National Exams December 2019

17-Ind-A1, Operations Research

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Any non-communicating calculator is permitted. This is an Open Book exam. Note to candidates: You must indicate the type of calculator being used; i.e. write the name and model designation of the calculator, on the first left hand sheet of the exam workbook.

3. All questions are worth 15 marks; therefore, the total value of all 9 questions is 135 marks. Any marks achieved will be considered toward the 100 total requirements.

15 1. Joe is selling Christmas trees to pay for his college tuition. He purchases trees for $10 each and sells them for $25 each. The number of trees he can sell is normally distributed with a mean of 100 and a standard deviation of 30. How many trees should Joe purchase?

15 2. You own a jewellery store with one designated parking spot. All customers arrive by car and use the designated parking spot. For security reasons the store can only accommodate one customer at a time. The salesperson monitors the parking spot, and immediately upon arrival she meets each customer at the car, serves them and returns them to their car at end of the session. She is suggesting that provision be made for one additional spot at $100/day where a customer can wait in the car in case there is already another customer in the store. Your records show that on average the salesperson serves 10 customers per 7-hr day, and on average spends ½ hour with each customer. 15% of customer visits result in sales, and the average profit per sale is $1000. Based on the assessment that 10% of customers who find the parking spot full never return, should you proceed to arrange for a second parking spot?

15 3. Suppose that a new car costs $10,000 and that the annual operating cost and resale value of the car are as shown below. If you have a new car now, determine a replacement policy that minimizes the net cost of owning and operating a car for the next six years.

Age of Car (in years)	Resale Value	Operating cost	
1	7000	300	**In year 1**
2	6000	500	In year 2
3	4000	800	In year 3
4	3000	1200	In year 4
5	2000	1600	In year 5
6	1000	2200	In year 6

15 4. We are thinking of filming the Don Harnett story. We know that if the film is a flop, we will lose $4 million, and if the film is a success, we will earn $15 million. Beforehand we believe that there is a 10% chance that the Don Harnett story will be a hit. Before filming we have the option of paying the noted movie critic Roger Alert $1 million for his view of the film. In the past Alert has predicted 60% of all actual hits to be hits and 90% of all actual flops to be flops. Use a decision tree to determine the best strategy and maximum expected profit.

15 5. A company assembles two products: A and B. Product A sells for $11 per unit, and product B sells for $23 per unit. A unit of product A requires 2 hours on assembly line 1 and 1 unit of raw material. A unit of Product B requires 2 units of raw material, 1 unit of A and 2 hours on assembly line 2. For line 1, 1300 hours of time are available, and 500 hours are available on line 2. A unit of raw material may be bought for $5, or produced at no cost by using 2 hours on line 1. Formulate a linear programming model to maximize the profit. No solution is required.

6. Consider the following problem

Maximize $z = 21x_1 + 9x_2 + 4x_3$ (profit)

Subject to

$$\begin{aligned} 2x_1 + x_2 + x_3 &\leq 31 && \text{(resource constraint 1)} \\ 3x_1 + 2x_2 + x_3 &\leq 60 && \text{(resource constraint 2)} \\ x_1 + 2x_2 + x_3 &\geq 50 && \text{(requirement constraint)} \\ x_1 &\geq 0 \\ x_2 &\geq 0 \end{aligned}$$

The simplex method yields the following final set of equations

$$\begin{aligned} z + (1/2)x_3 + (2/3)x_4 + x_6 &= 291 \\ x_1 + (1/3)x_3 + (2/3)x_4 + (1/3)x_6 &= 4 \\ x_2 + (1/3)x_3 - (1/3)x_4 - (2/3)x_6 &= 23 \\ x_5 - (2/3)x_3 - (4/3)x_4 + (1/3)x6 &= 2 \end{aligned}$$

where x_4 is the slack variable for resource constraint 1, x_5 is the slack variable for resource constraint 2, and x_6 is the slack variable for the requirement constraint.

a. What is the optimal solution, the maximum profit, the marginal values of resources 1 and 2, and the marginal cost of the requirement? 5

b. How much can the coefficient of x_2 in the objective function vary without affecting the optimal solution? 5

c. By how much would the profit be increased if 5 more units of resource 1 were available? What would be the new solution? 5

7. The best-selling college statistics text, *The Thrill of Statistics*, sells 5 million copies every fall. Some users keep the book, and some sell it back to the bookstore. Suppose that 90% of all students who buy a new book sell it back, 80% of all students who buy a once-used book sell it back, and 60% of all students who buy a twice-used book sell it back. If a book has been used four or more times the cover falls off, and it cannot be sold back. In the steady state how many new copies of the book will the publisher be able to sell each year? 15

8. Before a new product can be introduced the following activities must be completed (all times are in weeks):

Activity	Predecessor	Duration	Speed-up cost in $/wk
A	-	6	80
B	-	5	60
C	A	3	30
D	C	2	60
E	A,D	3	40
F	B	2	30
G	E	4	20
H	G, F	2	-

a. Draw the project diagram, including all critical paths and critical activities, and the total float and free float for each activity — 3
b. Set up an LP that can be used to determine the critical path — 4
c. Formulate a minimum cost network flow problem that can be used to find the critical path — 4
d. It is now 12 weeks before Christmas. The duration of each activity can be reduced by up to two weeks by incurring the Speed-up costs shown in the table. Formulate a Linear Programming model that will minimize the cost of getting the product into the stores before Christmas. — 4

9. You have been assigned to arrange the songs on a cassette version of one of Madonna's albums. The songs on each side of the cassette must total between 14 and 16 minutes in length. The length and type of each song are as follows — 15

Song	Type	Duration
1	Ballad	4
2	Hit	5
3	Ballad	3
4	Hit	2
5	Ballad	4
6	Hit	3
7		5
8	Ballad and Hit	4

You need to satisfy the following conditions: each side must have exactly two ballads, side 1 must have at least 3 hit songs, either song 5 or song 6 must be on side 1, and if song 2 and 4 are on side 1, then song 5 must be on side 2

Formulate an integer programming model which would determine if there is an arrangement of songs that can satisfy all these conditions.