National Exams December 2018

17-Ind-A1, Operations Research

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Any non-communicating calculator is permitted. This is an Open Book exam. Note to candidates: You must indicate the type of calculator being used. i.e. write the name and model designation of the calculator, on the first left hand sheet of the exam workbook.

3. There may be more questions than you are able to answer in the allotted time. Although the total value of the questions is 170, any marks achieved will be considered toward the 100 total requirements.

1. A company buys tractors at a cost of $6,500 and sells them for $10,000. A charge of $2500 is incurred for each order of tractors, regardless of the size of the order. The company estimates the holding charge for one tractor to be 500 $/tractor/mth.

5 a. If there is a monthly demand for 15 tractors what is the optimal ordering policy, and the resulting monthly inventory (i.e. holding and ordering) cost?

15 b. If instead the demand is not constant and the forecast for the next 4 months is 20, 25, 12 and 3, use dynamic programming to determine the optimal ordering policy and the minimal cost over the four-month period.

15 2. A company has contracted to sell certain quantities of a particular product over the next four months. Because of variations in the size of the labour force, the production capacity and costs vary from month to month. Storage costs are incurred on any item carried over to a later month. (If the item is sold in the same month as production, no storage cost is incurred.) These costs are summarized below:

Month	Contracted Sales	Production Capacity	Production Cost per Unit	Storage Cost per Unit
1	60	90	70	2
2	70	60	72	1
3	90	80	70	1
4	70	100	65	3

Formulate, but do not solve, the linear programming model to minimize costs.

3. Consider the following LP problem:

Maximize: $5x_1 + 6x_2$

subject to: $4x_1 + 2x_2 \leq 16$
$3x_1 + 3x_2 \geq 18$
$0 \leq x_2 \leq 5$
$x_1 \geq 0$

10 a. Solve the problem graphically.

10 b. State, but do not solve, the dual problem.

4. Consider the following problem

Maximize $z = 21x_1 + 9x_2 + 4x_3$ (profit)

Subject to

$2x_1 + x_2 + x_3 \leq 31$ (resource constraint 1)
$3x_1 + 2x_2 + x_3 \leq 60$ (resource constraint 2)
$x_1 + 2x_2 + x_3 \geq 50$ (requirement constraint)
$x_1 \geq 0$
$x_2 \geq 0$

The simplex method yields the following final set of equations

$z + (1/2)x_3 + (2/3)x_4 + x_6 = 291$
$x_1 + (1/3)x_3 + (2/3)x_4 + (1/3)x_6 = 4$
$x_2 + (1/3)x_3 - (1/3)x_4 - (2/3)x_6 = 23$
$x_5 - (2/3)x_3 - (4/3)x_4 + (1/3)x_6 = 2$

where x_4 is the slack variable for resource constraint 1, x_5 is the slack variable for resource constraint 2, and x_6 is the slack variable for the requirement constraint.

5 a. What is the optimal solution, the maximum profit, the marginal values of resources 1 and 2, and the marginal cost of the requirement?

5 b. How much can the coefficient of x_2 in the objective function vary without affecting the optimal solution?

5 c. By how much would the profit be increased if 5 more units of resource 1 where available? What would be the new solution?

5. In a particular department there are two mechanics to service 4 machines. All machines have identical exponential distributions for their running times after repair. The average is 6 days. The time needed to repair any of the machines is also exponentially distributed, with an average of 1 day.

10 a. What is the steady state probability that all machines are running?

10 b. On average, how many machines wait for an available mechanic?

6. A credit manager needs to decide if she should extend $100,000 credit to a potential new customer. She has 3 categories for the credit-worthiness of a customer: poor risk, average risk or good risk. Extending credit to a poor risk customer results in an expected loss of $15,000. Extending credit to an average or good risk customer results in an expected profit of $10,000 and $20,000 respectively. Based on experience the credit manager believes that the new customer has a 20% chance of turning out to be a poor risk, a 50% of being an average risk customer, and a 30% chance of being a good risk customer. For $5,000 she can use the services of a credit-rating service whose track record is given by the following table:

	Actual Credit Record		
Credit Evaluation	Poor	Average	Good
Poor	50%	40%	20%
Average	40%	50%	40%
Good	10%	10%	40%

5 a. Assuming she decides NOT to use the credit-rating service, based on an expected value criterion, should she extend credit to the new potential customer? What is the expected profit?

5 b. Assuming she decides to use the credit rating service, and they give the new potential customer a poor rating, should she extend credit? What is the expected profit?

5 c. Should the credit manager use the credit rating service? Explain.

20 7. The number of people entering a specialty shop during a particular hour exhibits some randomness. There is a 50% probability that there will be no arrivals in the particular hour, a 30% probability of one arrival, and a 20% probability of 2 arrivals. Each person entering the store independently decides to make a purchase with probability 0.7. If he makes a purchase the value of the purchase is $10 with probability 0.2, $100 with probability 0.6 and $1000 with probability 0.2. Using an appropriate flow chart and the partial listing of random numbers which follows, obtain one simulated value for the number of purchases made during the hour, outputting also the number of purchases per hour, and the average dollar volume of business for that hour.

Random number list: 9508 3702 9250 2946 1044 6559
1097 7416 2166 3561 5975 1787.

15 8. A meat-processing firm prepares a batch of spicy sausages every Wednesday for sale by butchers and supermarkets prior to the weekend. The sausages are sold by weight. Any sausages not sold by the weekend are sold to a pet food manufacturer on Monday. The pet food manufacturer accepts any amount of old sausages. The gross profit (i.e. the manufacturing price less manufacturing cost) is 50 cents/kg of fresh sausages sold. The gross loss per kg on sales to the pet food manufacturer (i.e. manufacturing cost less proceeds) amounts to 10 cents. The weekly demand for spicy sausages is a random variable with a probability density function uniformly distributed between 1000 and 2000 kg. Determine the size of the weekly batch that minimizes the expected total net profit (i.e. gross profit minus losses).

15 9. At a machine tool plant five jobs must be completed each day. The time it takes to process each job varies with the machine used to do the job. Also the first time a machine is used a setup time is required. The relevant processing and setup times (in minutes) are as follows:

Machine	Job 1	Job 2	Job 3	Job 4	Job 5	Setup
1	42	70	93	-	-	30
2	-	85	45	-	-	40
3	58	-	-	37	-	50
4	58	-	55	-	38	60
5	-	60	-	54	-	20

Formulate, but do not solve, an integer programming model to minimize the overall processing and setup time to complete the five jobs.

15 10. Solve the following integer LP problem using the branch and bound method.

Maximize $x_1 + 2x_2$

subject to

$$x_1 + 3x_2 \leq 16$$
$$x_1 + x_2 \leq 7$$
$$x_1 \geq 0, x_2 \geq 0,$$
$$x_1 \text{ and } x_2 \text{ integers}$$

Note: The graphical LP method may be used within the branch and bound solution method.