

National Exams December 2018

17-Ind-B1, Applied Probability & Statistics

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. This is a closed book exam. Candidates are permitted to use one of the two permitted calculators (Sharp or Casio models).
3. Candidates are permitted to have an aid sheet consisting of one 8.5" x 11.0" sheet of paper. Writing is permitted on both sides of the paper.
4. This exam consists of three sections (A→C). Within each section, candidates will be given a choice of questions to answer. Please read the instructions for each section carefully. A breakdown of questions and marks is as follows:

Section A:	Do 2 of 4 Questions.	Total marks: 20
Section B:	Do 2 of 3 Questions.	Total marks: 40
Section C:	Do 1 of 2 Questions.	Total marks: 20

Exam:	5 Questions.	Total marks: 100
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4. The value of each question is listed in the exam. Remember to check the instructions for each section. **DO NOT ATTEMPT TO DO ALL QUESTIONS.**
5. Statistical tables are provided. If, for any reason, a value is missing, please note this and use an approximate value.

Section A – Basic Concepts: Complete two of the following four questions. This section is worth a total of 30 marks. Do not attempt all questions.

1. An Industrial Engineer analyzing a production process involving several assembly steps.
 - a. One particular assembly step involves attaching two steel plates with a screw. If the process step is assumed to be $N(12, 3)$ seconds, what is the probability that an assembly requires more than 15 seconds to complete?
 - b. If a second step is added to the assembly process that is assumed to be $N(10, 5)$ seconds, would it be considered unusual if the total time to complete the assembly took more than 28 seconds?
 - c. Is the normal distribution a reasonable assumption in this instance where we are looking at a task completion time? You may assume that the task will be repeated many times.

10 Marks

2. An industrial process is to undergo a design change. Prior to the design change, a sample of size 5 is taken, yielding an average of 10 with a standard deviation of 3. After the design change another sample of size 5 is taken, yielding an average of 4 with a standard deviation of 5. You may assume the process values are normally distributed.

- a. Test the following hypothesis using an α of 0.05

$$\begin{aligned} H_0: & \quad \bar{x}_1 = \bar{x}_2 \\ H_1: & \quad \bar{x}_1 \neq \bar{x}_2 \end{aligned}$$

- b. Test the following hypothesis using an α of 0.05

$$\begin{aligned} H_0: & \quad \bar{x}_1 = \bar{x}_2 \\ H_1: & \quad \bar{x}_1 > \bar{x}_2 \end{aligned}$$

- c. Reconcile your results from (a) and (b) and provide guidance if management wishes to determine if the process change has resulted in a process improvement of at least 1, where lower values are considered to be better.

10 Marks

3. A firm uses two shipping companies to transport its finished goods from Canada to Europe. A sample of 5 shipping completion times from each company appears below, with the shipping time in days.

A	B
35	38
30	26

33	37
27	32
32	32

- Analyze the samples using whatever method you feel is appropriate and determine if one shipping firm provides faster deliveries than the other.
- Discuss the issues around appropriate statistical tests for this instance. What statistical elements should management be aware of when conducting a comparison?
- Assume you have been tasked with completing the analysis of the two shipping companies. Briefly describe how you would complete the analysis. I am not looking for a rehash of the method from your answer to (a), but rather a *plan* for collecting and analyzing data that reflects the issues identified in (b).

10 Marks

4. Suppose a continuous random variable x has the probability density function $f(x)$ given below:

$$f(x) = \begin{cases} 0 & f \ x \leq 0 \\ \frac{2x}{R^2} & f \ 0 < x < R \\ 0 & otherwise \end{cases}$$

- Prove that this function is a valid probability density function
- Find the mean of the function
- Find the variance of the function

10 Marks

Section B – Intermediate Concepts: Complete two of the following three questions. This section is worth a total of 30 marks. Do not attempt all questions.

1. As part of a recent study of patients undergoing a particular medical treatment, researchers recorded the demographics of patients in their study, specifically looking at the relationship between body mass index (BMI) and age. The data is below:

Patient	1	2	3	4	5	6	7	8	9
BMI	20	21	30	20	26	22	24	18	36
Age	45	34	40	32	28	30	52	33	47

- Fit a regression model for age (x) versus BMI (y) of the form $y = ax + b$
- Test for the significance of the regression using $\alpha = 0.05$.
- Determine whether the coefficient for x is significant.
- Give the expected BMI for a 42 year-old individual and determine if we would consider a 42 year old with a BMI of 28 to be a statistical anomaly.

