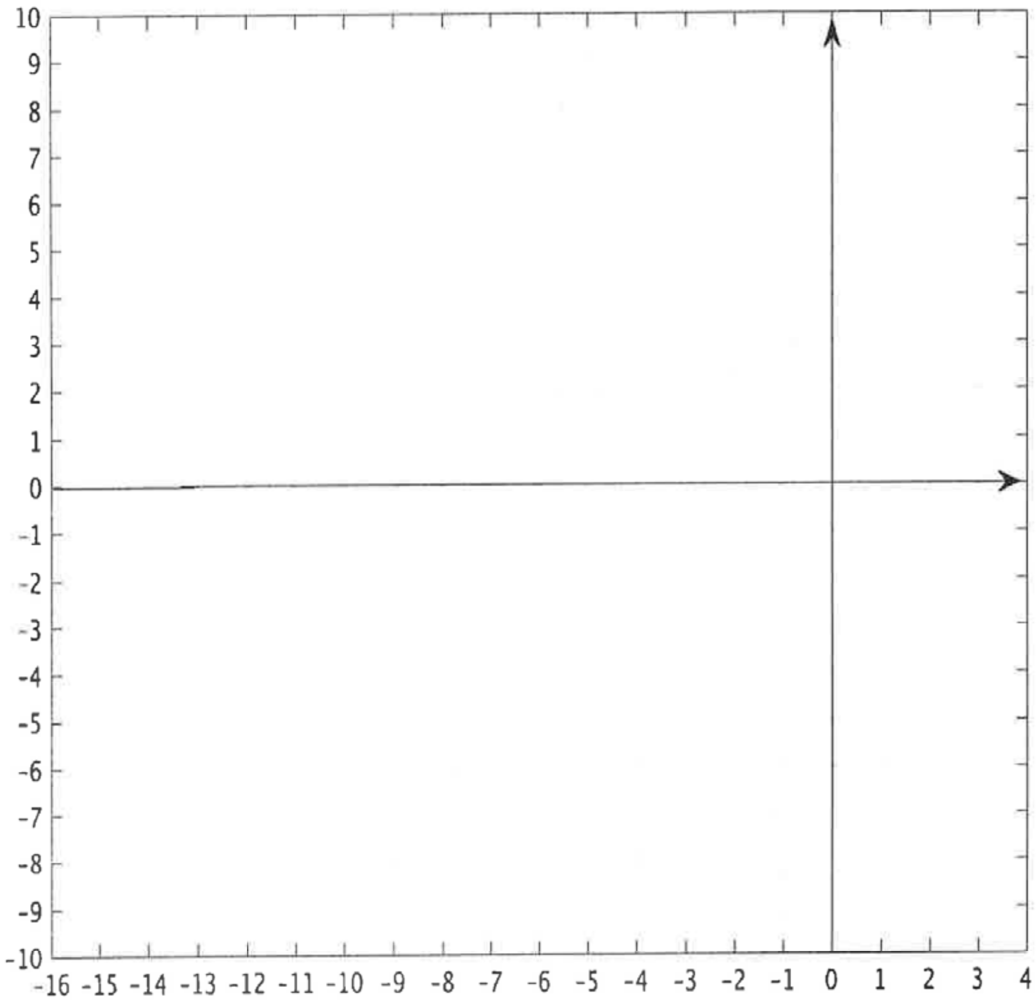


Figure Q2.2 – Sketch Here Your Root Locus of the System in Question 2

Question 3

Signal Flow Diagrams – Mason's Gain Formula

Consider a certain multi-feedback closed loop system, as shown in Figure Q3.1 below.

