National Exams December 2019

16-Mec-A3, System Analysis and Control

3 hours duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Candidates may use a Casio **or** Sharp approved calculator. This is a **closed book** exam. No aids other than semi-log graph papers are permitted.

3. Any four (4) questions constitute a complete paper. Only the first four (4) questions as they appear in your answer book will be marked.

4. All questions are of equal value.

Question 1:

Obtain the direct Laplace transform for the following differential and integral equations:

a) $L\ di(t)/dt + Ri(t) + 1/C \int i(t)\ dt = e(t)$

b) $M\ d^2x(t)/dt^2 + B\ dx(t)/dt + K\,x(t) = 3\,t$

c) $J\ d^2\Theta(t)/dt^2 + B\ d\Theta(t)/dt + K\ \Theta(t) = 10\ \sin \omega t$

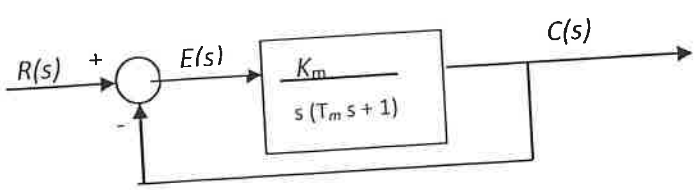
Question 2:

A two-phase ac induction motor is used in a simple positioning system as shown in Fig 1.0. Assume that a difference amplifier, whose gain is 10, is used as the error detector and also supplies power to the control field.

a) What are the undamped natural frequency ω_n and the damping factor ζ?

b) What are the percent overshoot and time to peak resulting from the application of a unit step input?

Figure 1.0



Question 3:

For the system shown in Fig 2.0, determine the following:

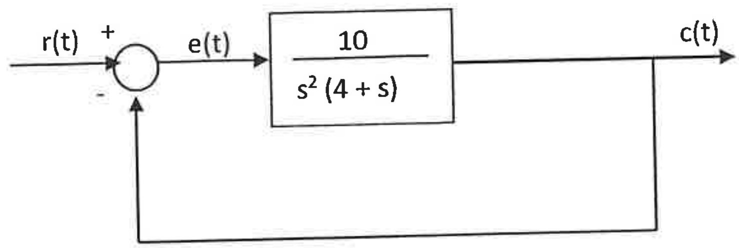
a) Steady-state error resulting from an input

r(t) = 10t

b) Steady-state error resulting from an input

$r(t) = 4 + 6t + 3t^2$

Figure 2.0



Question 4:

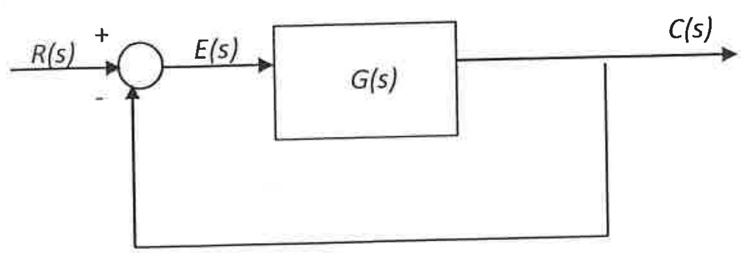
Using the Routh-Hurwitz stability criterion, determine if the feedback control system shown in figure 3.0 is stable for the following transfer functions:

a) $G(s) = 100/\, s\,(s^2 + 8s + 24)$

b) $G(s) = (3s + 1)\,/\,s^2\,(300s^2 + 600s + 50)$

c) $G(s) = 24\,/\,s\,(s + 2)\,(s + 4)$

Figure 3.0



Question 5:

A second-order servomechanism has a forward transfer function given by

$$G(s) = 16 / s(2 + s)$$

The feedback function is unity.

a) Draw the Bode diagram showing the magnitude and the phase characteristics as a function of frequency.
b) Can this system ever be unstable no matter how large the forward gain is made? Explain.

Question 6:

A unity negative feedback control system has a forward transfer function given by

$$G(s) = \frac{K}{s(1 + 0.1s)(1 + s)}$$

a) Sketch the root locus giving all pertinent characteristics of the locus

b) At what value of gain does the system become unstable?

Table of Laplace Transforms

$f(t)$	$\mathcal{L}[f(t)] = F(s)$		$f(t)$	$\mathcal{L}[f(t)] = F(s)$	
1	$\frac{1}{s}$	(1)	$\frac{ae^{at} - be^{bt}}{a-b}$	$\frac{s}{(s-a)(s-b)}$	(19)
$e^{at}f(t)$	$F(s-a)$	(2)	te^{at}	$\frac{1}{(s-a)^2}$	(20)
$\mathcal{U}(t-a)$	$\frac{e^{-as}}{s}$	(3)	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$	(21)
$f(t-a)\mathcal{U}(t-a)$	$e^{-as}F(s)$	(4)	$e^{at}\sin kt$	$\frac{k}{(s-a)^2+k^2}$	(22)
$\delta(t)$	1	(5)	$e^{at}\cos kt$	$\frac{s-a}{(s-a)^2+k^2}$	(23)
$\delta(t-t_0)$	e^{-st_0}	(6)	$e^{at}\sinh kt$	$\frac{k}{(s-a)^2-k^2}$	(24)
$t^n f(t)$	$(-1)^n \frac{d^nF(s)}{ds^n}$	(7)	$e^{at}\cosh kt$	$\frac{s-a}{(s-a)^2-k^2}$	(25)
$f'(t)$	$sF(s) - f(0)$	(8)	$t\sin kt$	$\frac{2ks}{(s^2+k^2)^2}$	(26)
$f^n(t)$	$s^nF(s) - s^{(n-1)}f(0) - \cdots - f^{(n-1)}(0)$	(9)	$t\cos kt$	$\frac{s^2-k^2}{(s^2+k^2)^2}$	(27)
$\int_0^t f(x)g(t-x)dx$	$F(s)G(s)$	(10)	$t\sinh kt$	$\frac{2ks}{(s^2-k^2)^2}$	(28)
$t^n \; (n = 0, 1, 2, \ldots)$	$\frac{n!}{s^{n+1}}$	(11)	$t\cosh kt$	$\frac{s^2-k^2}{(s^2-k^2)^2}$	(29)
$t^x \; (x \geq -1 \in \mathbb{R})$	$\frac{\Gamma(x+1)}{s^{x+1}}$	(12)	$\frac{\sin at}{t}$	$\arctan\frac{a}{s}$	(30)
$\sin kt$	$\frac{k}{s^2+k^2}$	(13)	$\frac{1}{\sqrt{\pi t}}e^{-a^2/4t}$	$\frac{e^{-a\sqrt{s}}}{\sqrt{s}}$	(31)
$\cos kt$	$\frac{s}{s^2+k^2}$	(14)	$\frac{a}{2\sqrt{\pi t^3}}e^{-a^2/4t}$	$e^{-a\sqrt{s}}$	(32)
e^{at}	$\frac{1}{s-a}$	(15)	$\operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right)$	$\frac{e^{-a\sqrt{s}}}{s}$	(33)
$\sinh kt$	$\frac{k}{s^2-k^2}$	(16)			
$\cosh kt$	$\frac{s}{s^2-k^2}$	(17)			
$\frac{e^{at} - e^{bt}}{a-b}$	$\frac{1}{(s-a)(s-b)}$	(18)			

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