20 Marks

2. Each year the Bluenose Marathon is run in Halifax, Nova Scotia in May. The route includes a loop on the Halifax peninsula and a loop across the harbour in Dartmouth, with runners crossing a bridge between the two loops. The organizers have traditionally alternated the loops: one year the Halifax loop is done first; the next year the Dartmouth loop is done first. The course is quite hilly, with particularly bad hills on the Dartmouth loop. The weather in May can be quite variable as well.

A random sample of actual data from 10 finishers from two years of the race is presented below:

Halifax 1st	Dartmouth 1st
203	207
209	222
225	229
239	240
255	247
260	247
261	250
262	252
263	260
279	265

- a. Determine, using $\alpha = 0.01$ whether the variances from the two populations are the same. (You may assume that the populations are approximately normal distributed.)
- b. Using this information, develop a hypothesis test to test whether the difference in finishing time between the two routes is actually 0. Use $\alpha = 0.1$
- c. State any assumptions that you have made in developing your hypothesis test.
- d. Find the approximate p-value for this test and provide interpretation about what the p-value means.
- e. Regardless of the statistical significance you found in (b), describe two or three potential sources of bias in the data sample and suggest a mechanism (or mechanisms) for resolving them.

20 Marks

3. Edison Motors is manufacturing a new line of electric scooters. They have three manufacturing facilities producing the same vehicles. Below is a sample of the number of labour hours required to complete a vehicle (in hours).

A	B	C
118	158	227
118	155	215
135	152	215
126	167	221
130	157	217

- a) Use Bartlett's test to test the hypothesis at the 0.05 level of significance that the population variances of the three growth records are equal.
- b) Assuming that labour hours is normally distributed, determine using ANOVA whether the plant significantly affects time to build. You may assume, for ease of calculations, that SST is 23047.6 and SSE is a number approximately equal to 460 (it is in fact not exactly 460 – please calculate SSE if possible. If you cannot find SSE, use the value 460).
- c) Using Tukey's test ($\alpha = 0.05$) to determine which, if any, of the plants has a significantly different growth pattern than the other.

20 Marks

Section C – Advanced Concepts: Complete **one** of the following **two** questions. This section is worth a total of 20 marks. Do not attempt all questions.

1. A variable (y) was observed at different combinations of values of controlled variables x_1, x_2, x_3 . The results are shown below.

x_1	x_2	x_3	y
85	12	7	2.15
66	18	10	1.09
74	17	8	1.26
80	15	6	1.57
85	16	7	1.92

The relation between y and (x_1, x_2) is assumed to be linear in the region of interest, find the least squares regression model for this data set. To aid you in your calculations assume the following:

$(X'X)$:

5	390	78	38
390	30682	6026	2922
78	6026	1238	602
38	2922	*	**

- a. Determine the missing values for the $(X'X)$ matrix.
- b. If $(X'X)^{-1}$ is given below:

322.8483	-2.5213	-4.0284	-8.3081
-2.5213	0.0204	0.0273	0.0661
-4.0284	0.0273	0.1197	0.0047
-8.3081	0.0661	0.0047	0.4055

Find the least squares linear regression model for this data set.

- c. Complete the ANOVA table for this regression model and determine whether the linear model is a significant predictor of y . For ease of calculations, you may assume that $SST = 0.7815$ and $SSR = 0.7749$
- d. Determine if the x_2 coefficient in your model is significant at the 95% level.
- e. Develop a 95% Confidence Interval and a 95% Prediction Interval for $(x_1 = 80, x_2 = 15, x_3 = 5)$. Explain the difference between the two (i.e. explain the difference between a CI and a PI).

20 Marks

2. You are an IE employed by the Northern Regional Health Authority (NRHA) to develop a simulation of the Albert Memorial Hospital in Bapaume, Nova Scotia. You are preparing to conduct a series of screening runs, using a design of experiments approach, to test patient flow policies via a simulation model. Your screening experiment includes two factors under management control: off-servicing and discharge planning. Off-servicing refers to the policy of allowing patients of one service (i.e. medicine) to be housed in a bed of another service (i.e. surgery). Discharge planning is a management policy which specifies that certain nursing staff members are assigned full time responsibility for planning a patient's discharge on admission (i.e. arranging home care, searching for space in nursing homes, etc.) Both controls are binary: they can be implemented or not. You may assume that implemented is the high setting and not implemented is the low setting.