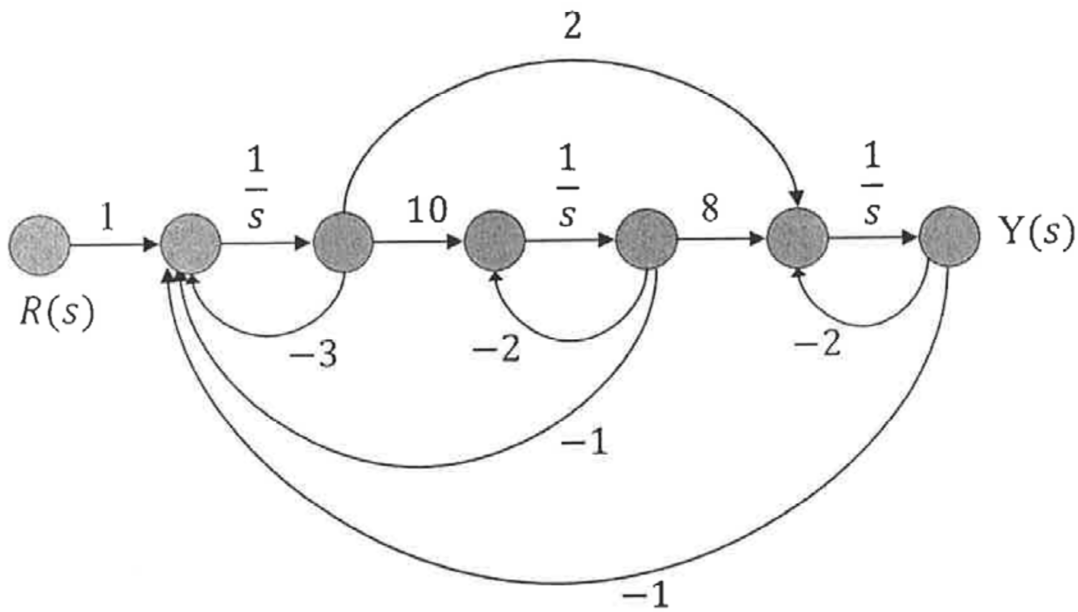


Figure Q3.1: Signal Flow Graph Representing a Closed Loop System in Q3

1) **(5 marks)** Complete the table below:

How many loops?	How many non-touching loops taken 2 at a time?	How many non-touching loops taken 3 at a time?	How many paths?

2) **(5 marks)** Clearly label the loops and paths on the diagram, then write the Mason's Gain formula in **general terms**, using only L_i for loops, and P_j for paths. For now, do not substitute values for loop and path gains, just write them below in a general form, as shown:

$$G(s) = \frac{P_1 \cdot (1 - \cdots) + P_2 \cdot (1 - \cdots) + \cdots}{1 - (\sum L_i) + (\sum L_j\, L_k) - (\sum L_l\, L_n L_m) + \cdots}$$

3) **(10 marks)** Next, substitute all the loop and path gains, and simplify the resulting transfer function to a ratio of two polynomials in descending orders of the s-operator.

Question 4

Second Order Dominant Poles Model in s-Domain and in Frequency Domain (Open and Closed Loop), Step Response Specifications.

Consider again the closed loop system in Question 3. Your task is to determine an appropriate 2nd order dominant poles model to represent it. The transfer function $G_{cl}(s)$ obtained from the signal flow graph in Question 3 is as shown below:

$$G_{cl}(s) = \frac{2(s + 42)}{(s + 5.67)(s^2 + 1.33s + 20.46)}$$

The step response of the closed loop transfer function $G_{cl}(s)$ is shown in Figure Q4.1, and the magnitude plot of a frequency response of $G_{cl}(j\omega)$ is shown in Figure Q4.2.

