National Exams May 2013

07-Mec-A3, SYSTEM ANALYSIS AND CONTROLS

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. Candidates may use one of two calculators, a Casio or Sharp approved calculator. This is a **closed book** exam. No aids other than semi-log graph papers are permitted.
3. Any four questions constitute a complete paper. Only the first four (4) questions as they appear in your answer book will be marked.
4. All questions are of equal value.

Question 1:

A control system with unity feedback has an open-loop transfer function:

$$G(s) = \frac{K(s+12)}{s^2+25}$$

where K is a scalar gain.

Calculate the closed-loop denominator and find the range of values for K over which the closed-loop poles are real. With $K = 24$, find the step response of the closed-loop system and evaluate the steady-state error which follows a step response.

Question 2:

A unity feedback closed-loop system contains an open-loop transfer function:

$$G(s) = \frac{K}{s(s+25)}$$

Find the value of K which will provide a steady-state ramp error of 5%, i.e. $K_r = 20$. For this value of K obtain (a) the steady-state error to a unit step input, and the transient errors produced by (b) a unit step input, and (c) a unit ramp input.

Question 3:

A unity feedback control system has an open-loop transfer function:

$$G(s) = \frac{K(s+2)}{s(s+1)(s^2+2s+2)}$$

Find the range of values for K over which the closed-loop system is stable.

Question 4:

The open-loop transfer function of a unity feedback system is:

$$G(s) = \frac{K}{s(0.5s+1)(2s+1)}$$

Determine the range of values for K over which the closed-loop system will be stable.

Show that if the feedback is made equal to $(s + 3)$ rather than unity, then the stability range will be increased.

Question 5:

Draw the root locus diagram for the system

$$G(s) = \frac{K}{s\,(s+1)\,(s+2)}$$

and find the value of K which gives the complex closed-loop pole pair a damping ratio of $\zeta = 0.5$.

Question 6:

Given the open-loop transfer function

$$G(s) = \frac{3\,(s+1)}{s\,(s+a)}$$

By means of a Bode plot, find the value of a which results in a gain crossover frequency of $\omega = 100$ rad/s, when the system is connected within a unity feedback loop.

Laplace Transform Table

Laplace Transform $F(s)$	Time function $f(t)$
1	Unit-impulse function $\delta(t)$
$\frac{1}{s}$	Unit-step function $u_s(t)$
$\frac{1}{s^2}$	Unit-ramp function t
$\frac{n!}{s^{n+1}}$	t^n (n = positive integer)
$\frac{1}{s+\alpha}$	$e^{-\alpha t}$
$\frac{1}{(s+\alpha)^2}$	$te^{-\alpha t}$
$\frac{n!}{(s+\alpha)^{n+1}}$	$t^n e^{-\alpha t}$ (n = positive integer)
$\frac{1}{(s+\alpha)(s+\beta)}$	$\frac{1}{\beta-\alpha}(e^{-\alpha t}-e^{-\beta t})\ (\alpha \neq \beta)$
$\frac{s}{(s+\alpha)(s+\beta)}$	$\frac{1}{\beta-\alpha}(\beta e^{-\beta t}-\alpha e^{-\alpha t})\ (\alpha \neq \beta)$
$\frac{1}{s(s+\alpha)}$	$\frac{1}{\alpha}(1-e^{-\alpha t})$
$\frac{1}{s(s+\alpha)^2}$	$\frac{1}{\alpha^2}(1-e^{-\alpha t}-\alpha t e^{-\alpha t})$
$\frac{1}{s^2(s+\alpha)}$	$\frac{1}{\alpha^2}(\alpha t-1+e^{-\alpha t})$
$\frac{1}{s^2(s+\alpha)^2}$	$\frac{1}{\alpha^2}\left[t-\frac{1}{\alpha}+\left(t+\frac{2}{\alpha}\right)e^{-\alpha t}\right]$

Laplace Transform Table (continued)

Laplace Transform $F(s)$	Time Function $f(t)$
$\frac{s}{(s+\alpha)^2}$	$(1-\alpha t)e^{-\alpha t}$
$\frac{\omega_n^2}{s^2+\omega_n^2}$	$\sin \omega_n t$
$\frac{s}{s^2+\omega_n^2}$	$\cos \omega_n t$
$\frac{\omega_n^2}{s(s^2+\omega_n^2)}$	$1-\cos \omega_n t$
$\frac{\omega_n^2(s+\alpha)}{s^2+\omega_n^2}$	$\omega_n\sqrt{\alpha^2+\omega_n^2}\sin(\omega_n t+\theta)$ where $\theta = \tan^{-1}(\omega_n/\alpha)$
$\frac{\omega_n}{(s+\alpha)(s^2+\omega_n^2)}$	$\frac{\omega_n}{\alpha^2+\omega_n^2}e^{-\alpha t}+\frac{1}{\sqrt{\alpha^2+\omega_n^2}}\sin(\omega_n t-\theta)$ where $\theta = \tan^{-1}(\omega_n/\alpha)$
$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\omega_n\sqrt{1-\zeta^2}\,t \quad (\zeta<1)$
$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\theta)$ where $\theta = \cos^{-1}\zeta \quad (\zeta<1)$
$\frac{s\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{-\omega_n^2}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t-\theta)$ where $\theta = \cos^{-1}\zeta \quad (\zeta<1)$
$\frac{\omega_n^2(s+\alpha)}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\omega_n\sqrt{\frac{\alpha^2-2\alpha\zeta\omega_n+\omega_n^2}{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\theta)$ where $\theta = \tan^{-1}\frac{\omega_n\sqrt{1-\zeta^2}}{\alpha-\zeta\omega_n} \quad (\zeta<1)$
$\frac{\omega_n^2}{s^2(s^2+2\zeta\omega_n s+\omega_n^2)}$	$t-\frac{2\zeta}{\omega_n}+\frac{1}{\omega_n\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\theta)$ where $\theta = \cos^{-1}(2\zeta^2-1) \quad (\zeta<1)$

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Semi-Logarithmic
4 Cycles x 10 to the inch