The partial design matrix for this problem is listed below:

Run	Factors	
	A	B
1	-1	-1
2	1	-1
3	-1	1
4	1	1

- a. Complete the design matrix for this experiment and interpret the meaning of the settings (i.e. tell me what the signs mean in terms of the control factor settings, the main effects, and the interaction effects). You may assume that Factor A refers to the off-servicing policy, while Factor B refers to the discharge planning policy.
- b. Assume that you have completed five simulation replications of each run listed in the design matrix above. The following output gives the average number of beds occupied:

Run	Factors		Rep 1	Rep 2	Rep 3	Rep 4	Rep 5	Average
	A	B						
1	-1	-1	73.93	70.02	72.97	70.13	71.71	71.75
2	1	-1	69.53	68.80	71.40	71.52	71.53	70.56
3	-1	1	71.08	73.39	73.78	72.23	73.16	72.73
4	1	1	68.51	69.16	71.96	70.88	69.58	70.02

Complete an analysis of variance (ANOVA) on this output and determine which, if any, factors are significant. You may assume that $SST = 53.17$ and an alpha value of 0.05.

20 Marks

Areas Under the Normal Curve

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.40	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.30	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.20	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.10	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.00	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.90	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.80	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.70	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.60	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.50	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.40	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.30	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.20	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.10	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.00	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.90	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.80	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.70	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.60	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.50	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.40	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.30	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.20	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.10	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.00	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.90	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.80	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.70	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.60	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.50	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.40	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.30	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.20	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.10	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.00	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Areas Under the Normal Curve

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.10	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.20	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.30	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.40	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.50	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.60	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.70	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.80	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.90	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.00	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.10	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.20	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.30	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.40	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.50	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.60	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.70	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.80	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.90	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.00	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.10	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.20	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.30	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.40	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.50	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.60	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.70	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.80	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.90	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.00	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.10	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.20	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.30	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.40	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Critical Values of the t-Distribution

v	α						
	0.40	0.30	0.20	0.15	0.10	0.05	0.025
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
50	0.255	0.528	0.849	1.047	1.299	1.676	2.009
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
70	0.254	0.527	0.847	1.044	1.294	1.667	1.994
80	0.254	0.526	0.846	1.043	1.292	1.664	1.990
90	0.254	0.526	0.846	1.042	1.291	1.662	1.987
100	0.254	0.526	0.845	1.042	1.290	1.660	1.984
110	0.254	0.526	0.845	1.041	1.289	1.659	1.982
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
130	0.254	0.526	0.844	1.041	1.288	1.657	1.978
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

Critical Values of the t-Distribution

v	α						
	0.02	0.015	0.01	0.0075	0.005	0.0025	0.0005
1	15.895	21.205	31.821	42.433	63.657	127.321	636.619
2	4.849	5.643	6.965	8.073	9.925	14.089	31.599
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	2.197	2.336	2.528	2.661	2.845	3.153	3.850
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	2.158	2.291	2.473	2.598	2.771	3.057	3.690
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	2.150	2.282	2.462	2.586	2.756	3.038	3.659
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551
50	2.109	2.234	2.403	2.519	2.678	2.937	3.496
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
70	2.093	2.215	2.381	2.494	2.648	2.899	3.435
80	2.088	2.209	2.374	2.486	2.639	2.887	3.416
90	2.084	2.205	2.368	2.480	2.632	2.878	3.402
100	2.081	2.201	2.364	2.475	2.626	2.871	3.390
110	2.078	2.199	2.361	2.471	2.621	2.865	3.381
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373
130	2.075	2.194	2.355	2.465	2.614	2.856	3.367
∞	2.054	2.170	2.327	2.433	2.576	2.808	3.291