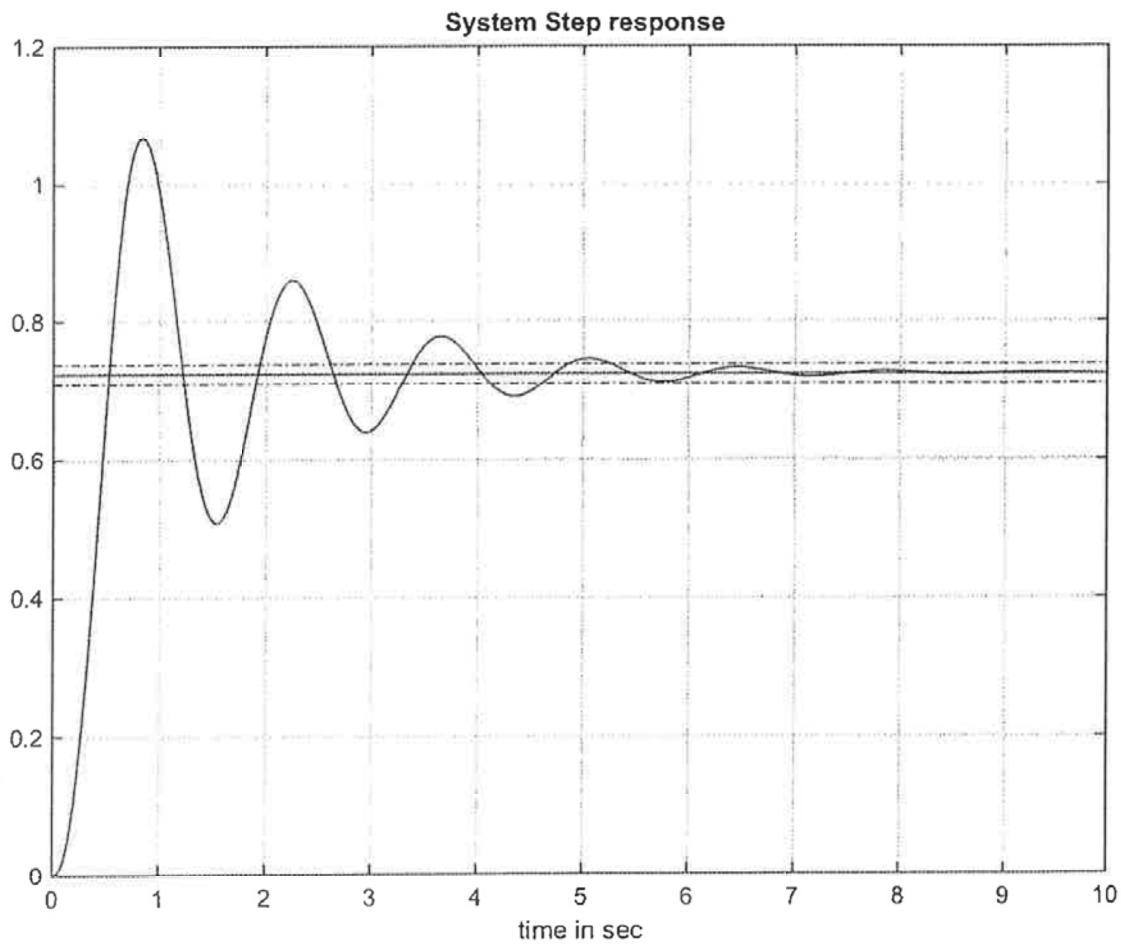


Figure Q4.1: Step Response of the Closed Loop Transfer Function $G_{cl}(s)$ in Question 4.

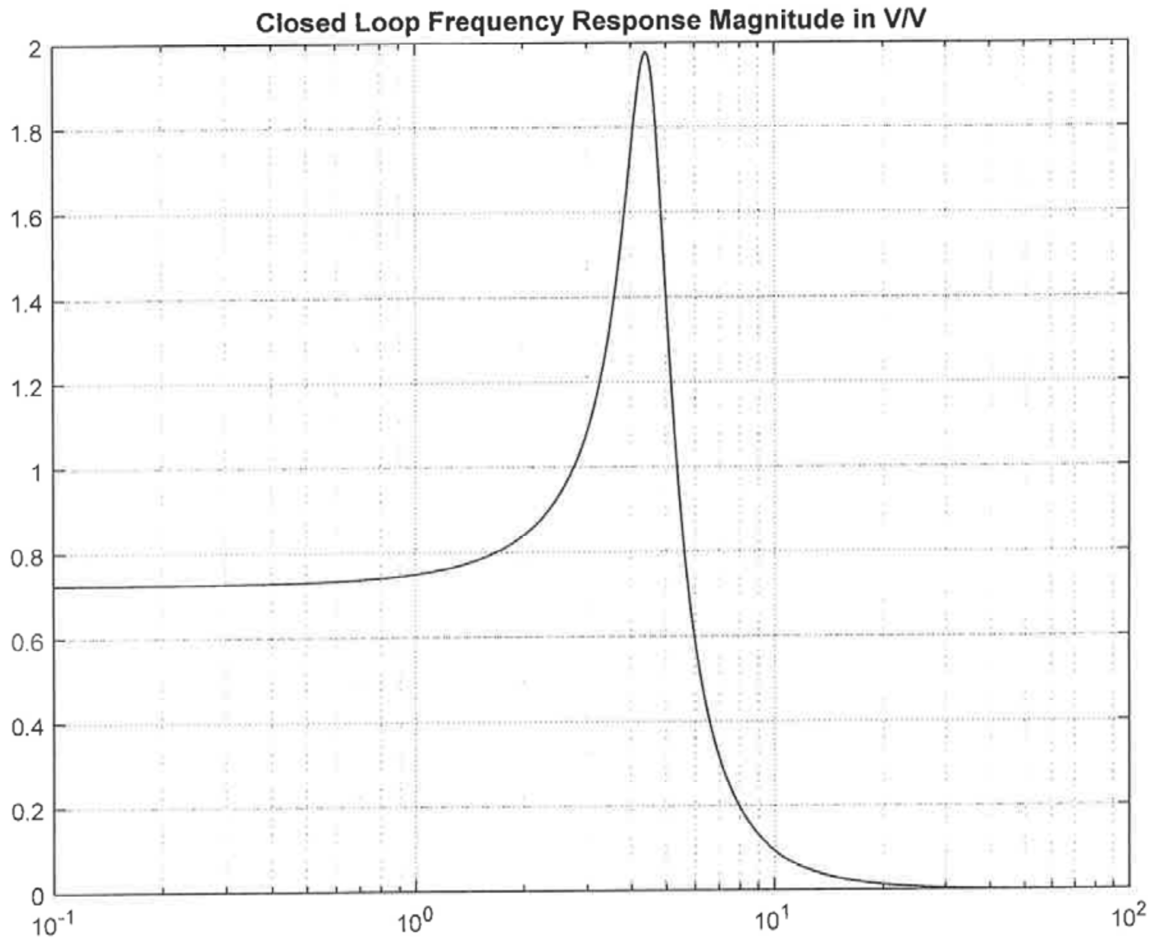


Figure Q4.2 – Closed Loop Frequency Response (Magnitude) Plot of $|G_{cl}(j\omega)|$

1) **(5 marks)** Explain why the second order dominant poles model is appropriate for the closed loop transfer function shown above. Next, determine the appropriate model for it, and write the transfer function of this model, $G_{m1}(s)$.

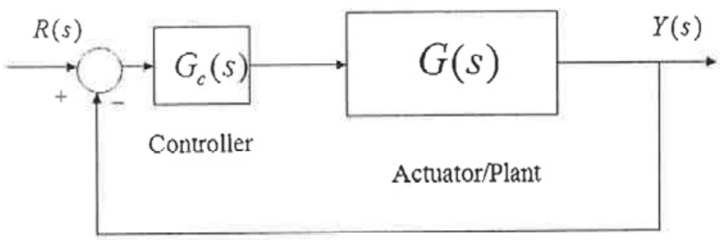
2) **(5 marks)** Determine the model parameters from the step response plot, and write the transfer function of this model, $G_{m2}(s)$.

3) **(5 marks)** Determine the model parameters from the plot of $|G_{cl}(j\omega)|$, and write the transfer function of this model, $G_{m3}(s)$.

4) **(5 marks)** Comment on any discrepancies between the three models, and on which of the three models you would consider the most accurate.

Question 5

Controller Design in Frequency Domain – Lead Controller, Step Response Specifications.



Consider a unit feedback closed loop control system, as shown on the left.

The system is to operate under **Lead Control.** The Lead Controller transfer function is as follows:

$$G_c(s) = K_c \cdot \frac{\tau s + 1}{\alpha \tau s + 1} = \frac{a_1 s + a_0}{b_1 s + 1}$$

Where τ is the so-called Lead Time Constant and $\alpha < 1$.

The process transfer function G(s) is as follows:

$$G(s) = \frac{100(s + 0.8)}{(s + 0.5)(s + 1)^2(s + 15)}$$

Frequency response plots of $G(j\omega)$ are shown in Figure Q5.1. Design requirements for the compensated closed loop system are:

- Steady State Error for the unit step input is to be no more than 4%;
- Percent Overshoot of the compensated closed loop system is to be no more than 15%;
- The Settling Time, $T_{settle(\pm 2\%)}$, is to be no more than 0.7 seconds.
- The Rise Time, $T_{rise(0-100\%)}$, is to be no more than 0.3 seconds.

1) **(5 marks)** Read off the Phase Margin of the uncompensated system (Φ_{m_u}) and the crossover frequency of the uncompensated system (ω_{cp_u}). Next, estimate the **uncompensated** closed loop step response specs: PO, $e_{ss(step\%)}$, $T_{rise(0-100\%)}$ and $T_{settle(\pm 2\%)}$.