Critical Values of the Chi-Squared Distribution

v	α									
	0.995	0.99	0.98	0.975	0.95	0.90	0.80	0.75	0.70	0.50
1	0.000	0.000	0.001	0.001	0.004	0.016	0.064	0.102	0.148	0.455
2	0.010	0.020	0.040	0.051	0.103	0.211	0.446	0.575	0.713	1.386
3	0.072	0.115	0.185	0.216	0.352	0.584	1.005	1.213	1.424	2.366
4	0.207	0.297	0.429	0.484	0.711	1.064	1.649	1.923	2.195	3.357
5	0.412	0.554	0.752	0.831	1.145	1.610	2.343	2.675	3.000	4.351
6	0.676	0.872	1.134	1.237	1.635	2.204	3.070	3.455	3.828	5.348
7	0.989	1.239	1.564	1.690	2.167	2.833	3.822	4.255	4.671	6.346
8	1.344	1.646	2.032	2.180	2.733	3.490	4.594	5.071	5.527	7.344
9	1.735	2.088	2.532	2.700	3.325	4.168	5.380	5.899	6.393	8.343
10	2.156	2.558	3.059	3.247	3.940	4.865	6.179	6.737	7.267	9.342
11	2.603	3.053	3.609	3.816	4.575	5.578	6.989	7.584	8.148	10.341
12	3.074	3.571	4.178	4.404	5.226	6.304	7.807	8.438	9.034	11.340
13	3.565	4.107	4.765	5.009	5.892	7.042	8.634	9.299	9.926	12.340
14	4.075	4.660	5.368	5.629	6.571	7.790	9.467	10.165	10.821	13.339
15	4.601	5.229	5.985	6.262	7.261	8.547	10.307	11.037	11.721	14.339
16	5.142	5.812	6.614	6.908	7.962	9.312	11.152	11.912	12.624	15.338
17	5.697	6.408	7.255	7.564	8.672	10.085	12.002	12.792	13.531	16.338
18	6.265	7.015	7.906	8.231	9.390	10.865	12.857	13.675	14.440	17.338
19	6.844	7.633	8.567	8.907	10.117	11.651	13.716	14.562	15.352	18.338
20	7.434	8.260	9.237	9.591	10.851	12.443	14.578	15.452	16.266	19.337
21	8.034	8.897	9.915	10.283	11.591	13.240	15.445	16.344	17.182	20.337
22	8.643	9.542	10.600	10.982	12.338	14.041	16.314	17.240	18.101	21.337
23	9.260	10.196	11.293	11.689	13.091	14.848	17.187	18.137	19.021	22.337
24	9.886	10.856	11.992	12.401	13.848	15.659	18.062	19.037	19.943	23.337
25	10.520	11.524	12.697	13.120	14.611	16.473	18.940	19.939	20.867	24.337
26	11.160	12.198	13.409	13.844	15.379	17.292	19.820	20.843	21.792	25.336
27	11.808	12.879	14.125	14.573	16.151	18.114	20.703	21.749	22.719	26.336
28	12.461	13.565	14.847	15.308	16.928	18.939	21.588	22.657	23.647	27.336
29	13.121	14.256	15.574	16.047	17.708	19.768	22.475	23.567	24.577	28.336
30	13.787	14.953	16.306	16.791	18.493	20.599	23.364	24.478	25.508	29.336

Critical Values of the Chi-Squared Distribution

v	α									
	0.3	0.25	0.2	0.1	0.05	0.025	0.02	0.01	0.005	0.001
1	1.074	1.323	1.642	2.706	3.841	5.024	5.412	6.635	7.879	10.828
2	2.408	2.773	3.219	4.605	5.991	7.378	7.824	9.210	10.597	13.816
3	3.665	4.108	4.642	6.251	7.815	9.348	9.837	11.345	12.838	16.266
4	4.878	5.385	5.989	7.779	9.488	11.143	11.668	13.277	14.860	18.467
5	6.064	6.626	7.289	9.236	11.070	12.833	13.388	15.086	16.750	20.515
6	7.231	7.841	8.558	10.645	12.592	14.449	15.033	16.812	18.548	22.458
7	8.383	9.037	9.803	12.017	14.067	16.013	16.622	18.475	20.278	24.322
8	9.524	10.219	11.030	13.362	15.507	17.535	18.168	20.090	21.955	26.124
9	10.656	11.389	12.242	14.684	16.919	19.023	19.679	21.666	23.589	27.877
10	11.781	12.549	13.442	15.987	18.307	20.483	21.161	23.209	25.188	29.588
11	12.899	13.701	14.631	17.275	19.675	21.920	22.618	24.725	26.757	31.264
12	14.011	14.845	15.812	18.549	21.026	23.337	24.054	26.217	28.300	32.909
13	15.119	15.984	16.985	19.812	22.362	24.736	25.472	27.688	29.819	34.528
14	16.222	17.117	18.151	21.064	23.685	26.119	26.873	29.141	31.319	36.123
15	17.322	18.245	19.311	22.307	24.996	27.488	28.259	30.578	32.801	37.697
16	18.418	19.369	20.465	23.542	26.296	28.845	29.633	32.000	34.267	39.252
17	19.511	20.489	21.615	24.769	27.587	30.191	30.995	33.409	35.718	40.790
18	20.601	21.605	22.760	25.989	28.869	31.526	32.346	34.805	37.156	42.312
19	21.689	22.718	23.900	27.204	30.144	32.852	33.687	36.191	38.582	43.820
20	22.775	23.828	25.038	28.412	31.410	34.170	35.020	37.566	39.997	45.315
21	23.858	24.935	26.171	29.615	32.671	35.479	36.343	38.932	41.401	46.797
22	24.939	26.039	27.301	30.813	33.924	36.781	37.659	40.289	42.796	48.268
23	26.018	27.141	28.429	32.007	35.172	38.076	38.968	41.638	44.181	49.728
24	27.096	28.241	29.553	33.196	36.415	39.364	40.270	42.980	45.559	51.179
25	28.172	29.339	30.675	34.382	37.652	40.646	41.566	44.314	46.928	52.620
26	29.246	30.435	31.795	35.563	38.885	41.923	42.856	45.642	48.290	54.052
27	30.319	31.528	32.912	36.741	40.113	43.195	44.140	46.963	49.645	55.476
28	31.391	32.620	34.027	37.916	41.337	44.461	45.419	48.278	50.993	56.892
29	32.461	33.711	35.139	39.087	42.557	45.722	46.693	49.588	52.336	58.301
30	33.530	34.800	36.250	40.256	43.773	46.979	47.962	50.892	53.672	59.703

Critical Values of the F Distribution

$f_{0.05(v1, v2)}$

v2	v1									
	1	2	3	4	5	6	7	8	9	10
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83

Critical Values of the F Distribution

$f_{0.05(v1,v2)}$

v2	v1									
	10	12	15	20	24	30	40	60	120	∞
1	241.88	243.91	245.95	248.01	249.05	250.10	251.14	252.20	253.25	254.30
2	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37
6	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.41
12	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
50	2.03	1.95	1.87	1.78	1.74	1.69	1.63	1.58	1.51	1.44
60	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.26
∞	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.01

Critical Values of the F Distribution

		$f_{0.01(v1, v2)}$									
v2	v1										
	1	2	3	4	5	6	7	8	9	10	
1	4052.18	4999.50	5403.35	5624.58	5763.65	5858.99	5928.36	5981.07	6022.47	6055.85	
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	
50	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78	2.70	
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	

Critical Values of the F Distribution

		$f_{0.01}(v_1, v_2)$									
		v1									
v2	10	12	15	20	24	30	40	60	120	∞	
1	6055.85	6106.32	6157.28	6208.73	6234.63	6260.65	6286.78	6313.03	6339.39	6365.55	
2	99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.49	99.50	
3	27.23	27.05	26.87	26.69	26.60	26.50	26.41	26.32	26.22	26.13	
4	14.55	14.37	14.20	14.02	13.93	13.84	13.75	13.65	13.56	13.46	
5	10.05	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02	
6	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88	
7	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65	
8	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86	
9	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31	
10	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91	
11	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60	
12	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36	
13	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17	
14	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.01	
15	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87	
16	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75	
17	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65	
18	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57	
19	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49	
20	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42	
21	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36	
22	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31	
23	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26	
24	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21	
25	3.13	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27	2.17	
26	3.09	2.96	2.81	2.66	2.58	2.50	2.42	2.33	2.23	2.13	
27	3.06	2.93	2.78	2.63	2.55	2.47	2.38	2.29	2.20	2.10	
28	3.03	2.90	2.75	2.60	2.52	2.44	2.35	2.26	2.17	2.07	
29	3.00	2.87	2.73	2.57	2.49	2.41	2.33	2.23	2.14	2.04	
30	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01	
40	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.81	
50	2.70	2.56	2.42	2.27	2.18	2.10	2.01	1.91	1.80	1.68	
60	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60	
120	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38	
∞	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.01	