2) **(5 marks)** Calculate the Position Constant for the uncompensated system (K_{pos_u}), then the Position Constant for the compensated system (K_{pos_c}) that would meet the design requirements. Decide what value of the Phase Margin for the compensated system (Φ_{m_c}) would meet the design requirements. Decide what value of the crossover frequency for the compensated system (ω_{cp_c}) would meet the design requirements.

3) **(10 marks)** Calculate the appropriate Lead Controller parameters and clearly write the Lead Controller transfer function $G_c(s)$.

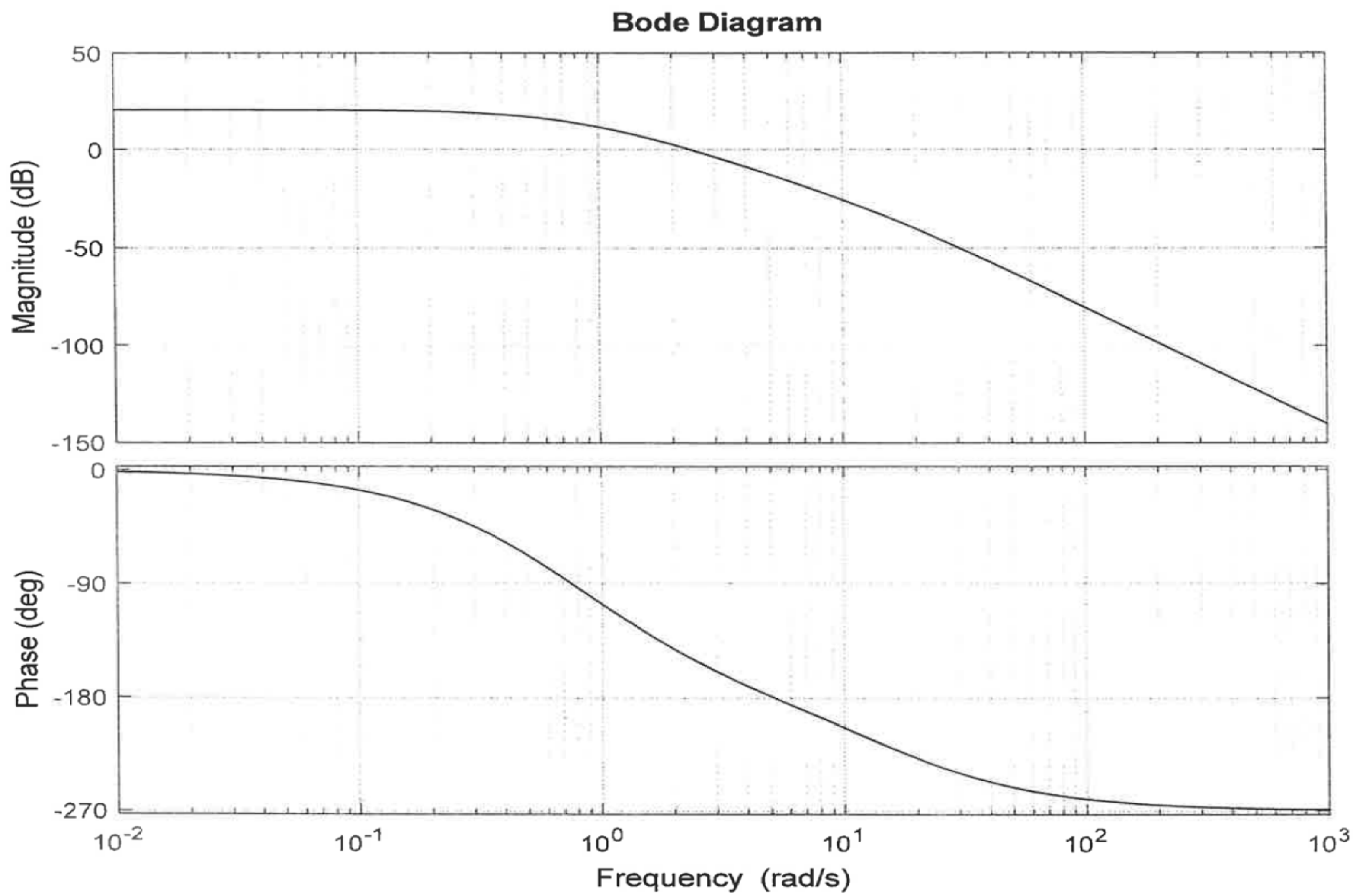


Figure Q5.1 - Frequency Response Plots of the Process $G(j\omega)$ in Question 5

Question 6

Root Locus Analysis and Gain Selection, Stability, Second Order Model, Step Response Specifications.

A unit feedback control system is to be stabilized using a Proportional Controller, as shown in Figure Q6.1.

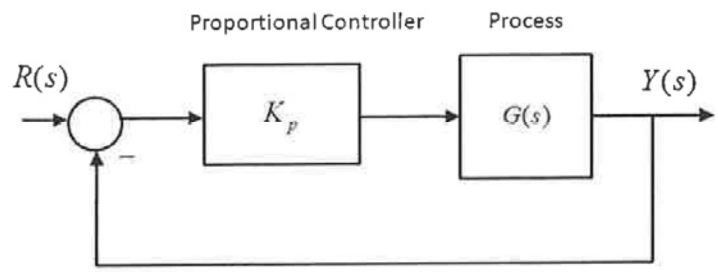


Figure Q6.1

The process transfer function is described as follows:

$$G(s) = \frac{(s+3)(s+6)}{s^2(s+2)}$$

1) **(10 marks)** Sketch the Root Locus for the system, in the space provided in Figure Q6.2. Calculate all relevant coordinates, such as: asymptotic angles, break-in/away points, the location of the centroid as well as the coordinates of the crossover with the Imaginary axis, i.e. ω_{osc} and the corresponding value of the critical gain, K_{crit}, at which the system becomes marginally stable, if applicable.

2) **(7 marks)** It is required that the unit step response of the Closed Loop system exhibits Percent Overshoot of approximately 5%. Determine the corresponding Proportional Gain value, K_{op}, and calculate estimates of the following specs: Settling Time, $T_{settle(\pm 5\%)}$, Rise Time, $T_{rise(0-100\%)}$, and Steady State Error, $e_{ss(step\%)}$.

3) **(3 marks)** Finally, briefly comment on any possible differences between the expected system response (i.e. of the dominant poles model) and the actual system response.

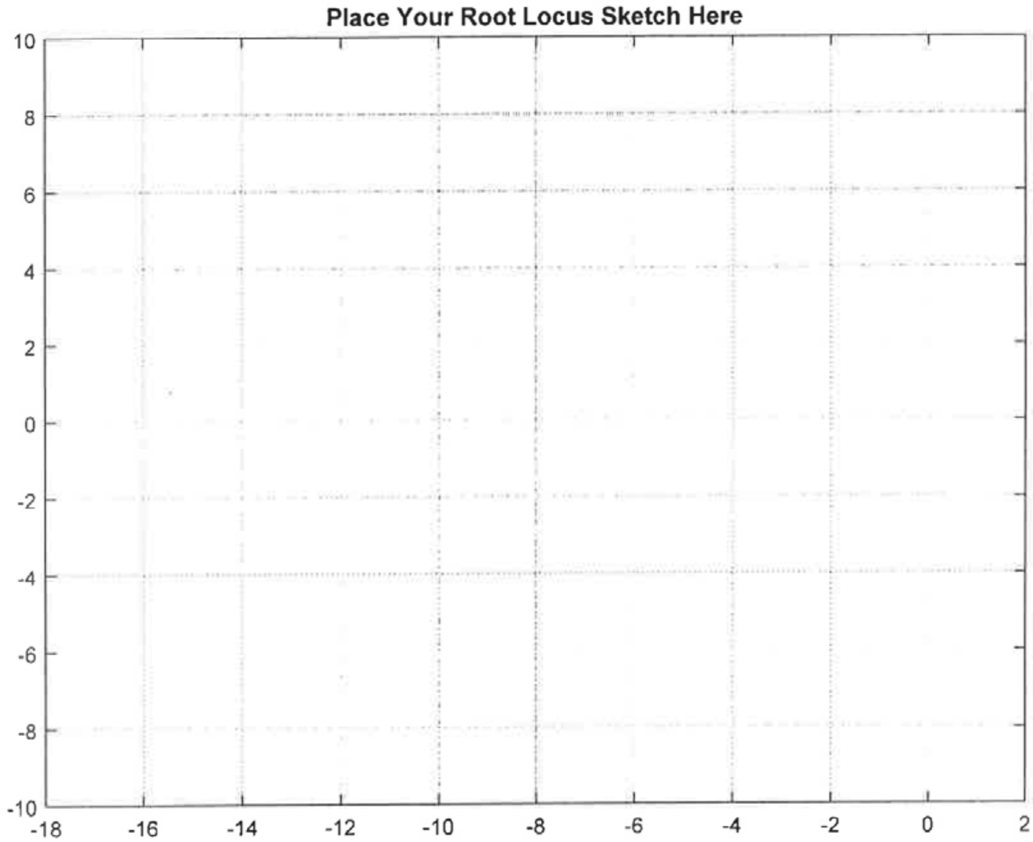


Figure Q6.2 – Root Locus of the System in Question 6

Question 7

Controller Design by Pole Placement, Response Specifications, Second Order Model.