Critical Values for Bartlett's Test

$b_k(0.01;n)$									
Number of populations, k									
n	2	3	4	5	6	7	8	9	10
3	0.1411	0.1672							
4	0.2843	0.3165	0.3475	0.3729	0.3937	0.4110			
5	0.3984	0.4304	0.4607	0.4850	0.5046	0.5207	0.5343	0.5458	0.5558
6	0.4850	0.5149	0.5430	0.5653	0.5832	0.5975	0.6100	0.6204	0.6293
7	0.5512	0.5787	0.6045	0.6248	0.6410	0.6542	0.6652	0.6744	0.6824
8	0.6031	0.6282	0.6518	0.6704	0.6851	0.6970	0.7069	0.7153	0.7225
9	0.6445	0.6676	0.6892	0.7062	0.7197	0.7305	0.7395	0.7471	0.7536
10	0.6783	0.6996	0.7195	0.7352	0.7475	0.7575	0.7657	0.7726	0.7786
11	0.7063	0.7260	0.7445	0.7590	0.7703	0.7795	0.7871	0.7935	0.7990
12	0.7299	0.7483	0.7654	0.7789	0.7894	0.7980	0.8050	0.8109	0.8160
13	0.7501	0.7672	0.7832	0.7958	0.8056	0.8135	0.8201	0.8256	0.8303
14	0.7674	0.7835	0.7985	0.8103	0.8195	0.8269	0.8330	0.8382	0.8426
15	0.7825	0.7977	0.8118	0.8229	0.8315	0.8385	0.8443	0.8491	0.8532
16	0.7958	0.8101	0.8235	0.8339	0.8421	0.8486	0.8541	0.8586	0.8625
17	0.8076	0.8211	0.8338	0.8436	0.8514	0.8576	0.8627	0.8670	0.8707
18	0.8181	0.8309	0.8429	0.8523	0.8596	0.8655	0.8704	0.8745	0.8780
19	0.8275	0.8397	0.8512	0.8601	0.8670	0.8727	0.8773	0.8811	0.8845
20	0.8360	0.8476	0.8586	0.8671	0.8737	0.8791	0.8835	0.8871	0.8903

Values for Tukey's Test
Upper Percentage Points of the Studentized Range Distribution: Values of $q(0.05, k, \nu)$

Degrees of Freedom ν	Number of Treatments k								
	2	3	4	5	6	7	8	9	10
1	18.00	27.00	32.80	37.20	40.50	43.10	45.10	47.10	49.10
2	6.09	5.33	9.80	10.89	11.73	12.43	13.03	13.54	13.99
3	4.50	5.91	6.83	7.51	8.04	8.47	8.85	9.18	9.46
4	3.93	5.04	5.76	6.29	6.71	7.06	7.35	7.60	7.83
5	3.64	4.60	5.22	5.67	6.09	6.40	6.68	6.93	7.16
6	3.46	4.34	4.90	5.30	5.69	6.02	6.27	6.49	6.68
7	3.34	4.16	4.68	5.06	5.41	5.70	5.92	6.11	6.27
8	3.26	4.04	4.53	4.89	5.22	5.47	5.66	5.82	5.96
9	3.20	3.95	4.41	4.76	5.07	5.29	5.45	5.59	5.71
10	3.15	3.88	4.33	4.65	4.94	5.14	5.28	5.41	5.51
11	3.11	3.82	4.26	4.57	4.84	5.02	5.15	5.26	5.35
12	3.08	3.77	4.20	4.51	4.76	4.92	5.04	5.14	5.22
13	3.06	3.73	4.15	4.45	4.68	4.82	4.92	5.01	5.08
14	3.03	3.70	4.11	4.41	4.62	4.74	4.83	4.91	4.97
15	3.01	3.67	4.08	4.37	4.56	4.66	4.74	4.81	4.87
16	3.00	3.65	4.05	4.33	4.51	4.59	4.66	4.72	4.77
17	2.98	3.63	4.02	4.30	4.47	4.54	4.60	4.65	4.70
18	2.97	3.61	4.00	4.28	4.44	4.50	4.55	4.60	4.64
19	2.96	3.59	3.98	4.25	4.40	4.45	4.50	4.54	4.58
20	2.95	3.58	3.96	4.23	4.37	4.41	4.45	4.49	4.52
24	2.92	3.53	3.90	4.17	4.30	4.34	4.38	4.41	4.44
30	2.89	3.49	3.85	4.10	4.22	4.25	4.28	4.31	4.33
40	2.86	3.44	3.79	4.04	4.15	4.17	4.19	4.21	4.23
60	2.83	3.40	3.74	3.98	4.08	4.10	4.11	4.12	4.13
120	2.80	3.36	3.68	3.92	4.01	4.02	4.03	4.04	4.05
∞	2.77	3.31	3.63	3.86	3.94	3.95	3.96	3.97	3.98