Consider a closed loop positioning control system working under the Proportional + Integral + Rate Feedback Control, shown in Figure Q7.1. The compensated closed loop step response of this system is to have the following specifications: $PO = 5\%$, $T_{settle(\pm 2\%)} = 1$ sec, and $e_{ss(step(\%))} = 0\%$.

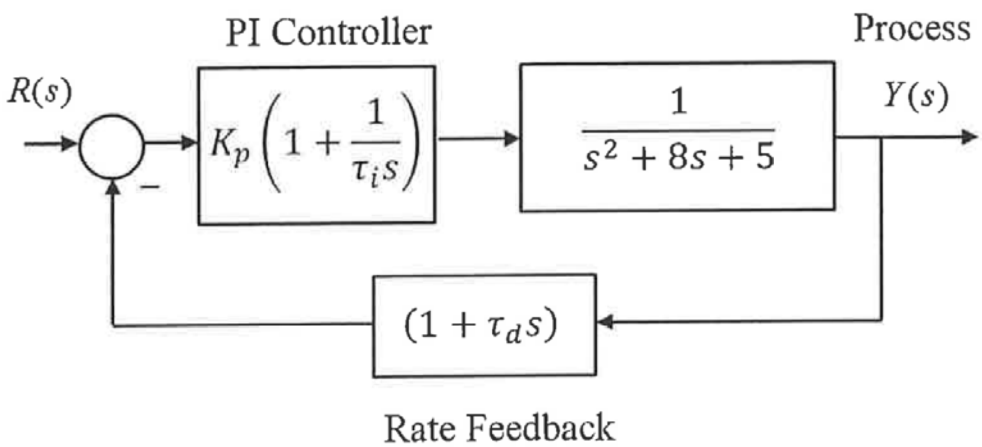


Figure Q7.1 – Closed loop System under PI+Rate Feedback Control

1) **(6 marks)** Derive the closed loop system transfer function in terms of Controller parameters, K_p, τ_d and τ_i, and write the system Characteristic Equation, $Q(s) = 0$.

2) **(6 marks)** Determine the closed loop system damping ratio, ζ, the frequency of natural oscillations, ω_n, and DC Gain, K_{dc}, to meet the transient and steady state response requirements.

3) **(8 marks)** Choose the pole locations for the closed loop system so that the system two complex conjugate "dominant" poles correspond to the desired second order closed loop model (above) and the third real pole is placed so that **a pole-zero cancellation** in the closed loop transfer function occurs. Compute the required Controller parameters, K_p, τ_d, and τ_i.

Question 8

State Space Model from Transfer Functions, Pole Placement by State Feedback Method, Steady State Errors to Step and Ramp Inputs.

Consider a linear open loop system described by the following state space model:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -8 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot u$$

$$y = \begin{bmatrix} 1 & 8 \end{bmatrix} \cdot \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + [0] \cdot u$$

1) **(4 marks)** Find the system eigenvalues. Is the open loop system stable?
2) **(4 marks)** Find the open loop system transfer function, $G_{open}(s) = \frac{Y(s)}{U(s)}$
3) **(4 marks)** Determine if the open loop system is observable and/or controllable.
4) **(4 marks)** Place the system in a closed loop configuration with the reference input **r** and assume the controller equation to be in the form:

$$u = K \cdot \left(r - \mathbf{k}^T \cdot \mathbf{x}\right)$$

Determine the values of the Proportional Gain K and the state feedback vector gains **k** so that the closed loop system will have poles at: -10 and -30, and the steady-state error to a step input will be zero.

5) **(4 marks)** Find the closed loop system transfer function, $G_{cl}(s) = \frac{Y(s)}{R(s)}$, the closed loop poles and zeros, and the closed loop DC gain.